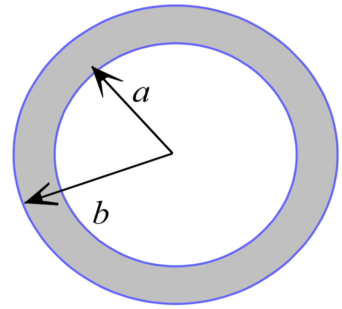
Phys. 221 – E&M I – Final – May 11, 2005

1. (17 pts) A hollow spherical shell carries a charge density

$$\rho(r) = \frac{A}{r} \quad \text{for } a \le r \le b \text{, where } A \text{ is a constant.}$$



a) Find the electric field in each of the three regions:
   (*i*) $r < a$, (*ii*) $a < r < b$, (*iii*) $r > b$.

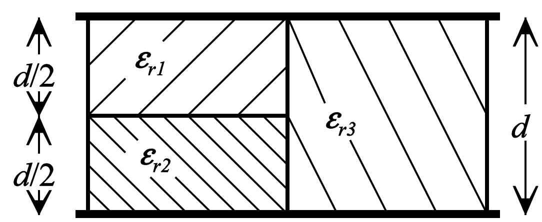
b) Find the energy stored in the distribution, *i.e.,* the work done to assemble the charge distribution.

2. (17 pts) An uncharged (grounded) metal sphere of radius R is placed in a uniform electric field given by $\vec{E} = E_0\hat{z}$.

a) Determine the potential inside and outside the metal sphere.

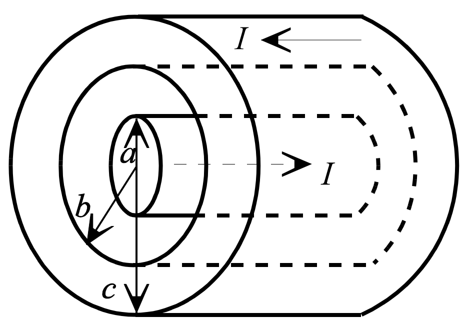
b) Determine the induced surface charge density $\sigma(\theta)$ on the metal sphere.

3. (17 pts) A parallel plate capacitor is filled with three dielectrics with dielectric constants ε_{r1}, ε_{r2}, and ε_{r3} as shown. Half the capacitor is filled with ε_{r3}. The other half is divided equally between ε_{r1} and ε_{r2}. The total area of a plate is A and d is the distance between the plates.



Determine the total capacitance of the arrangement in terms of the original capacitance C_0 with no dielectric material present $\left(C_0 = \varepsilon_0 A/d\right)$ if $\varepsilon_{r1} = 3$, $\varepsilon_{r2} = 6$, and $\varepsilon_{r3} = 4$.

4. (17 pts) Consider a long coaxial cable. The center conductor $s \le a$ carries a current I to the right and the outer conductor $b \le s \le c$ carries the return current I to the left. The currents are uniformly distributed in the conductors.



Use Ampere's Law to find the magnetic field $\vec{B}$ for all s.

5. (17 pts) A long circular cylinder of radius R carries a magnetization $\vec{M} = as^2\hat{\phi}$, where a is a constant, s is the distance from the axis, and $\hat{\phi}$ is the usual azimuthal unit vector.

a) Determine the bound currents, $\vec{J}_b$ and $\vec{K}_b$.

b) Use Ampere's Law with the magnetic field $\vec{B}$ to determine the field inside and outside the cylinder.

c) Use Ampere's Law with the auxiliary field $\vec{H}$ to determine the magnetic field inside and outside the cylinder.

6. (17 pts) A phonograph record of radius R, carrying a uniform surface charge σ, is rotating at constant angular velocity ω about its axis which is pointing in the z direction.

a) Determine the surface current density K at a distance s from the center.

b) Find its magnetic dipole moment.

c) What is the magnetic field $\vec{B}$ in the xy plane at a distance s from the origin at the center of the phonograph record if $s >> R$?

Electric field due to an electric dipole $\vec{E} = \frac{k_e}{r^3}[3(\vec{p}\bullet\hat{r})\hat{r} - \vec{p}]$

Bound charge $\rho_b = -\vec{\nabla}\bullet\vec{P}$ $\sigma_b = \vec{P}\bullet\hat{n}$

Magnetic field due to a magnetic dipole $\vec{B} = \frac{k_m}{r^3}\left[3(\vec{m}\bullet\hat{r})\hat{r} - \vec{m}\right]$

Bound currents $\vec{J}_b = \nabla\times\vec{M}$ $\vec{K}_b = \vec{M}\times\hat{n}$

Biot-Savart Law $\vec{B}(\vec{r}) = k_m\int\frac{\vec{I}\times(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}dl'$