Phys 208 – Theoretical Physics – Test 2 (March 9, 2012)

1.(20 pts) Consider an AC circuit which has a resistor, inductor, and capacitor in series. The voltage source is given by $V(t) = V_0 \sin(\omega t)$, where $V_0 = 20$ volts and $\omega = 100$ rad/sec. The resistor is 2 ohms, the inductor is 60 milli-henries, and the capacitor is 2.5 milli-farads.

a) What is the complex impedance of the circuit?

b) Find the current amplitude and the phase angle for the circuit. Does the current lead or lag the voltage source?

c) Determine the average power transferred to the circuit.

d) Determine the actual physical voltage across the inductor as a function of time, $V_L(t)$.

2.(20 pts) Given the integral $\int_0^\infty e^{-ax^2} dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$, evaluate the integral $\int_0^\infty x^4 e^{-ax^2} dx$.

3.(20 pts) Consider the integral $I = \int_0^\infty \int_0^\infty \frac{(x^2+y^2)x^2y^2}{1+(x^2-y^2)^2} e^{-ax^2y^2} dxdy$. Make the change of variables as follows: let $u = x^2 - y^2$ and $v = xy$. Determine the integration range for the new variables *u* and *v* and evaluate the integral.

4.(20 pts) Given the function $f(x,y,z) = x^2y - y^2z$ and the point $P = (1,1,1)$.

a) Determine the gradient of f at the point *P*.

b) What is the direction of the most rapid increase of f at the point *P* ?

c) Determine the directional derivative of f at the point *P* in the direction $\hat{i} - 2\hat{j} + \hat{k}$.

5.(20 pts) Find the derivative with respect to *x* of the integral $I(x) = \int_x^{x^2} \frac{\sin(xt)}{t} dt$.