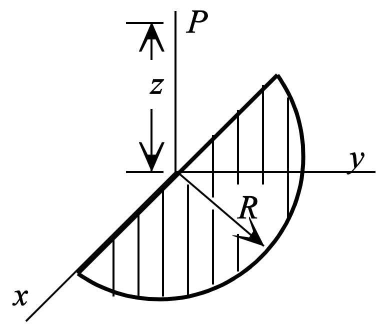
Phys. 221 – E&M I – Final Exam – May 16, 2002

1. (25 pts) A sphere of radius R carries a charge density $\rho(r) = Ar^3$, where A is a constant.

a) Determine the electric field inside and outside the sphere.

b) Find the electrostatic energy stored in the sphere.

2. (25 pts) Find the electric potential at a distance z above a charge distribution which consists of a half disk of radius R lying in the xy plane. The half disk has a surface charge distribution given by $\sigma = \sigma_0 \sin(3\phi)$, where σ_0 is a constant.



3. (25 pts) A thick spherical shell (inner radius a, outer radius b) is made of dielectric material with a "frozen-in" polarization $\vec{P}(\vec{r}) = Ar^3\hat{r}$, where A is a constant and r is the distance from the center. There is no free charge in the problem.

a) Calculate all the bound charges and then use Gauss's Law for $\vec{E}$ to calculate the field in all three regions.

b) Determine the potential at the center of the thick spherical shell, *i.e.*, at $r = 0$.

4. (25 pts) A neutral (or grounded) conducting sphere of radius R is placed in an otherwise uniform electric field $\vec{E}_0 = E_0\hat{z}$.

a) Find the electric potential inside and outside the conducting sphere.

b) Determine the induced dipole moment on the conducting sphere.

Recall the general solution to Laplace's equation in spherical coordinates is

$$V(r) = \sum_{\ell=0}^{\infty}\left(A_\ell r^\ell + \frac{B_\ell}{r^{\ell+1}}\right)P_\ell(\cos\theta) \text{ ,where } P_0 = 1, \quad P_1 = \cos\theta,$$

$$P_2 = \tfrac{1}{2}(3\cos^2\theta - 1), \quad P_3 = \tfrac{1}{2}(5\cos^3\theta - 3\cos\theta)$$

5. (25 pts) A steady current I flows down a long cylindrical wire of radius R. The current is distributed in such a way that J is proportional to s, where s is the distance from the axis.

a) Determine the constant of proportionality.

b) Find the magnetic field, both inside and outside the wire.

6. (25 pts) A phonograph record of radius R, carrying a uniform surface charge σ, is rotating at constant angular velocity ω. Find its magnetic dipole moment.