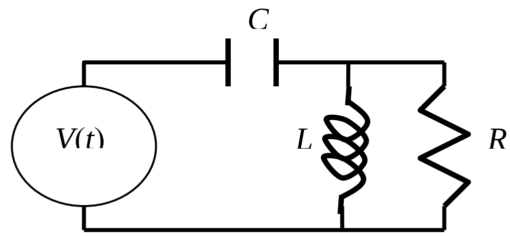
Phys 208 – Theoretical Physics – Final (May 10, 2010)

1.(17 pts) An AC voltage source has a voltage amplitude of 8 volts. It is connected to a capacitor, an inductor, and a 1Ω resistor as shown. Assume the frequency of the source is such that $\omega L = 2\ \Omega$ and $1/(\omega C) = 6/5\ \Omega$ or $1.2\ \Omega$.



a) Find the impedance of the circuit.

b) Find the current amplitude.

c) Determine the phase angle for the circuit. Does the current lead or lag the applied voltage?

d) Determine the average power transferred to the circuit.

e) Determine the physical (not complex) voltage across the capacitor as a function of time.

2.(17 pts) a) Determine the Fourier transform of the function $f(x) = e^{-b|x|}$. Use the unsymmetric form of the Fourier transform.

b) Write $f(x) = \int_{-\infty}^{\infty} g(\alpha) e^{i\alpha x} d\alpha$ and determine the integral $\int_0^{\infty} \frac{\cos(\alpha)}{\alpha^2 + b^2} d\alpha$.

c) Use Parseval's theorem for Fourier transforms to evaluate the integral $\int_0^{\infty} \frac{d\alpha}{(\alpha^2 + b^2)^2}$.

3.(17 pts) A ball is thrown straight up and falls straight back down.

a) Find the probability density function $f(h)$ so that $f(h)\,dh$ is the probability of finding the ball between height h and $h + dh$.

b) Determine the expectation (or average) value of h.

4.(17 pts) A hydrogen-like atom has a nucleus of charge Ze and an electron of charge $-e$. Z is a constant and the attractive potential is the Coulomb potential between the nuclear charge and the electron charge. The distance between the nucleus and the electron is the radial coordinate r. The radial Hamiltonian for this hydrogen-like atom is given by

$$H\,R(r) = -\frac{\hbar^2}{2mr^2}\frac{d}{dr}\left(r^2\frac{dR(r)}{dr}\right) - \frac{Ze^2}{4\pi\varepsilon_0 r}R(r) = E\,R(r),$$

where $\hbar$ is Planck's constant divided by 2π, m is the effective electron mass, E is the bound state energy, and $R(r)$ is the bound state wavefunction.

a) Show that $R(r) = Ne^{-ar}$ (N is a constant) is a solution. Hint: You need to choose a so that there is no $1/r$ dependence in the equation. Using your result for a, what value do you obtain for E? You may find it useful to define a constant $a_0 = \dfrac{\hbar^2 4\pi\varepsilon_0}{me^2}$.

b) Recall the radial probability density function is defined such that $f(r)dr = r^2[R(r)]^2\,dr$ is the probability that the electron is between r and $r + dr$ and r varies from 0 to ∞. Use this information to determine N.

c) Calculate $\Delta r = \sqrt{\langle r^2\rangle - \langle r\rangle^2}$ for this wavefunction.

5.(17 pts) Solve the following differential equation using the power series method.

$$y'' + x^2 y = 0$$

a) Determine the recursion relation for the coefficients a_n if $y(x) = \sum_{n=0}^{\infty} a_n x^n$.

Hint: Shift all powers of x to the highest exponent and take out the terms, *i.e.*, 'outliers', that do not match the general form.

b) Determine the solution $y(x)$ for at least the first six (6) nonzero terms (lowest powers) of the series.

6.(17 pts) A ball of mass M and radius R rolls without slipping down an inclined plane under the action of gravity. The incline plane is at an angle α. The moment of inertia of the ball about an axis through its center is given by $I = \frac{2}{5}MR^2$.

a) Determine a Lagrangian which describes the motion of the ball.

b) Determine Lagrange's equation of motion for the ball.

c) Determine the acceleration of the center-of-mass of the ball down the incline plane.