Phys 221 - E\&M I - Test 3 - April 26, 2002

1. ( 25 pts ) Three charges lie in the $y z$ plane as shown. There is a charge $-q$ at $y=-a$, a charge $2 q$ at $y=a$, and a charge $2 q$ at $z=a$.
a) Calculate the monopole and dipole moments for this distribution.

b) Find the approximate potential at points far from the distribution. Give your results in spherical coordinates.
2. ( 25 pts ) A dipole $\vec{p}$ is situated a distance $z$ above an infinite grounded conducting plane. The dipole makes an angle $\theta$ with the perpendicular to the plane.
a) Draw the image dipole.

b) Find the torque on the dipole due to the image dipole.
3. (25 pts) A spherical conductor, of radius $a$, carries a charge $Q$. It is surrounded by linear dielectric material with a dielectric constant $\varepsilon_{\mathrm{r}}$, out to a radius $b$.
a) Determine the electric field in all three regions, $r<a ; \quad a<r<b ; \quad r>b$.

b) Determine the polarization in all three regions.
c) Determine the bound volume and surface charge densities. Note that the dielectric material has both an inner and outer surface.
4. (25 pts) A parallel plate capacitor is filled with three dielectrics with dielectric constants $\varepsilon_{r 1}, \varepsilon_{r 2}$, and $\varepsilon_{r 3}$ as shown. Half the capacitor is filled with $\varepsilon_{r 3}$. The other half is divided equally between $\boldsymbol{\varepsilon}_{r 1}$ and $\boldsymbol{\varepsilon}_{r 2}$. The total area of a plate is $A$ and $d$ is the distance between the
 plates.
Determine the total capacitance of the arrangement in terms of the original capacitance $C_{0}$ with no dielectric material present $\left(C_{0}=\varepsilon_{0} A / d\right)$ if $\boldsymbol{\varepsilon}_{r 1}=2, \varepsilon_{r 2}=3$, and $\varepsilon_{r 3}=4$.

Electric field due to a dipole $\vec{E}=\frac{k_{e}}{r^{3}}[3(\vec{p} \cdot \hat{r}) \hat{r}-\vec{p}]$

Bound charge $\rho_{b}=-\vec{\nabla} \cdot \vec{P} \quad \sigma_{b}=\vec{P} \cdot \hat{n}$

$$
\vec{D}=\varepsilon_{0} \vec{E}+\vec{P}=\varepsilon_{0} \vec{E}+\varepsilon_{0} \chi_{e} \vec{E}=\varepsilon \vec{E}=\varepsilon_{r} \varepsilon_{0} \vec{E}
$$

