Phys 208 – Theoretical Physics – Test 2 (April 19, 2010)

1.(17 pts) Find the general form for the integral $I_n = \int_0^\infty x^n e^{-ax} dx$ for n = 1, 2, 3, … , given that $I_0 = \int_0^\infty e^{-ax} dx = \frac{1}{a}$. Determine the recursion, that is, how I_{n+1} is related to I_n.

2.(17 pts) Consider the integral $I = \int_0^\infty \int_0^\infty \frac{(x^2+y^2)xy}{1+(x^2-y^2)^2} e^{-axy} dx dy$. Make the change of variables as follows: let $u = x^2 - y^2$ and $v = xy$ and evaluate *I*. Determine the integration range for the new variables *u* and *v*.

3.(17 pts) Calculate the curl of the vector, $\vec{A}(r,\theta,\phi) = (r\cos^2\theta)\,\hat{e}_r - (r\cos\theta\sin\theta)\,\hat{e}_\theta + 3r\,\hat{e}_\phi$, *i.e.*, $\vec{\nabla}\times\vec{A}$, in spherical coordinates.

4.(17 pts) A function over one period is given as $f(x) = \begin{cases} 0, & -\frac{1}{2} < x < 0, \\ 1, & 0 < x < \frac{1}{2} \end{cases}$

a) Expand it in a sine-cosine Fourier Series.

b) To what values will the Fourier series converge at $x = 0, \pm\frac{1}{4}, \pm\frac{1}{2}$?

5.(17 pts) In your homework problem 7.5.2 you were given the periodic function on the interval $(-\pi,\pi)$ defined by $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ 1 & 0 < x < \pi/2 \\ 0 & \pi/2 < x < \pi \end{cases}$. You should have obtained the following result

$$f(x) = \frac{1}{4} + \frac{1}{\pi}\sum_{n\ odd}^{\infty} \frac{(-1)^{1+n}\cos(nx)}{n} + \frac{1}{\pi}\sum_{n\ odd}^{\infty} \frac{\sin(nx)}{n} + \frac{1}{\pi}\sum_{n\ odd}^{\infty} \frac{\sin(2nx)}{n}.$$

Use Parseval's theorem to obtain the series $\sum_{n\ odd}^{\infty} \frac{1}{n^2}$.

6.(17 pts) a) Determine the Fourier transform of the function $f(x) = e^{-b|x|}$. Use the unsymmetric form of the transform.

b) Write $f(x) = \int_{-\infty}^{\infty} g(\alpha) e^{i\alpha x} d\alpha$ and determine the integral $\int_0^\infty \frac{\cos(\alpha)}{\alpha^2 + b^2} d\alpha$.

c) Use Parseval's theorem for Fourier transforms to evaluate the integral $\int_0^\infty \frac{d\alpha}{(\alpha^2+b^2)^2}$.