Phys. 221 – E&M I – Final – May 11, 2007

1. (17 pts) A sphere of radius *R* carries a charge density $\rho(r) = Ar^2$, where A is a constant.

a) Determine the electric field as a function of r inside and outside the sphere.

b) Determine the electric potential as a function of r inside and outside the sphere.

c) Determine the energy stored in the charge distribution, *i.e.*, the work done to assemble the spherical charge distribution.

2. (17 pts) A neutral (or grounded) metal sphere of radius *R* is placed in an otherwise uniform electric field given by $\vec{E} = E_0\hat{z}$.

a) Determine the electric potential inside and outside the metal sphere.

b) Determine the induced surface charge density on the metal sphere.

c) How would the electric potential inside and outside the metal sphere change if the sphere initially had a charge of *Q* on it?

d) What would the surface charge density be if the sphere initially had a charge of *Q* on it?

3. (17 pts) Consider a metal sphere, radius *a*, surrounded by a concentric metal spherical shell of radius *c*. The space between *a* and *b* is filled with material of dielectric constant ε_r and the space between *b* and *c* is air ($a < b < c$). Assume there is a charge *q* at *r* = *a* and $-q$ at *r* = *c*.

a) Determine the displacement $\vec{D}$ and the electric field $\vec{E}$ in all four regions, *i.e.*, $r < a$, $a < r < b$, $b < r < c$, and $r > c$.

b) Find the capacitance of this spherical capacitor.

4.(17 pts) A current *I* flows down a wire of radius *a.* The volume current density *J* is given as $J = Cs^2$.

a) Determine the constant *C*.

b) Determine the magnetic field inside and outside the wire.

5. (17 pts) a) Find the vector potential a distance s from an infinite straight wire carrying a current I.

b) Check that your vector potential satisfies the Coulomb gauge, *i.e.*, that $\vec{\nabla} \cdot \vec{A} = 0$.

6.(17 pts) A spherical shell of radius R, carrying a uniform surface charge σ, is rotating at a constant angular velocity ω, which is pointing in the z direction.

a) Determine its magnetic moment.

b) What is the magnetic field $\vec{B}$ in the xy-plane at a distance r from the center of the sphere if $r \gg R$?