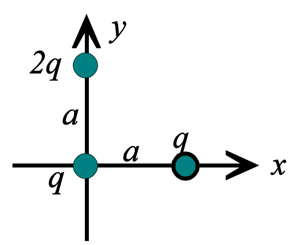
Phys 221 – Test 3 – April 16, 2004

1.(25 pts) A surface charge density $\sigma(\phi)$ is glued over the surface of an infinite cylinder of radius *R*. The surface charge density is given by $\sigma(\phi) = \sigma_0 \sin(3\phi)$ where σ_0 is a constant.

a) Determine the total charge on the cylinder.

b) Find the electric potential $V(s,\phi)$ inside and outside the cylinder.

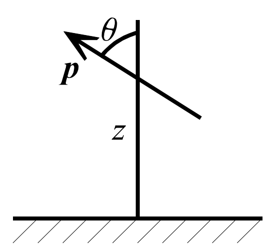
2. (25 pts) Three charges lie in the *xy* plane as shown. There is a charge q at $y = 0$, a charge $2q$ at $y = a$, and a charge q at $x = a$.



a) Calculate the monopole and dipole moments for this distribution.

b) Find the approximate potential at points far from the distribution. Give your results in spherical coordinates.

3. (25 pts) A dipole $\vec{p}$ is situated a distance *z* above an infinite grounded conducting plane. The dipole makes an angle θ with the perpendicular to the plane.



a) Draw the image dipole.

b) Find the torque on the dipole due to the image dipole.

4. (25 pts) A certain coaxial cable consists of a copper wire, radius *a*, surrounded by a concentric copper tube of inner radius *c*. The space from *a* to *b* is filled with material of dielectric constant ε_{r1} and the space from *b* to *c* is filled with material of dielectric constant ε_{r2}. The radii are such that $b = 2a$ and $c = 4a$. In addition $\varepsilon_{r1} = 2$ and $\varepsilon_{r2} = 1.5$. Assume the inner cylindrical conductor of radius *a* has charge per unit length λ.

a) Determine the displacement $\vec{D}$ in each region.

b) Determine the electric field $\vec{E}$ and the polarization $\vec{P}$ in each region.

c) Find the capacitance per unit length of the coaxial cable.

Solution to Laplace's equation, $\nabla^2 V = 0$

Cylindrical Coordinates (no z dependence):

$$V(s,\phi) = A_0 + B_0 \ln s + \sum_{m=1}^{\infty} s^m (A_m \cos m\phi + B_m \sin m\phi) + \sum_{m=1}^{\infty} \frac{1}{s^m} (C_m \cos m\phi + D_m \sin m\phi)$$

If $f(\phi) = \sum_{m=1}^{\infty} \{A_m \cos(m\phi) + B_m \sin(m\phi)\}$, then

$$A_m = \tfrac{1}{\pi}\int_0^{2\pi} f(\phi)\cos(m\phi)d\phi \quad \text{and} \quad B_m = \tfrac{1}{\pi}\int_0^{2\pi} f(\phi)\sin(m\phi)d\phi \ .$$

Electric field due to a dipole $\vec{E} = \frac{k_e}{r^3}[3(\vec{p} \bullet \hat{r})\hat{r} - \vec{p}]$

Bound charge $\rho_b = -\vec{\nabla} \bullet \vec{P}$ $\qquad \sigma_b = \vec{P} \bullet \hat{n}$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 \vec{E} + \varepsilon_0 \chi_e \vec{E} = \varepsilon \vec{E} = \varepsilon_r \varepsilon_0 \vec{E}$$