Phys 221 – Final Exam – May 11, 2004

1. (17 pts) A long cylinder of radius R carries a volume charge density that is proportional to the distance from the axis squared. The volume charge density is given by

$\rho(s) = As^2$; $0 < s < R$, where A is a constant.

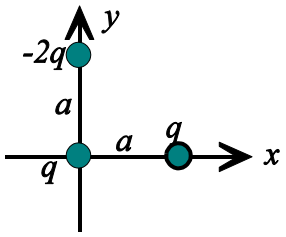
a) Find the electric field as a function of s inside and outside the long cylinder.

b) Find the electric potential as a function of s inside and outside the cylinder. Use $s = R$ as the reference point for the electric potential.

c) If the charge per unit length on the long cylinder is λ, determine the constant A.

2. (17 pts) A metal sphere of radius R carries a total charge Q. What is the force of repulsion between the "northern" hemisphere and the "southern" hemisphere?

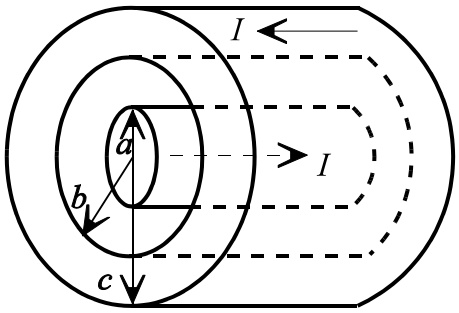
3. (17 pts) Three charges lie in the xy plane as shown. There is a charge q at $y = 0$, a charge $-2q$ at $y = a$, and a charge q at $x = a$.



a) Calculate the monopole and dipole moments for this distribution.

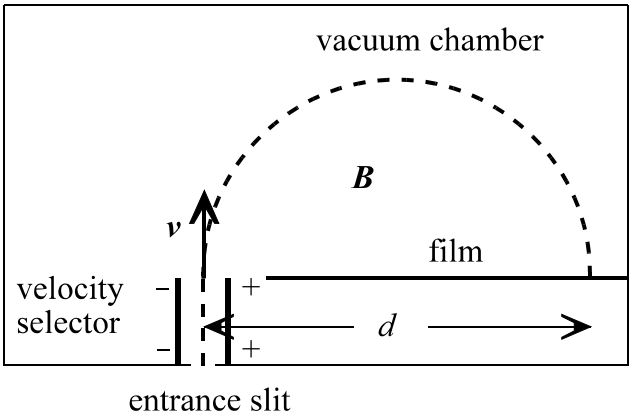
b) Find the approximate potential at points far from the distribution. Give your results in spherical coordinates.

4. (17 pts) Consider a long coaxial cable. The center conductor $s \le a$ carries a current I to the right and the outer conductor $b \le s \le c$ carries the return current I to the left. The currents are uniformly distributed in the conductors.



Use Ampere's Law to find the magnetic field $\vec{B}$ for all s.

5. (17 pts) The figure shows a schematic version of a mass spectrograph of K. T. Bainbridge. It has a velocity selector charged as shown in order to produce an electric field perpendicular to the magnetic field. The positive ions pass through the velocity selector and move in the magnetic field until they hit the photographic film and make a spot on the film a distance *d* away from the center of the velocity selector.



a) Give the direction of the electric field, right or left, in the figure.

b) The magnetic field covers the whole region of the vacuum chamber. If the ions are positively charged, does the magnetic field point into the paper or out of the paper?

c) If the ions have a positive charge *q*, determine an expression for the mass of the ions that impinge on the photographic plate a distance *d* from the slit.

6. (17 pts) A charged annular disk with inner radius *a* and outer radius *b*, and surface charge density σ, lies in the *xy* plane with its center at the origin. It rotates about the *z* axis with angular velocity ω.

a) Use the Biot-Savart Law to find $B_z(0,0,z)$ for all values of *z*.

b) What is the magnetic moment $\vec{m}$?

c) What is $\vec{B}$ in the *xy* plane at distance *s* from the origin, where $s >> a,b$?

Recall the Biot-Savart Law is given by $\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{I}\,dl \times (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3}$

and the magnetic moment is given by $\vec{m} = \frac{1}{2}\oint \vec{r} \times \vec{I}\,dl$

Note: $\vec{I}\,dl$; $\vec{K}da$; $\vec{J}d\tau$ Are equivalent forms.