Exam I Review

Lectures 1-7

Forces, Fields, Energy and Potential

$\vec{F} = k\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12}$	$\vec{F} = q\vec{E}$	$\vec{E} = k\frac{q}{r^2}\hat{r}$
$\Delta U = -\int \vec{F}\cdot d\vec{s}$		$\Delta V = -\int \vec{E}\cdot d\vec{s}$
$F_x = -\frac{\partial U}{\partial x}$		$E_x = -\frac{\partial V}{\partial x}$
$U = k\frac{q_1 q_2}{r_{12}}$	$U = qV$	$V = k\frac{q}{r}$

Forces, Fields, Energy and Potential
Continuous Forms

$\vec{F} = \int k\frac{dq_1 q_2}{r_{12}^2}\hat{r}_{12}$	$\vec{F} = q\vec{E}$	$\vec{E} = \int k\frac{dq}{r^2}\hat{r}$
$\Delta U = -\int \vec{F}\cdot d\vec{s}$		$\Delta V = -\int \vec{E}\cdot d\vec{s}$
$F_x = -\frac{\partial U}{\partial x}$		$E_x = -\frac{\partial V}{\partial x}$
$U = \int k\frac{dq_1 q_2}{r_{12}}$	$U = qV$	$V = \int k\frac{dq}{r}$

Forces, Fields, Energy and Potential
Continuous Forms

$\vec{F} = \int k \frac{dq_1 q_2}{r_{12}^2} \hat{r}_{12}$ $\vec{E} = \int k \frac{dq}{r^2} \hat{r}$ $U = \int k \frac{dq_1 q_2}{r_{12}}$ $V = \int k \frac{dq}{r}$

Procedure:
- Find dq
- Find r (and $\hat{r}$ for $\vec{F}$ or $\vec{E}$)
- Substitute into integral
- Solve integral

Forces, Fields, Energy and Potential
Continuous Forms

$\vec{F} = \int k \frac{dq_1 q_2}{r_{12}^2} \hat{r}_{12}$ $\vec{E} = \int k \frac{dq}{r^2} \hat{r}$ $U = \int k \frac{dq_1 q_2}{r_{12}}$ $V = \int k \frac{dq}{r}$

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dq examples

Line of charge:

$$dq = \lambda dx = \frac{Q}{L} dx$$

Arc of charge:

$$dq = \lambda r d\phi = \frac{Q}{r\theta} r d\phi = \frac{Q}{\theta} d\phi$$

Forces, Fields, Energy and Potential
Continuous Forms

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Procedure:
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r examples

Line of charge

$$\vec{r} = r_x \hat{\imath} + r_y \hat{\jmath}$$

$$r = \sqrt{r_x^2 + r_y^2}$$

$$\hat{r} = \frac{\vec{r}}{r} = \frac{r_x}{\sqrt{r_x^2 + r_y^2}} \hat{\imath} + \frac{r_y}{\sqrt{r_x^2 + r_y^2}} \hat{\jmath}$$

Forces, Fields, Energy and Potential
Continuous Forms

$\vec{F} = \int k \frac{dq_1 q_2}{r_{12}^2} \hat{r}_{12}$ $\vec{E} = \int k \frac{dq}{r^2} \hat{r}$ $U = \int k \frac{dq_1 q_2}{r_{12}}$ $V = \int k \frac{dq}{r}$

Procedure:
- Find dq
- Find r (and $\hat{r}$ for $\vec{F}$ or $\vec{E}$)
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r examples

Arc of charge

$$\vec{r} = -a\cos\phi\,\hat{\imath} - a\sin\phi\,\hat{\jmath}$$

$$r = a$$

$$\hat{r} = -\cos\phi\,\hat{\imath} - \sin\phi\,\hat{\jmath}$$

(Assumption: finding $\vec{F}$ or $\vec{E}$ at origin)

Finding Equilibria for Charges Constrained to Line

Procedure:
- Consider all possible regions
- For each region, express location with a variable
- Express forces in terms of variable
- Set total force on object to zero
- Solve for location
- Accept solution only if in selected region
- Determine if stable or unstable

Finding Equilibria for Charges Constrained to Line

Procedure:
- Determine if stable or unstable
  - Consider small displacements from equilibrium
  - If net force is towards equilibrium, then stable
  - If net force is away from equilibrium, then unstable

Motion in a Uniform Electric Field

Find $\vec{a}$ and apply Newton's equations of motion for constant acceleration from Physics I.

$$\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m}$$

$$x = x_0 + v_0 t + \frac{1}{2} a_x t^2 \qquad y = y_0 + v_0 t + \frac{1}{2} a_y t^2$$

$$v_x = v_{x0} + a_x t \qquad v_y = v_{y0} + a_y t$$

$$v_x^2 = v_{x0}^2 + 2a_x \Delta x \qquad v_y^2 = v_{y0}^2 + 2a_y \Delta y$$

Gauss's Law

Procedure:
- Divide into distinct regions
- Write out law, $\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}$
- Sketch Gaussian surface
- Factor out E and substitute for Gaussian area
- Substitute for enclosed charge
- In non-conducting regions, solve for E and add direction
- In conducting regions, solve for surface charge

Gauss's Law

Procedure:
- Factor out E and substitute for Gaussian area

Areas:
- Spherical Symmetry
  - $E(4\pi r^2) =$
- Cylindrial Symmetry
  - $E(2\pi rL) =$
- Planar Symmetry
  - $E(2A) =$

Gauss's Law

Procedure:
- In conducting regions, solve for surface charge

Inner Surface:
$$Q_{\text{inner}} = -\sum q_{\text{other enclosed}}$$
Outer Surface:
$$Q_{\text{outer}} = Q_{\text{conductor}} - Q_{\text{inner}}$$

Dipoles in Uniform Field

$\vec{p} = q\vec{d}$ (from – to +)

$$\vec{F} = 0$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$U = -\vec{p} \cdot \vec{E}$$

Finding Capacitance

Procedure:
- Find $\vec{E}$ using Gauss's Law $\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}$
- Find V using $V = |\Delta V| = \left|-\int \vec{E} \cdot d\vec{s}\right|$
- Find C using $C = \left|\frac{Q}{V}\right|$

Capacitors in Circuits

In Series	In Parallel
$C_T = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}$	$C_T = C_1 + C_2$
$V_T = V_1 + V_2$	$V_T = V_1 = V_2$
$Q_T = Q_1 = Q_2$	$Q_T = Q_1 + Q_2$

Capacitors in Circuits

In Series	In Parallel
$C_T = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}$	$C_T = C_1 + C_2$
$V_T = V_1 + V_2$	$V_T = V_1 = V_2$
$Q_T = Q_1 = Q_2$	$Q_T = Q_1 + Q_2$

Problems involving circuits: Use these equations and $C = \frac{Q}{V}$

Capacitors in Circuits

Additional topics:
- What forces are there when two objects are brought near each other? Consider both charged and neutral and both conductors and insulators.
- What forces are on a charged object in an electric field?
- How does the force, field, energy or potential change as an object is moved from one location to another in the presence of a field or other charges?