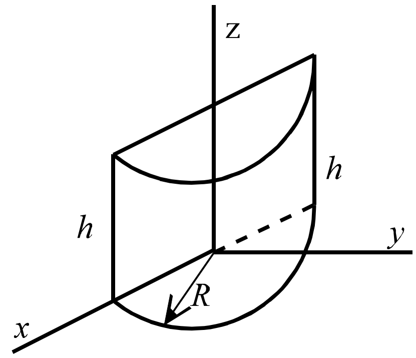
Phys. 221 – E & M- I – Test 1 – Feb. 19, 2001

1. (25 pts) a) Compute the divergence of the function

$$\vec{v} = \left(s^2 \sin\phi\right)\hat{s} + \left(s^2 \cos\phi\right)\hat{\phi} + \left(2z^2\right)\hat{z} \quad .$$



b) Check the divergence theorem for this function, using as your volume the half-cylinder of radius R and height h whose base is on the xy plane and centered about the y-axis.

2. (25 pts) An electrostatic field is given by $\vec{E} = k[2xy\,\hat{x} + (2yz + x^2)\,\hat{y} + y^2\,\hat{z}]$, where k is a constant with the appropriate units.

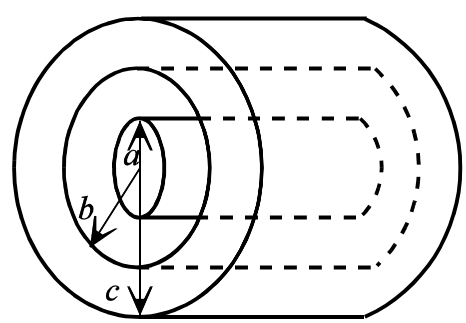
a) Verify that this is a possible electrostatic field.

b) Find the potential, using the *origin* as your reference point.

3. (25 pts) A long coaxial cable consists of an inner cylinder of radius a and a thick outer cylinder of inner radius b and outer radius c (Note: $a < b < c$). The volume charge densities on the inner and outer cylinders are given by $\rho(s) = As^2; \quad 0 < s < a$ and $\rho(s) = -B/s^2; \quad b < s < c$, where A and B are both positive constants. Assume the cable as a whole is electrically neutral.

Find the electric field in each of the four regions:

(*i*) inside the inner cylinder, $s < a$.
(*ii*) between the cylinders, $a < s < b$.
(*iii*) inside the thick outer cylinder, $b < s < c$.
(*iv*) outside the cable, $s > c$.



4. (25 pts) A uniformly charged solid sphere has a radius R and a total charge q.

a) Find the electric field inside and outside the sphere, *i.e.*, as a function of r.

b) Find the electric potential inside and outside the sphere, *i.e.*, as a function of r. Use infinity as your reference point.