Forces, Fields, Energy and Potential

$\vec{F} = k\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}$	$\vec{F} = q\vec{E}$	$\vec{E} = k\frac{q}{r^2}\hat{r}$
$\Delta U = -\int \vec{F} \cdot d\vec{s}$		$\Delta V = -\int \vec{E} \cdot d\vec{s}$
$F_x = -\frac{\partial U}{\partial x}$		$E_x = -\frac{\partial V}{\partial x}$
$U = k\frac{q_1 q_2}{r_{12}}$	$U = qV$	$V = k\frac{q}{r}$

Forces, Fields, Energy and Potential
Continuous Forms

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Forces, Fields, Energy and Potential
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Procedure:

- Find dq
- Find r (and $\hat{r}$ for $\vec{F}$ or $\vec{E}$)
- Substitute into integral
- Solve integral

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dq examples

Line of charge:

$$dq = \lambda dx = \frac{Q}{L} dx$$

Arc of charge:

$$dq = \lambda r d\phi = \frac{Q}{r\theta} r d\phi = \frac{Q}{\theta} d\phi$$

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Continuous Forms

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r examples

Line of charge

$$\vec{r} = r_x \hat{\imath} + r_y \hat{\jmath}$$

$$r = \sqrt{r_x^2 + r_y^2}$$

$$\hat{r} = \frac{\vec{r}}{r} = \frac{r_x}{\sqrt{r_x^2 + r_y^2}} \hat{\imath} + \frac{r_y}{\sqrt{r_x^2 + r_y^2}} \hat{\jmath}$$

Forces, Fields, Energy and Potential
Continuous Forms

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- Find dq
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r examples

Arc of charge

$$\vec{r} = -a\cos\phi\,\hat{\imath} - a\sin\phi\,\hat{\jmath}$$

$$r = a$$

$$\hat{r} = -\cos\phi\,\hat{\imath} - \sin\phi\,\hat{\jmath}$$

(Assumption: finding $\vec{F}$ or $\vec{E}$ at origin)

Finding Equilibria for Charges Constrained to Line

Procedure:

- Consider all possible regions
- For each region, express location with a variable
- Express forces in terms of variable
- Set total force on object to zero
- Solve for location
- Accept solution only if in selected region
- Determine if stable or unstable

Finding Equilibria for Charges Constrained to Line

Procedure:

- Determine if stable or unstable
  - Consider small displacements from equilibrium
  - If net force is towards equilibrium, then stable
  - If net force is away from equilibrium, then unstable

Motion in a Uniform Electric Field

Find $\vec{a}$ and apply Newton's equations of motion for constant acceleration from Physics I.

$$\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m}$$

$$x = x_0 + v_0 t + \frac{1}{2} a_x t^2$$

$$v_x = v_{x0} + a_x t$$

$$v_x^2 = v_{x0}^2 + 2a_x \Delta x$$

$$y = y_0 + v_0 t + \frac{1}{2} a_y t^2$$

$$v_y = v_{y0} + a_y t$$

$$v_y^2 = v_{y0}^2 + 2a_y \Delta y$$

Gauss's Law

Procedure:

- Divide into distinct regions
- Write out law, $\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}$
- Sketch Gaussian surface
- Factor out E and substitute for Gaussian area
- Substitute for enclosed charge
- In non-conducting regions, solve for E and add direction
- In conducting regions, solve for surface charge

Gauss's Law

Procedure:

- Factor out E and substitute for Gaussian area

Areas:

Spherical Symmetry

$$E(4\pi r^2) =$$

Cylindrial Symmetry

$$E(2\pi rL) =$$

Planar Symmetry

$$E(2A) =$$

Gauss's Law

Procedure:

- In conducting regions, solve for surface charge

Inner Surface:

$$Q_{\text{inner}} = -\sum q_{\text{other enclosed}}$$

Outer Surface:

$$Q_{\text{outer}} = Q_{\text{conductor}} - Q_{\text{inner}}$$

Dipoles in Uniform Field

$\vec{p} = q\vec{d}$ (from – to +)

$$\vec{F} = 0$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$U = -\vec{p} \cdot \vec{E}$$

Finding Capacitance

Procedure:

- Find $\vec{E}$ using Gauss's Law $\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}$
- Find V using $V = |\Delta V| = \left|- \int \vec{E} \cdot d\vec{s}\right|$
- Find C using $C = \left|\frac{Q}{V}\right|$

Capacitors in Circuits

In Series	In Parallel
$C_T = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}$	$C_T = C_1 + C_2$
$V_T = V_1 + V_2$	$V_T = V_1 = V_2$
$Q_T = Q_1 = Q_2$	$Q_T = Q_1 + Q_2$

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$Q_T = Q_1 = Q_2$	$Q_T = Q_1 + Q_2$

Problems involving circuits: Use these equations and $C = \frac{Q}{V}$

Capacitors in Circuits

Additional topics:

- What forces are there when two objects are brought near each other? Consider both charged and neutral and both conductors and insulators.
- What forces are on a charged object in an electric field? How does the force, field, energy or potential change as an object is moved from one location to another in the presence of a field or other charges?

Example: A uniformly positively charged insulator is brought near a positive point charge. Describe any force that might be experienced by the insulator.

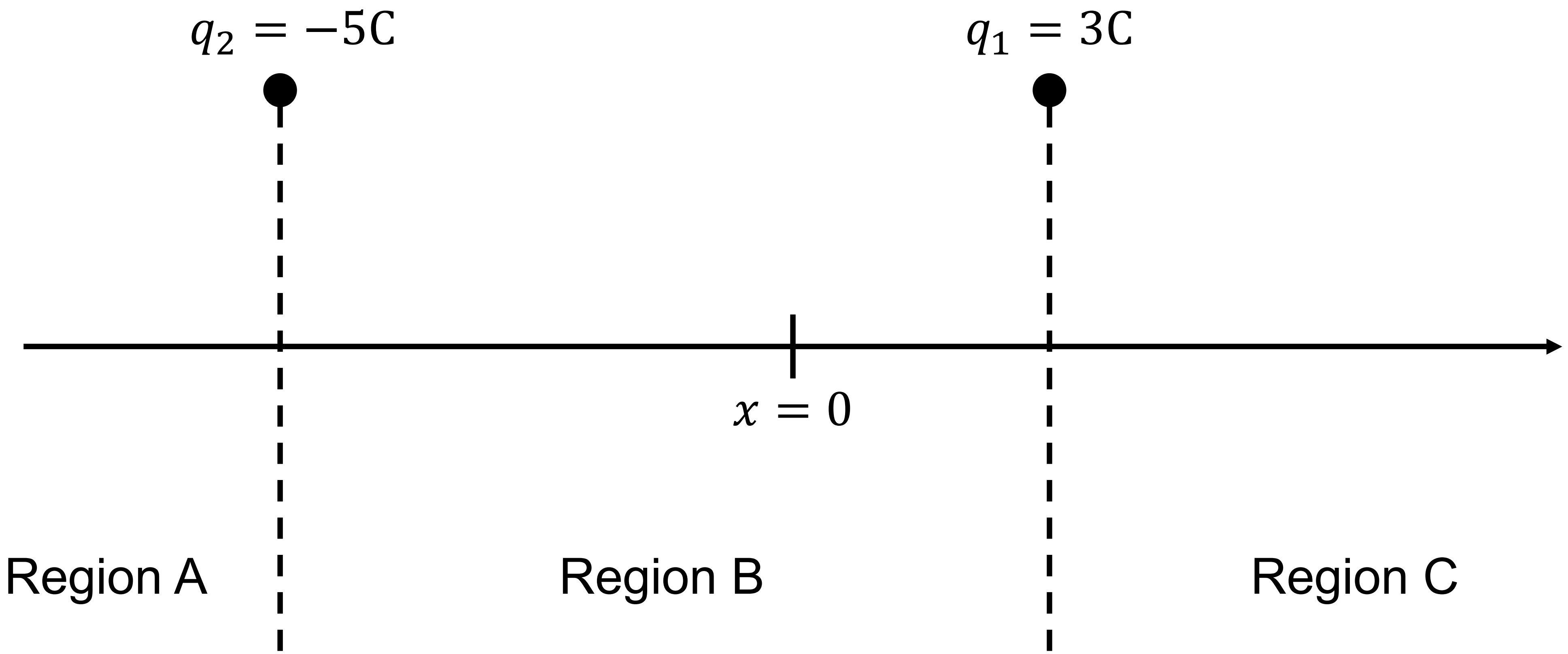
Example: A dipole is located in a uniform electric field.

a) Illustrate the orientation of the dipole that will result in the maximum potential energy.

b) Illustrate the orientation of the dipole that will result in the maximum torque.

c) Illustrate the orientation of the dipole that will result in the maximum force.

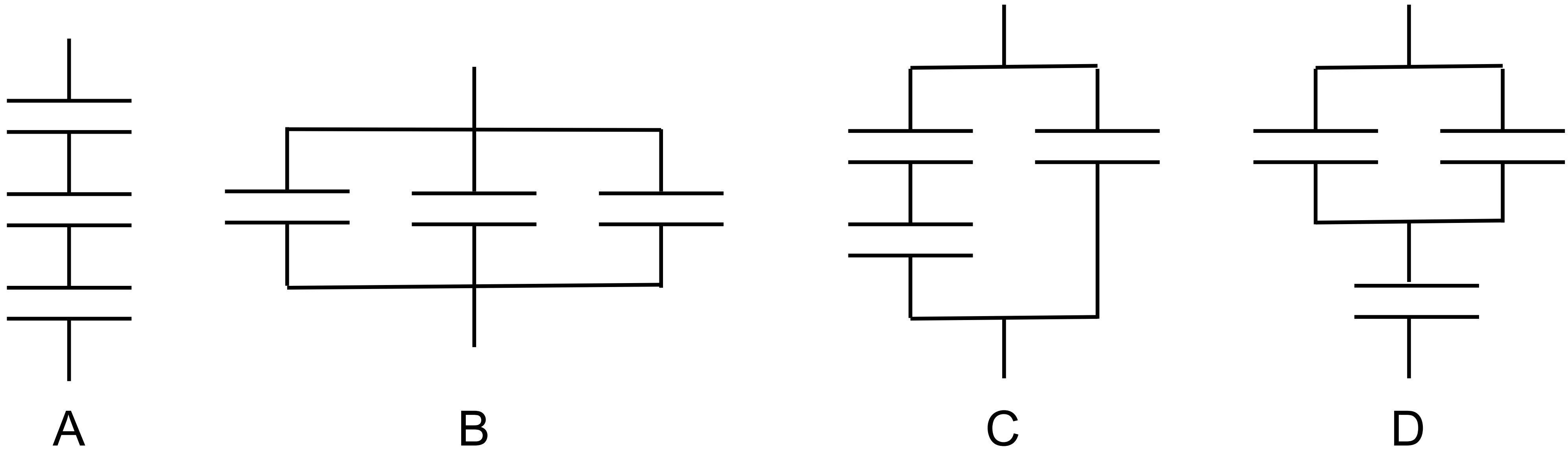
Example: $q_3 = 2\text{C}$ is constrained to move along the x-axis. $q_1 = 3\text{C}$ at $(2\text{cm}, 2\text{cm})$ and $q_2 = -5\text{C}$ at $(-4\text{cm}, 2\text{cm})$ are held fixed. In which region, is there an equilibrium for q_3?



Example: A pair of charges have an energy U_0. The distance between the charges is doubled. What is the energy of the modified arrangement?

Example: A charge experiences a force F_0 due to another nearby charge. The charge is moved so that it now experiences a force $2F_0$ due to the nearby charge. What must have happened when the charge was moved?

Example: Three identical capacitors may be arranged as illustrated. Rank the three arrangements in order from least capacitance to greatest capacitance.



Example: A charge Q is uniformly distributed along an arc of radius a as illustrated. One wishes to find the force on the arc due to a charge q located at the origin.

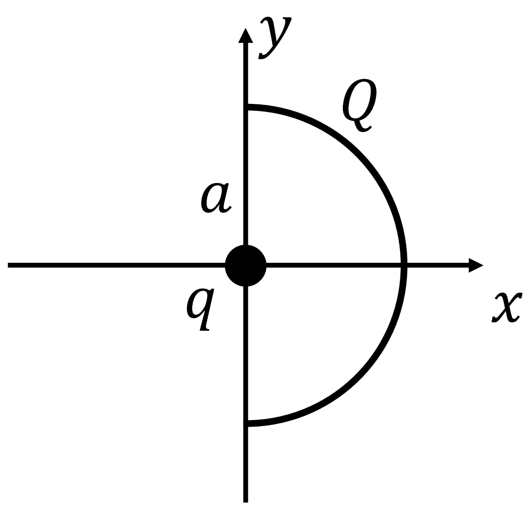
a) Determine dQ.

b) Determine r.

c) Determine $\hat{r}$.

d) Determine the limits of integration.

e) Write the integral.



Example: A charge Q is uniformly distributed along a line from $(a, 0)$ to (a, b). One wishes to find the electric field at $(0, b)$ due to the line of charge.

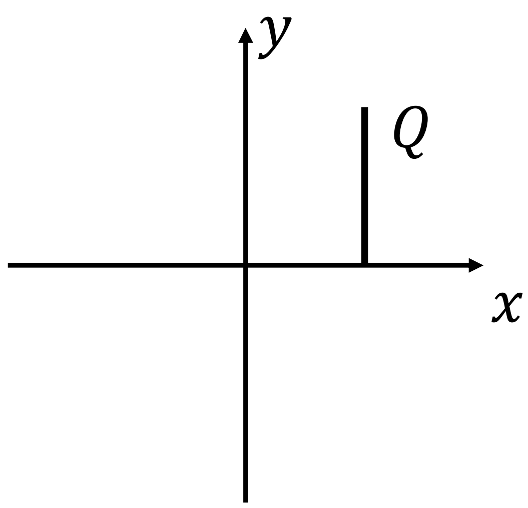
a) Determine dQ.

b) Determine r.

c) Determine $\hat{r}$.

d) Determine the limits of integration.

e) Write the integral.



Example: A point charge q_0 is located at the center of a sphere of radius a with a uniform surface charge distribution σ. Concentric with the sphere is a conducting spherical shell with inner radius $b > a$ and outer radius c. The conducting shell has a charge Q.

a) Determine the electric field in the region $a < r < b$.

b) Determine the surface charge on the conductor at $r = b$.

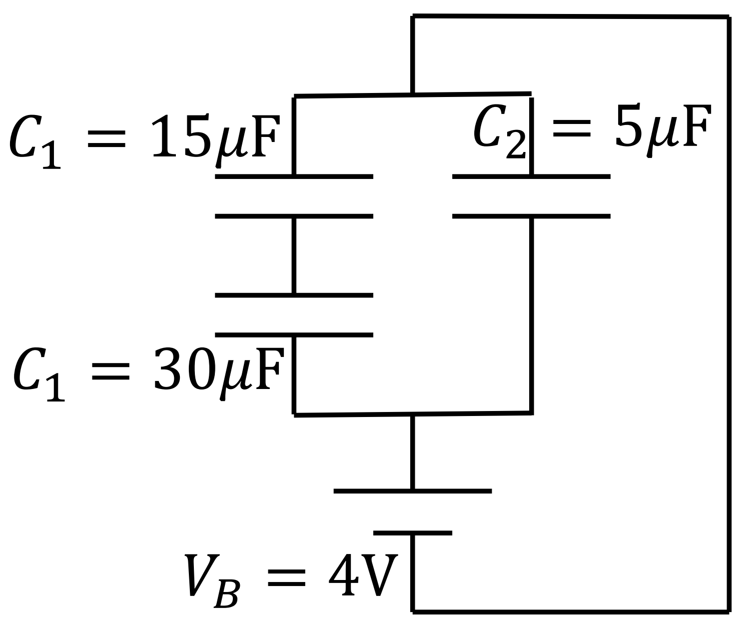
c) Determine the surface charge density on the conductor at $r = c$.

Example: Consider the illustrated circuit.

a) Determine the total capacitance of the circuit.

b) Determine the charge on each capacitor.

c) Determine the potential difference across each capacitor.



The gap of square parallel plate capacitor is completely filled by an insulator with dielectric constant κ. The capacitor has sides of length a and a distance between the plates d. The isolated capacitor holds charge Q.

- What is the change in charge on the capacitor?
- The dielectric is removed from the capacitor. What is the change in energy stored on the capacitor?
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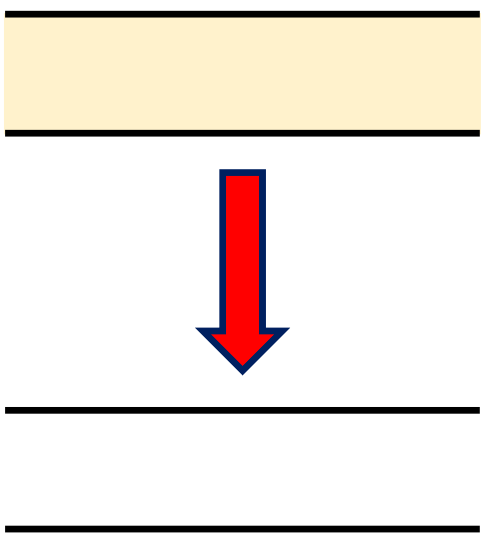

Let C_0 be without dielectric: $C_i = \kappa C_0$ and $C_f = C_0$

$$Q_f = Q_i = Q$$

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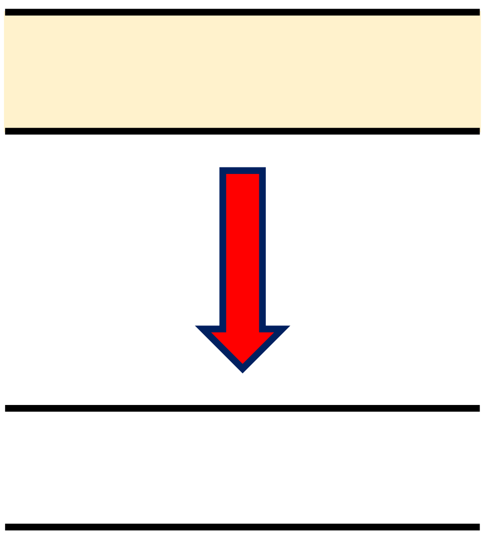

$$U_i = \frac{1}{2}\frac{Q^2}{C_i} = \frac{1}{2}\frac{Q^2}{\kappa C_0} = \frac{1}{\kappa}\left[\frac{1}{2}\frac{Q^2}{C_0}\right] = \frac{1}{\kappa}\left[\frac{1}{2}\frac{Q^2}{C_f}\right] = \frac{1}{\kappa}U_f$$

$$\Delta U = U_f\left(1 - \frac{1}{\kappa}\right) = \left(\frac{1}{2}\frac{Q^2}{C_0}\right)\left(1 - \frac{1}{\kappa}\right) = \frac{Q^2 d}{2\epsilon_0 a^2}\left(1 - \frac{1}{\kappa}\right)$$

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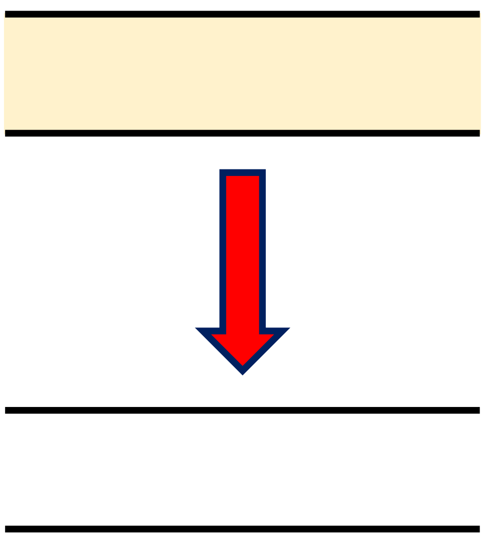

$$V_i = \frac{Q_i}{C_i} = \frac{Q_f}{\kappa C_0} = \frac{1}{\kappa}\left(\frac{Q_f}{C_0}\right) = \frac{1}{\kappa}\left(\frac{Q_f}{C_f}\right) = \frac{1}{\kappa} V_f$$

$$\Delta V = \left(1 - \frac{1}{\kappa}\right) V_f = \left(1 - \frac{1}{\kappa}\right)\left(\frac{Q_f}{C_0}\right) = \left(1 - \frac{1}{\kappa}\right)\left(\frac{Qd}{\epsilon_0 a^2}\right)$$

A cylindrical wire has a diameter d and length L and is made of a metal with a conductivity σ at temperature T_0.

- What is the resistance of the wire?
- What is the resistance of the wire at temperature T if the temperature coefficient of resistivity is α?

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$$R = \rho \frac{L}{A} = \rho_0 [1 + \alpha(T - T_0)] \frac{L}{A} = R_0 [1 + \alpha(T - T_0)] = \frac{4L}{\sigma \pi d^2} [1 + \alpha(T - T_0)]$$

A wire of radius r and length L is connected across a potential difference ΔV. The wire is made of a material with free electron density n.

- What is the current in the wire?
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$$I = JA = \sigma EA = \frac{1}{\rho}\frac{\Delta V}{L}A = \frac{\Delta V \pi r^2}{\rho L} \qquad \text{OR} \qquad I = \frac{\Delta V}{R} = \frac{\Delta V \pi r^2}{\rho L}$$

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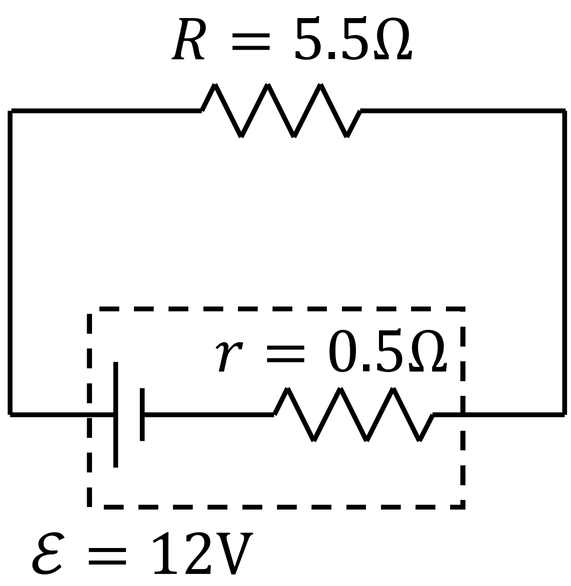
OR

$$I = \frac{\Delta V}{R} = \frac{\Delta V \pi r^2}{\rho L}$$

$$v_d = \frac{J}{ne} = \frac{\sigma E}{ne} = \frac{E}{\rho ne} = \frac{\Delta V}{pneL}$$

A 12V battery with an internal resistance $r = 0.5\Omega$ is connected to a resistor with resistance $R = 5.5\Omega$.

- What is the current in the circuit?
- What is the terminal voltage across the battery?
- What is the rate at which chemical energy is converted to electrical energy in the battery?



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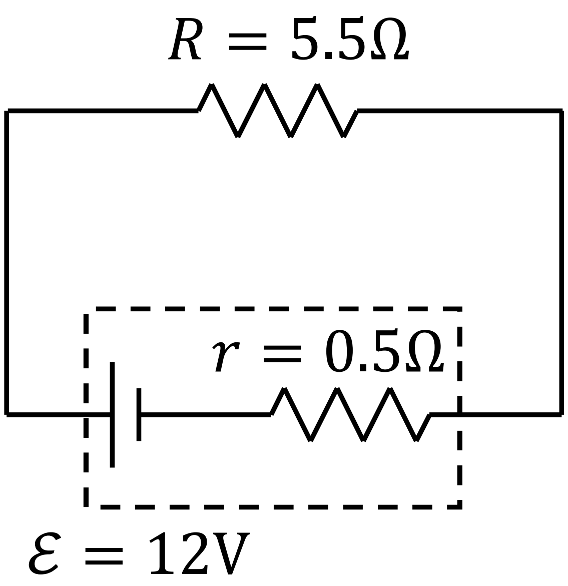

$$I = \frac{\mathcal{E}}{R + r} = \frac{12\text{V}}{5.5\Omega + 0.5\Omega} = \frac{12\text{V}}{6\Omega} = 2\text{A}$$

A $12V$ battery with an internal resistance $r = 0.5\Omega$ is connected to a resistor with resistance $R = 5.5\Omega$.

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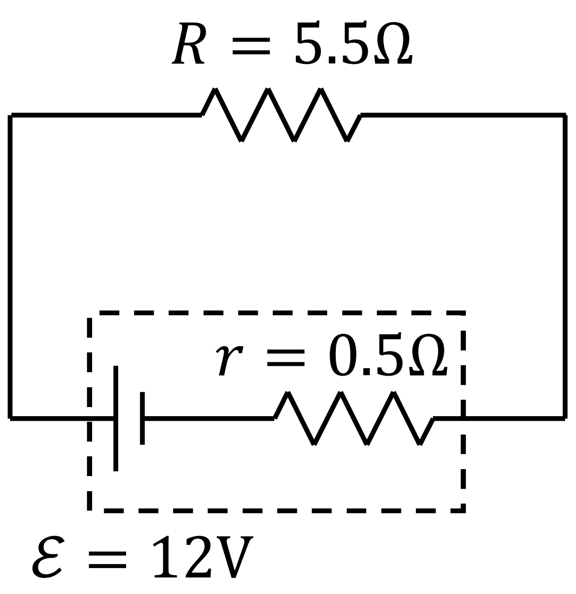

$$I = \frac{\mathcal{E}}{R + r} = \frac{12V}{5.5\Omega + 0.5\Omega} = \frac{12V}{6\Omega} = 2A$$

$$V_{terminal} = \mathcal{E} - Ir = 12V - (2A)(0.5\Omega) = 11V$$

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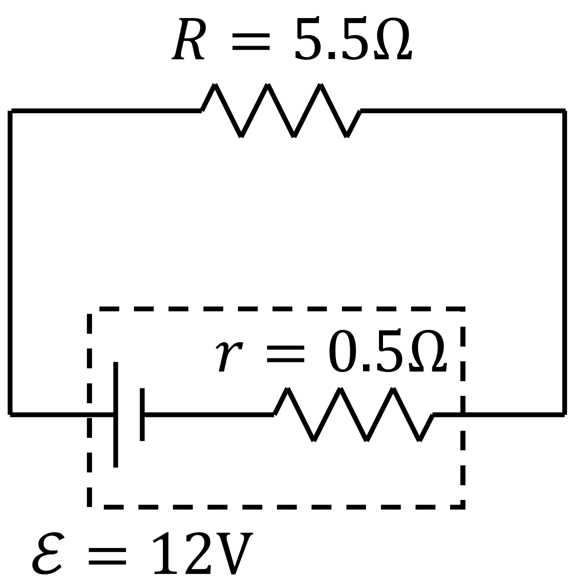

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$$V_{terminal} = \mathcal{E} - Ir = 12\text{V} - (2\text{A})(0.5\Omega) = 11\text{V}$$

$$P_{\mathcal{E}} = I\mathcal{E} = (2\text{A})(12\text{V}) = 24\text{W}$$