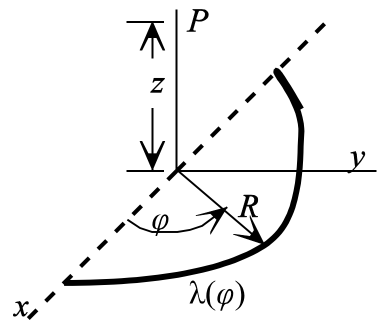
Phys. 221 – E & M I – Test 1 – Feb. 21, 2003

1. (25 pts) A half of a circular loop of radius R lies in the xy plane and carries a line charge described by

$\lambda(\phi) = \lambda_0 \sin\phi; \quad 0 \le \phi \le \pi$, where λ_0 is a constant and ϕ is measured from the x axis. Take the origin to be where the center of a complete loop would be.

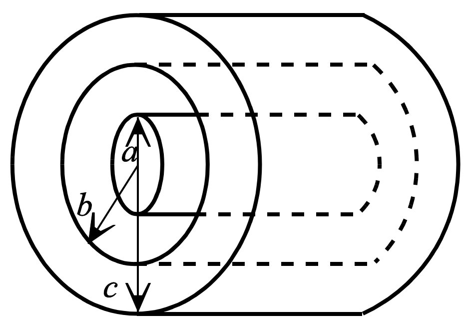


Find the electric field a distance z above the origin.

You may recall:

$$\sin^2 x = \tfrac{1}{2}(1 - \cos 2x) \quad \cos^2 x = \tfrac{1}{2}(1 + \cos 2x)$$

2. (25 pts) A long coaxial cable consists of a conducting inner cylinder of radius a and a thick outer conducting cylinder of inner radius b and outer radius c (Note: $a < b < c$). The surface charge density on the inner cylinder is σ .



a) Find the surface charge densities σ_b, σ_c.

b) Find the electric field in each of the four regions:
   (i) inside the inner cylinder, $s < a$.
   (ii) between the cylinders, $a < s < b$.
   (iii) inside the thick outer cylinder, $b < s < c$.
   (iv) outside the cable, $s > c$.

b) If the outer cylinder is now grounded, find the capacitance per unit length of the arrangement.

3. (25 pts) A sphere of radius R carries a charge density $\rho(r) = Ar^2$, where A is a constant.

a) Determine the electric field inside and outside the sphere.

b) Find the electrostatic energy stored in the sphere.

4. (25 pts) a) Compute the divergence of the function

$$\vec{v} = \left(r\cos\phi\right)\hat{r} + (r\sin\theta)\hat{\theta} + r\hat{\phi}\ .$$

b) Check the divergence theorem for this function, using as your volume half of an inverted hemispherical bowl of radius R, resting on the xy plane and centered at the origin.