Phys 208 – Theoretical Physics – Test 1 (February 20, 2009)

1.(16 pts) Find the interval of convergence for $\sum_{n=1}^{\infty} \frac{(-1)^n (x+1)^n}{n}$. Don't forget to check the endpoints of the interval.

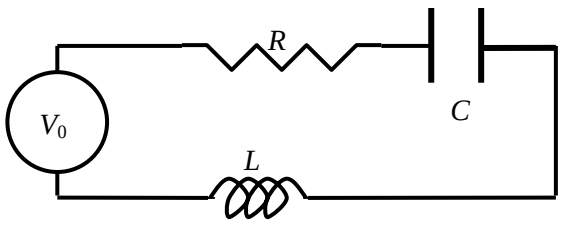
2.(16 pts) The arcsin(x) is defined by the integral $\arcsin(x) = \int_0^x \frac{dt}{\sqrt{1-t^2}}$. Determine the first three terms of the Maclaurin series for the arcsin(x) by expanding $1/\sqrt{1-t^2}$ in a binomial series and integrating term-by-term.

3.(16 pts) Describe geometrically the set of points in the complex plane satisfying $\mathrm{Re}(z^2) = 4$. Give a rough sketch of your result.

4.(16 pts) Find all the values of the indicated roots of $\sqrt[3]{i}$. Give your results in rectangular form, $(x+iy)$, and plot your results in the complex plane.

5.(16 pts) Evaluate $\cos(2i \ln i)$.

6.(20 pts) An AC voltage source has a voltage amplitude of 120 volts. It is connected to a resistor of 300 ohms and an inductor and capacitor as shown. The voltage source has an angular frequency $\omega = 1000$ rad/sec.
$L = 0.5\,H; \quad C = 5.0\,\mu F$



a) Find the impedance of the circuit.

b) Find the current amplitude.

c) Determine the phase angle for the circuit. Does the current lead or lag the applied voltage?

d) Determine the average power transferred to the circuit.

e) Determine the physical (not complex) voltage across the inductor as a function of time.

Some formulae: $\binom{p}{0} = 1 \qquad \binom{p}{n} = \frac{p(p-1)(p-2)\cdots(p-n+1)}{n!}$

$Z_R = R \qquad Z_L = i\omega L \qquad Z_C = \frac{1}{i\omega C} \qquad P = \frac{1}{2} I_0 V_0 \cos(\phi)$