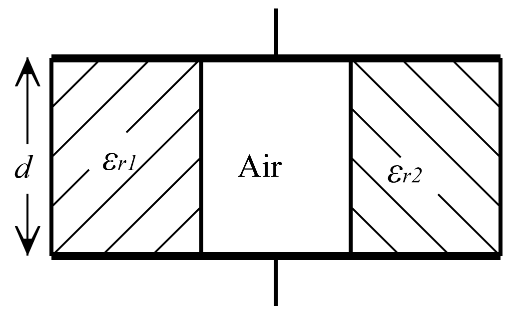
Phys 221 – E&M I – Test 3 – April 30, 2003

1. (25 pts) A thick spherical shell (inner radius *a*, outer radius *b*) is made of dielectric material with a "frozen-in" polarization $\vec{P}(r) = \frac{C}{r^4}\hat{r}$, where *C* is a constant and *r* is the distance from the center. There is no free charge in the problem.

a) Calculate all the bound charges and then use Gauss's Law for $\vec{E}$ to calculate the field in all three regions.

b) Determine the potential at the center of the thick spherical shell, *i.e.*, at $r = 0$.

2. (25 pts) A parallel plate capacitor is filled with two dielectrics with dielectric constants $\varepsilon_{r1} = 1.5$ and $\varepsilon_{r2} = 2$ as shown. A third of the capacitor is filled with ε_{r1} , a third of the capacitor is filled with air, and a third is filled with ε_{r2} .



a) Determine the capacitance of the arrangement in terms of the original capacitance C_0 with no dielectric material present $(C_0 = \varepsilon_0 A/d)$. The area of a plate is *A* and *d* is the distance between the plates.

b) For a given potential difference *V* between the plates, find the free and bound surface charge densities on all surfaces.

3. (25 pts) A steady current *I* flows down a long hollow cylindrical wire (inner radius *a*, outer radius *b*). The volume current density $\vec{J} = Cs^2\hat{z}$, where *C* is a constant and $\hat{z}$ is in the direction of the current *I*.

a) Determine the constant *C*.

b) Determine the magnetic field for all *s*, *i.e.*, $s < a$; $a < s < b$; $s > b$.

4. (25 pts) A spherical conductor, of radius *a*, carries a charge *Q*. It is surrounded by linear dielectric material with a dielectric constant ε_r, out to a radius *b*.

a) Determine the electric field and the displacement in all three regions, $r < a$; $a < r < b$; $r > b$.

b) Determine the energy of this configuration.