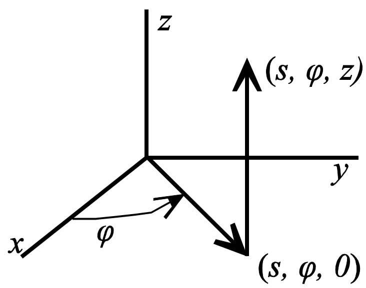
Phys. 221 – E & M- I – Test 1 – Feb. 20, 2002

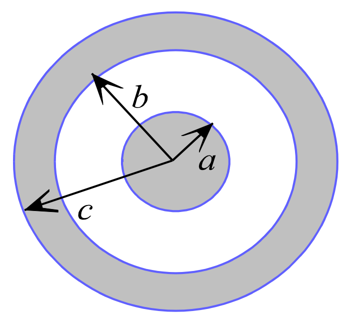
1. (25 pts) An electrostatic field is given by
$\vec{E} = C\left[s(2+\sin^2\phi)\hat{s} + s(\sin\phi\cos\phi)\hat{\phi} + 3z\hat{z}\right]$, where C is a constant with the appropriate units.



a) Verify that this is a possible electrostatic field.

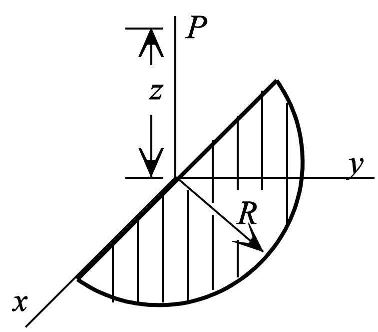
b) Find the potential, using the *origin* as your reference point. Use the indicated path from the origin to the point; that is, go from *0* to *s* along the path with *φ* fixed and then from *0* to *z* up to the point.

2. (25 pts) A spherical system consists of an inner spherical ball of radius *a* surrounded by a thick spherical shell of inner radius *b* and outer radius *c* (Note: $a < b < c$). The volume charge densities on the inner sphere and the outer shell are given by $\rho(r) = Ar^2;\quad 0 < r < a$ and $\rho(r) = -B/r^2;\quad b < r < c$, where *A* and *B* are both positive constants.



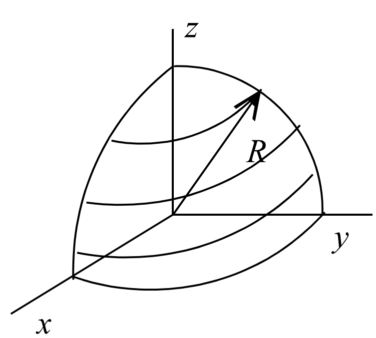
Find the electric field in each of the four regions:
(*i*) inside the inner sphere, $r < a$.
(*ii)* between the inner sphere and the outer shell, $a < r < b$.
(*iii*) inside the thick outer shell, $b < r < c$.
(*iv)* outside the spherical system, $r > c$.

3. (25 pts) Find the electric potential at a distance *z* above a charge distribution which consists of a half disk of radius R lying in the *xy* plane. The half disk has a surface charge distribution given by $\sigma = \sigma_0 \sin\phi$, where σ_0 is a constant.



4. (25 pts) a) Compute the divergence of the function

$$\vec{v} = (r\cos^2\theta)\hat{r} - (r\cos\theta\sin\theta)\hat{\theta} + 3r\hat{\phi}$$



b) Check the divergence theorem for this function, using as your volume the octant of a sphere of radius

R.