

Phys 208 – Theoretical Physics – Test 3 (April 24, 2009)

1. (25 pts) Let two dice be thrown. There are 36 possible outcomes or points in a uniform sample space. Let  $a, b$  represent an outcome where the number  $a$  is the number on the first die and the number  $b$  is the number on the second die. Thus all 36 possible outcomes will be

$$\begin{bmatrix} 1,1 & 1,2 & 1,3 & 1,4 & 1,5 & 1,6 \\ 2,1 & 2,2 & 2,3 & 2,4 & 2,5 & 2,6 \\ 3,1 & 3,2 & 3,3 & 3,4 & 3,5 & 3,6 \\ 4,1 & 4,2 & 4,3 & 4,4 & 4,5 & 4,6 \\ 5,1 & 5,2 & 5,3 & 5,4 & 5,5 & 5,6 \\ 6,1 & 6,2 & 6,3 & 6,4 & 6,5 & 6,6 \end{bmatrix}$$

Let  $x$  represent the gain in rolling the two dice; that is, the amount of money given to you if you roll a certain total number,  $a + b$ . Suppose  $x = \$20$  if the sum of the two dice is 11,  $x = \$10$  if the sum is 7,  $x = \$5$  if you roll a pair, *i.e.*,  $a = b$ , and  $x = \$0$  otherwise.

- Make a table  $x_i, p_i$  of the different  $x$  values and its probability  $p$ .
- Determine the expectation (or average) value of  $x$ .
- If you had to pay \$5 every time you rolled the dice, would the game be favorable to you?

2.(25 pts) It is shown in the kinetic theory of gases that the probability for the distance an atom travels between collisions to be between  $x$  and  $x + dx$ , is proportional to  $e^{-x/\lambda} dx$ , where  $\lambda$  is a constant called the mean free path.

- Determine the average distance between collisions.
- Determine the standard deviation of  $x$ .
- Find the probability that  $x$  lies between 0 and  $2\lambda$ .

3.(25 pts) a) Find the Fourier transform of the function ,  $f(x) = \begin{cases} e^{-bx} & 0 \leq x < \infty \\ 0 & x < 0 \end{cases}$ .

- List all the possible values of the integral  $\int_{-\infty}^{+\infty} g(\alpha)e^{i\alpha x} d\alpha$ .
- Use Parseval's theorem to determine the integral  $\int_{-\infty}^{+\infty} |g(\alpha)|^2 d\alpha$ .

4.(25 pts) The normalized ground and first excited state wave functions of the one-dimensional harmonic oscillator are given by:

$\psi_0(x) = \left(\frac{a}{\pi}\right)^{1/4} e^{-ax^2/2}$  and  $\psi_1(x) = \sqrt{2a} \left(\frac{a}{\pi}\right)^{1/4} x e^{-ax^2/2}$ , where  $-\infty < x < \infty$  and where  $a$  is a constant. These states have energies  $E_0 = \hbar\omega/2$  and  $E_1 = 3\hbar\omega/2$ .

a) Show that  $\langle x \rangle = 0$  in each of the stationary states,  $\psi_0(x)$  and  $\psi_1(x)$ . Hint: You do not need to do the integrals. Just use symmetry arguments.

b) Consider a time-dependent wave function defined as:

$$\Psi(x, t) = \frac{1}{\sqrt{2}} \left[ \psi_0(x) e^{-iE_0 t/\hbar} + \psi_1(x) e^{-iE_1 t/\hbar} \right]$$

Show that  $|\Psi(x, t)|^2 = \frac{1}{2} \left[ |\psi_0|^2 + |\psi_1|^2 + 2\psi_0\psi_1 \cos(\omega t) \right]$ .

c) Determine the average position of the particle as a function of time, *i.e.*,  $\langle x \rangle(t)$ .