Phys. 208 – Theoretical Physics – Final (May 11, 2009)

1.(17 pts) a) Find and plot the roots of $\sqrt[3]{-8i}$.

b) Evaluate $(-i)^i$ in Cartesian form, *i.e.*, $x+iy$ form.

c) Determine the points in the (*x*, *y*) plane that satisfy the equation $|z-2+3i|=4$.

2.(17 pts) Derive the general expression for the integral $I_n = \int_0^\infty x^{2n} e^{-ax^2} dx$, given that

$I_0 = \frac{1}{2}\sqrt{\frac{\pi}{a}}$ as follows:

a) By taking a derivative of I_n , determine the recursion relation for I_n ,*i.e.*, $I_{n+1} = [\cdots]\frac{dI_n}{da}$.

b) Use your recursion relation to find the general form for the integral I_n .

3.(17 pts) a) Find the Fourier <u>transform</u> of the function, $f(x) = \begin{cases} 1 & -2 < x < 0 \\ -1 & 0 < x < 2 \end{cases}$

b) Write $f(x)$ as an integral and use your result to evaluate $\int_0^\infty \frac{(\cos 2\alpha - 1)\sin 2\alpha}{\alpha} d\alpha$.

c) Use Parseval's theorem to evaluate the integral $\int_0^\infty \frac{[\cos 2\alpha - 1]^2}{\alpha^2} d\alpha$.

4.(17 pts) The normalized wave function for the one-dimensional harmonic oscillator is given

by $\psi(x) = Ae^{-ax^2/2}$, where $a = m\omega/\hbar$ and *x* is defined as $-\infty < x < \infty$.

a) Determine *A*.

b) Determine $\langle x \rangle$ and $\langle x^2 \rangle$.

c) Determine the uncertainty in *x*, *i.e.*, Δx or σ_x .

5.(17 pts) Solve the following differential equation using the power series method.

$$y'' + xy = 0$$

a) Determine the recursion relation for the coefficients a_n if $y(x) = \sum_{n=0}^{\infty} a_n x^n$.

Hint: Shift all powers of x to the highest exponent and take out the terms, *i.e.*, ‘outliers’, that do not match the general form.

b) Determine the solution $y(x)$ for at least the first six (6) lowest powers of nonzero terms of the series.

6.(17 pts) A ball of mass M and radius R rolls without slipping down an inclined plane under the action of gravity. The incline plane is at an angle α. The moment of inertia of the ball about an axis through its center is given by $I = \frac{2}{5} MR^2$.

a) Determine the Lagrangian which describes the motion of the ball.

b) Determine Lagrange’s equation of motion for the ball.

c) Determine the acceleration of the center-of-mass of the ball down the incline plane.