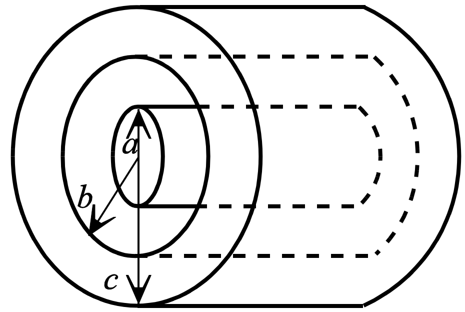
Phys 221 – E & M I – May 12, 2003

1. (25 pts) A long coaxial cable consists of a conducting inner cylinder of radius *a* and a thick outer conducting cylinder of inner radius *b* and outer radius *c* (Note: $a < b < c$). The surface charge density on the inner cylinder is σ .



a) Find the surface charge densities σ_b, σ_c.

b) Find the electric field in each of the four regions:
(*i*) inside the inner cylinder, $s < a$.
(*ii)* between the cylinders, $a < s < b$.
(*iii*) inside the thick outer cylinder, $b < s < c$.
(*iv)* outside the cable, $s > c$.

b) If the outer cylinder is now grounded, find the capacitance per unit length of the arrangement.

2. (25 pts) A surface charge density $\sigma(\phi) = \sigma_0 \cos(3\phi)$ is glued over the surface of an infinite cylinder of radius *R* (σ_0 is a constant).

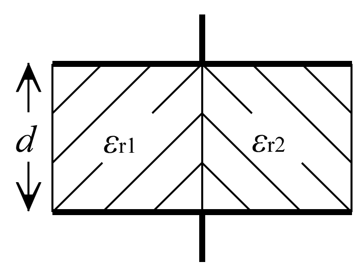
Find the electric potential $V(s,\phi)$ inside and outside the cylinder.

3. (25 pts) A steady current *I* flows down a long hollow cylindrical wire (inner radius *a*, outer radius *b*). The volume current density $\vec{J} = \frac{C}{s^3}\hat{z}$, where *C* is a constant and $\hat{z}$ is in the direction of the current *I.*

a) Determine the constant *C*.

b) Determine the magnetic field for all *s*, *i.e.,* $s < a$; $a < s < b$; $s > b$.

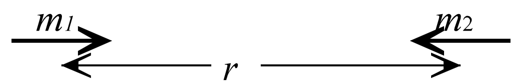
4. (25 pts) A parallel plate capacitor is filled half and half with two dielectrics with dielectric constants $\varepsilon_{r1} = 3$ and $\varepsilon_{r2} = 2$ as shown.



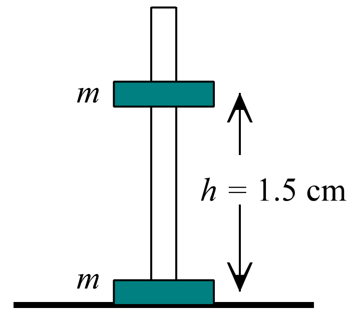
a) Determine the capacitance of the arrangement in terms of the original capacitance C_0 with no dielectric material present $\left(C_0 = \varepsilon_0 A/d\right)$. The area of a plate is *A* and *d* is the distance between the plates.

b) For a given potential difference *V* between the plates, find the free and bound surface charge densities on all surfaces.

5. (25 pts) a) Find the force of repulsion between two magnetic dipoles $\vec{m}_1$ and $\vec{m}_2$ oriented as shown a distance r apart.



b) Assume the two dipoles have the same dipole moment m and mass M = 30 gms. They are donut shaped so they can slide frictionlessly on a vertical rod. If the upper one "floats" at a distance of 1.5 cm above the lower one, determine the dipole moment m of the magnets.



The magnetic field of a pure dipole is given by:

$$\vec{B} = \frac{k_m}{r^3}\left[3(\vec{m} \bullet \hat{r})\hat{r} - \vec{m}\right]$$

6. (25 pts) A current flows down a long straight wire of radius R. The volume current density is given by: $\vec{J} = Cs\hat{z}$. The wire is made of linear material (copper, say, or aluminum) with magnetic susceptibility χ_m.

a) Determine the magnetic field a distance s from the axis.

b) Determine the bound currents.

c) Calculate the *net* bound current flowing down the wire.