The gap of square parallel plate capacitor is completely filled by an insulator with dielectric constant κ. The capacitor has sides of length a and a distance between the plates d. The isolated capacitor holds charge Q.

- What is the change in charge on the capacitor?
- The dielectric is removed from the capacitor. What is the change in energy stored on the capacitor?
- What is the change in potential across the capacitor?

1

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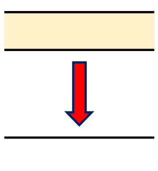

Let C_0 be without dielectric: $C_i = \kappa C_0$ and $C_f = C_0$

$$Q_f = Q_i = Q$$

2

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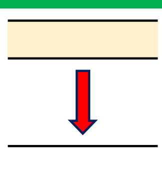

$$U_i = \frac{1}{2}\frac{Q^2}{C_i} = \frac{1}{2}\frac{Q^2}{\kappa C_0} = \frac{1}{\kappa}\left[\frac{1}{2}\frac{Q^2}{C_0}\right] = \frac{1}{\kappa}\left[\frac{1}{2}\frac{Q^2}{C_f}\right] = \frac{1}{\kappa}U_f$$

$$\Delta U = U_f\left(1 - \frac{1}{\kappa}\right) = \left(\frac{1}{2}\frac{Q^2}{C_0}\right)\left(1 - \frac{1}{\kappa}\right) = \frac{Q^2 d}{2\epsilon_0 a^2}\left(1 - \frac{1}{\kappa}\right)$$

3

The gap of square parallel plate capacitor is completely filled by an insulator with dielectric constant κ. The capacitor has sides of length a and a distance between the plates d. The isolated capacitor holds charge Q.

- What is the change in charge on the capacitor?
- The dielectric is removed from the capacitor. What is the change in energy stored on the capacitor?
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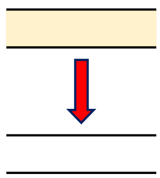

$$V_i = \frac{Q_i}{C_i} = \frac{Q_f}{\kappa C_0} = \frac{1}{\kappa}\left(\frac{Q_f}{C_0}\right) = \frac{1}{\kappa}\left(\frac{Q_f}{C_f}\right) = \frac{1}{\kappa}V_f$$

$$\Delta V = \left(1 - \frac{1}{\kappa}\right)V_f = \left(1 - \frac{1}{\kappa}\right)\left(\frac{Q_f}{C_0}\right) = \left(1 - \frac{1}{\kappa}\right)\left(\frac{Qd}{\epsilon_0 a^2}\right)$$

4

A cylindrical wire has a diameter d and length L and is made of a metal with a conductivity σ at temperature T_0.

- What is the resistance of the wire?
- What is the resistance of the wire at temperature T if the temperature coefficient of resistivity is α?

5

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$$R = \rho\frac{L}{A} = \frac{1}{\sigma}\frac{L}{\pi\left(\frac{d}{2}\right)^2} = \frac{4L}{\sigma\pi d^2}$$

6

A cylindrical wire has a diameter d and length L and is made of a metal with a conductivity σ at temperature T_0.

- What is the resistance of the wire?
- What is the resistance of the wire at temperature T if the temperature coefficient of resistivity is α?

$$R = \rho\frac{L}{A} = \frac{1}{\sigma}\frac{L}{\pi\left(\frac{d}{2}\right)^2} = \frac{4L}{\sigma\pi d^2}$$

$$R = \rho\frac{L}{A} = \rho_0[1+\alpha(T-T_0)]\frac{L}{A} = R_0[1+\alpha(T-T_0)] = \frac{4L}{\sigma\pi d^2}[1+\alpha(T-T_0)]$$

7

A wire of radius r and length L is connected across a potential difference ΔV. The wire is made of a material with free electron density n.

- What is the current in the wire?
- What is the drift velocity of the free electrons in the wire?

8

A wire of radius r and length L is connected across a potential difference ΔV. The wire is made of a material with free electron density n and resistivity ρ.

- What is the current in the wire?
- What is the drift velocity of the free electrons in the wire?

$$I = JA = \sigma EA = \frac{1}{\rho}\frac{\Delta V}{L}A = \frac{\Delta V\pi\ {}^2}{\rho L} \qquad \text{OR} \qquad I = \frac{\Delta V}{R} = \frac{\Delta V\pi\ {}^2}{\rho L}$$

9

A wire of radius r and length L is connected across a potential difference ΔV. The wire is made of a material with free electron density n and resistivity ρ.

- What is the current in the wire?
- What is the drift velocity of the free electrons in the wire?

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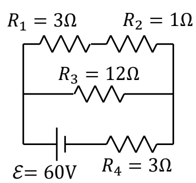
$$v_d = \frac{J}{ne} = \frac{\sigma E}{ne} = \frac{E}{\rho ne} = \frac{\Delta V}{pneL}$$

10

Consider the given circuit.

- What is the equivalent resistance for the circuit?
- What is the current through R_3?
- What is the power dissipated by R_2?



11

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- What is the current through R_3?
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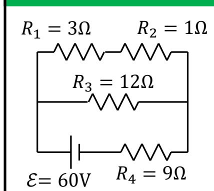


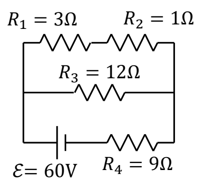
$$R_{12} = R_1 + R_2 = 4\Omega$$

$$R_{123} = \left(\frac{1}{R_{12}} + \frac{1}{R_3}\right)^{-1} = \left(\frac{1}{4\Omega} + \frac{1}{12\Omega}\right)^{-1} = 3\Omega$$

$$R_T = R_{123} + R_4 = 3\Omega + 9\Omega = 12\Omega$$

12

Consider the given circuit.
- What is the equivalent resistance for the circuit?
- What is the current through R_3?
- What is the power dissipated by R_2?



$$I_T = \frac{\mathcal{E}}{R_T} = \frac{60\text{V}}{12\Omega} = 5\text{A}$$

$$V_4 = I_4 R_4 = (5\text{A})(9\Omega) = 45\text{V}$$

$$I_3 = \frac{V_3}{R_3} = \frac{V_T - V_3}{R_3} = \frac{60\text{V} - 45\text{V}}{12\Omega} = 1.25\text{A}$$

13

Consider the given circuit.
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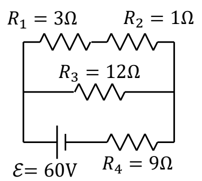

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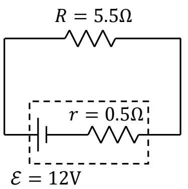
$$P_2 = I_2^2 R_2 = (I_T - I_3)^2 R_2 = (5\text{A} - 1.25\text{A})^2 (1\Omega) = 14\text{W}$$

14

A 12V battery with an internal resistance $r = 0.5\Omega$ is connected to a resistor with resistance $R = 5.5\Omega$.
- What is the current in the circuit?
- What is the terminal voltage across the battery?
- What is the rate at which chemical energy is converted to electrical energy in the battery?



15

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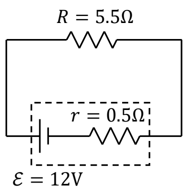

$$I = \frac{\mathcal{E}}{R + r} = \frac{12\text{V}}{5.5\Omega + 0.5\Omega} = \frac{12\text{V}}{6\Omega} = 2\text{A}$$

16

A 12V battery with an internal resistance $r = 0.5\Omega$ is connected to a resistor with resistance $R = 5.5\Omega$.

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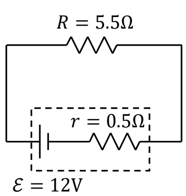

$$I = \frac{\mathcal{E}}{R + r} = \frac{12\text{V}}{5.5\Omega + 0.5\Omega} = \frac{12\text{V}}{6\Omega} = 2\text{A}$$

$$V_{terminal} = \mathcal{E} - Ir = 12\text{V} - (2\text{A})(0.5\Omega) = 11\text{V}$$

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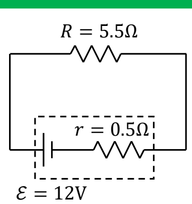


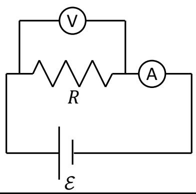
$$I = \frac{\mathcal{E}}{R + r} = \frac{12\text{V}}{5.5\Omega + 0.5\Omega} = \frac{12\text{V}}{6\Omega} = 2\text{A}$$

$$V_{terminal} = \mathcal{E} - Ir = 12\text{V} - (2\text{A})(0.5\Omega) = 11\text{V}$$

$$P_{\mathcal{E}} = I\mathcal{E} = (2\text{A})(12\text{V}) = 24\text{W}$$

18

A pair of galvanometers are used to make a voltmeter and an ammeter. They have an internal resistance r_G and experience full deflection with a current I_G.
- What resistance should be used for the series resistor in the voltmeter to get full deflection for a voltage $V_{\max}$?
- What resistance should be used for the shunt resistor in the ammeter to get full deflection for a current $I_{\max}$?



19

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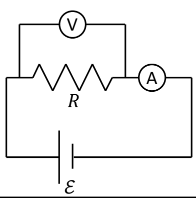

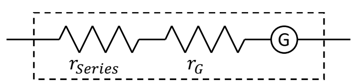

$$V_{\max} = I_G(r_{Series} + r_G)$$

$$r_{Series} = \frac{V_{\max}}{I_G} - r_G$$

20

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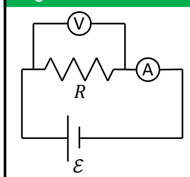
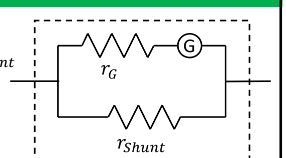


$$I_G r_G = (I_{\max} - I_G) r_{Shunt}$$

$$\frac{I_G}{I_{\max} - I_G} r_G = r_{Shunt}$$



21

A capacitor with capacitance C is connected in series with a resistor with resistance R and a battery with emf $\mathcal{E}$.

- What is the charge on the capacitor at time t?
- When is the charge on the capacitor 60% of its final charge?
- What is the current through the resistor at time t?
- What is the voltage across the resistor when the voltage across the capacitor is V_C?

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- What is the current through the resistor at time t?
- What is the voltage across the resistor when the voltage across the capacitor is V_C?

$$Q = Q_f\left(1 - e^{-t/RC}\right)$$

$$Q = C\mathcal{E}\left(1 - e^{-t/RC}\right)$$

23

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- What is the current through the resistor at time t?
- What is the voltage across the resistor when the voltage across the capacitor is V_C?

$$0.6Q_f = Q_f\left(1 - e^{-t/RC}\right)$$

$$e^{-t/RC} = 0.4$$

$$t = -RC\ln(0.4)$$

24

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- What is the current through the resistor at time t?
- What is the voltage across the resistor when the voltage across the capacitor is V_C?

$$I = \frac{dQ}{dt} = \frac{d}{dt}\left[\mathcal{E}C\left(1 - e^{-t/RC}\right)\right]$$

$$I = \frac{\mathcal{E}}{R}e^{-t/RC}$$

25

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- When is the charge on the capacitor 60% of its final charge?
- What is the current through the resistor at time t?
- What is the voltage across the resistor when the voltage across the capacitor is V_C?

$$0 = \mathcal{E} - V_C - V_R$$

$$V_R = \mathcal{E} - V_C$$

26

A particle with mass m, charge q and velocity $v_0\hat{\imath}$ enters a region with uniform magnetic field $B_0\hat{\jmath}$.
- What is the radius of the particle's trajectory?
- What is the rotational direction of the particle?

27

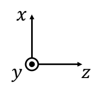
A particle with mass m, charge q and velocity $v_0\hat{\imath}$ enters a region with uniform magnetic field $B_0\hat{\jmath}$.

- What is the radius of the particle's trajectory?
- What is the rotational direction of the particle?

$$\frac{mv^2}{r} = \left|q\vec{v} \times \vec{B}\right|$$

$$\frac{mv^2}{r} = qvB$$

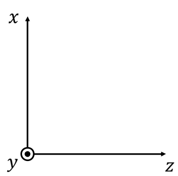
$$\frac{mv}{qB} = r$$



28

A uniform magnetic field $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath}$ exists in a region. Consider the flux through a rectangle of sides a and b.

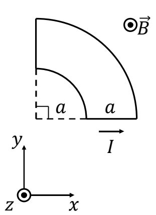
- The rectangle is in the xy-plane.
- The rectangle is in the yz-plane.



29

A uniform magnetic field $B_0\hat{k}$ exists in a region with the given current arrangement.

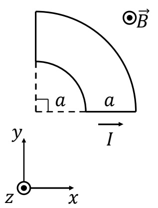
- What is the force on the straight sections of wire?
- What is the force on the arced sections of wire?



30

A uniform magnetic field $B_0\hat{k}$ exists in a region with the given current arrangement.
- What is the force on the straight sections of wire?
- What is the force on the arced sections of wire?

$\vec{F} = I\vec{L} \times \vec{B}$



Bottom: $\vec{F}_{bottom} = Ia\hat{\imath} \times B_0\hat{k}$

$$\vec{F}_{bottom} = -IaB_0\hat{\jmath}$$

Left: $\vec{F}_{left} = Ia(-\hat{\jmath}) \times B_0\hat{k}$

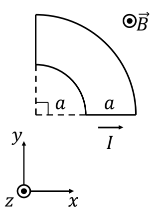
$$\vec{F}_{left} = -IaB_0\hat{\imath}$$

31

A uniform magnetic field $B_0\hat{k}$ exists in a region with the given current arrangement.
- What is the force on the straight sections of wire?
- What is the force on the arced sections of wire?

$\vec{F} = I\vec{L} \times \vec{B}$

$\vec{F} = \int I d\vec{s} \times \vec{B}$



Upper Arc: $\vec{F}_{ua} = \int_0^{\pi/2} I2ad\phi(-\sin\phi\,\hat{\imath} + \cos\phi\,\hat{\jmath}) \times B_0\hat{k}$

$$\vec{F}_{ua} = 2IaB_0 \int_0^{\pi/2} d\phi(\sin\phi\,\hat{\jmath} + \cos\phi\,\hat{\imath})$$

$$\vec{F}_{ua} = 2IaB_0[-\cos\phi\,\hat{\jmath} + \sin\phi\,\hat{\imath}]_0^{\pi/2}$$

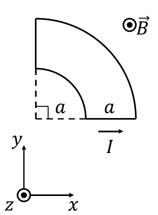
$$\vec{F}_{ua} = 2IaB_0(\hat{\imath} + \hat{\jmath})$$

32

A uniform magnetic field $B_0\hat{k}$ exists in a region with the given current arrangement.
- What is the force on the straight sections of wire?
- What is the force on the arced sections of wire?

$\vec{F} = I\vec{L} \times \vec{B}$

$\vec{F} = \int I d\vec{s} \times \vec{B}$



Lower Arc: $\vec{F}_{la} = \int_0^{\pi/2} Iad\phi(\sin\phi\,\hat{\imath} - \cos\phi\,\hat{\jmath}) \times B_0\hat{k}$

$$\vec{F}_{la} = IaB_0 \int_0^{\pi/2} d\phi(-\sin\phi\,\hat{\jmath} - \cos\phi\,\hat{\imath})$$

$$\vec{F}_{la} = IaB_0[\cos\phi\,\hat{\jmath} - \sin\phi\,\hat{\imath}]_0^{\pi/2}$$

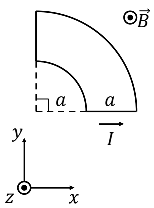
$$\vec{F}_{la} = -IaB_0(\hat{\imath} + \hat{\jmath})$$

33

A uniform magnetic field $B_0\hat{k}$ exists in a region with the given current arrangement.
- What is the force on the straight sections of wire?
- What is the force on the arced sections of wire?

$\vec{F} = I\vec{L} \times \vec{B}$

$\vec{F} = \int Id\vec{s} \times \vec{B}$



$\vec{F}_{bottom} = -IaB_0\hat{\jmath}$

$\vec{F}_{left} = -IaB_0\hat{\imath}$

$\vec{F}_{ua} = 2IaB_0(\hat{\imath} + \hat{\jmath})$

$\vec{F}_{la} = -IaB_0(\hat{\imath} + \hat{\jmath})$

$\rightarrow \vec{F}_{Total} = 0$

34

A coil of wire with N turns and a radius R lies in the xy-plane and carries a current I. There is a uniform magnetic field $\vec{B} = B_x\hat{\imath} + B_z\hat{k}$.
- What is the energy of the dipole?
- What is the torque on the dipole?

35

A coil of wire with N turns and a radius R lies in the xy-plane and carries a current I such that $\vec{A}$ is in the positive z-direction. There is a uniform magnetic field $\vec{B} = B_x\hat{\imath} + B_z\hat{k}$.
- What is the energy of the dipole?
- What is the torque on the dipole?

$U = -\vec{\mu} \cdot \vec{B}$

$U = -NI\vec{A} \cdot \vec{B}$

$U = -NI\pi R^2 B_z$

$\vec{\tau} = \vec{\mu} \times \vec{B}$

$\vec{\tau} = NIA\hat{k} \times \left(B_x\hat{\imath} + B_z\hat{k}\right)$

$\vec{\tau} = NIAB_x\hat{\jmath}$

36