Phys. 221 - E \& M I - Test 1 - Feb.12, 2007

1. (25 pts) Given the vector function:

$$
\vec{v}=\left(r \cos ^{2} \theta\right) \hat{r}-(r \cos \theta \sin \theta) \hat{\theta}+(3 r \cos \phi) \hat{\phi}
$$

Calculate the integral $\oint \vec{v} \bullet d \vec{a}$ using as your enclosed volume the octant of a sphere of radius $R$.

2.(25 pts) Find the electric potential at a distance $z$ above a charge distribution which consists of a half disk of radius $R$ lying in the $x y$ plane. The half disk has a surface charge density given by $\sigma(s)=A s^{2}$, where $A$ is a constant and $s$ is the distance from the $z$ axis.

3.(25 pts) A spherical system consists of an inner spherical ball of charge of radius $R$ surrounded by a thick spherical shell of charge of inner radius $a$ and outer radius $b$, so that $R<a<b$. The volume charge densities on the inner spherical ball and outer spherical shell are given by:

$$
\begin{array}{ll}
\rho(r)=A \quad 0<r<R \\
\rho(r)=-B / r^{2} \quad a<r<b
\end{array}
$$

where $A$ and $B$ are both positive constants chosen in such a way as to make the total charge on the whole system equal to zero.


Find the electric field in each of the four regions; $r<R ; \quad R<r<a ; \quad a<r<b ; \quad r>b$.
4. (25 pts) A sphere of radius $R$ carries a charge density $\rho(r)=A r^{2}$, where A is a constant.
a) Determine the electric field as a function of $r$ inside and outside the sphere.
b) Determine the electric potential as a function of $r$ inside and outside the sphere.
c) Determine the energy stored in the charge distribution, i.e., the work done to assemble the spherical charge distribution.

