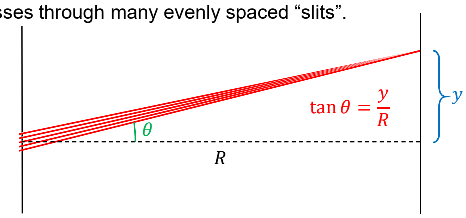
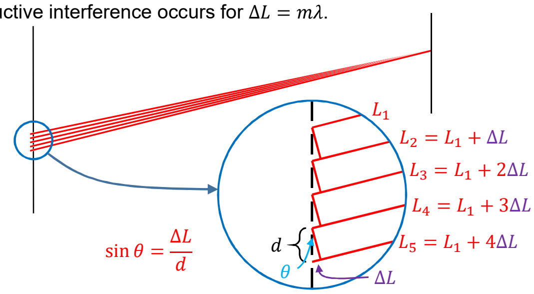
Diffraction Gratings

Light passes through many evenly spaced “slits”.



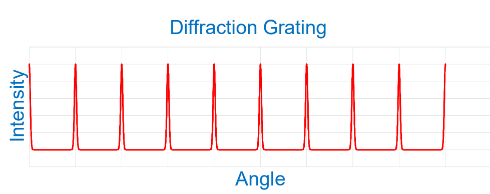
Diffraction Gratings

Constructive interference occurs for $\Delta L = m\lambda$.



Diffraction Gratings

Interference patterns due to many slits have evenly spaced narrow intense peaks.



Diffraction Gratings

Any set of evenly spaced lines that act as light sources in phase may be used to produce diffraction pattern.

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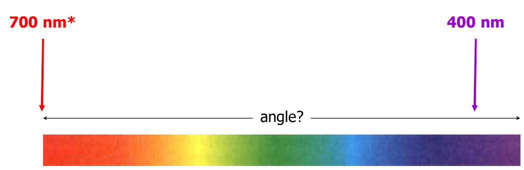
A common device is a transparent plate with evenly spaced lines scribed on the plate, usually characterized by lines per distance $\left(\frac{1}{d}\right)$ rather than by distance per line (d). This is a diffraction grating.

For example, a grating with $400\,\frac{\text{lines}}{mm}$ has a "slit" separation

$$d = \frac{1}{400}mm = 2.5\mu m.$$

Example: the wavelengths of visible light are from approximately 400 nm (violet) to 700 nm (red). Find the angular width of the first-order visible spectrum produced by a plane grating with 600 slits per millimeter when white light falls normally on the grating.



Example: the wavelengths of visible light are from approximately 400 nm (violet) to 700 nm (red). Find the angular width of the first-order visible spectrum produced by a plane grating with 600 slits per millimeter when white light falls normally on the grating.

Resolving Power

If the minimum wavelength difference that can be resolved by a diffraction grating is given by $\Delta\lambda$

then the resolving power is

$$R = \frac{\lambda}{\Delta\lambda}$$

where λ is the average of the two wavelengths.

It can be shown that

$$R = Nm$$

where N is the number of slits and m refers to the mth-order maxima.

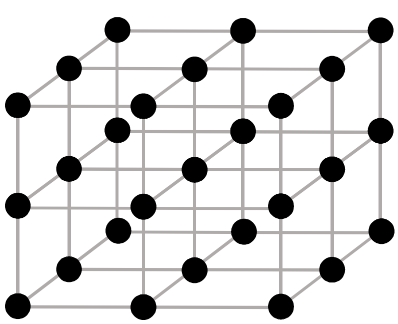
Example: Light from mercury vapor lamps contain several wavelengths in the visible region of the spectrum including two yellow lines at 577 and 579 nm. What must be the resolving power of a grating to distinguish these two lines?

Example: how many lines of the grating must be illuminated if these two wavelengths are to be resolved in the first-order spectrum?

X-ray Diffraction

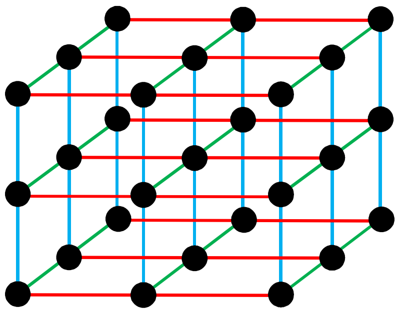
A grid of regularly spaced objects may also be used to produce a diffraction pattern.

- X-ray diffraction uses atoms in a crystal to create a diffraction pattern.
- Practitioners work backwards from the pattern to deduce the arrangement of atoms in the crystal.
- Multiple patterns arise from the regular spacing along different lines in the crystal.



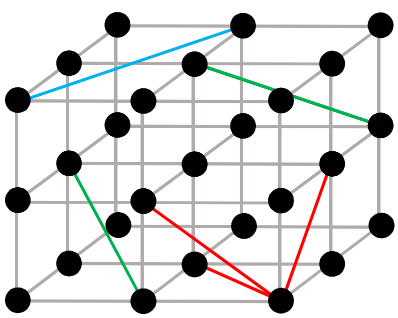
Consider atoms in a crystal, as illustrated.

Each regular spacing can produce a diffraction pattern.



Consider atoms in a crystal, as illustrated.

Spacings:
Right-Left
Up-Down
In-Out



Even if all three directions are the same there are still other spacings.

Spacings:
Adjacent
Face Diagonal
Cube Diagonal
Two Square Diagonal

Finding Maxima and Minima

Double slit maxima (constructive interference)

$$\frac{m\lambda}{d} = \sin\theta$$

Single slit minima (destructive interference)

$$\frac{m\lambda}{a} = \sin\theta$$

Diffraction grating maxima (constructive interference)

$$\frac{m\lambda}{d} = \sin\theta$$

Intensity

Maximum intensity corresponds to constructive interference (bright fringes).

Double slit

Phase difference:

$$\phi = 2\pi \left(\frac{\Delta L}{\lambda}\right)$$

Intensity:

$$I = I_0 \cos^2\left(\frac{\phi}{2}\right)$$

Single slit

Phase difference:

$$\beta = \frac{2\pi}{\lambda} a \sin\theta$$

Intensity:

$$I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2}\right]^2$$