Maxwell's Equations in English

Gauss's Law: $$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}$$

Gauss's Law for B: $$\oint \vec{B} \cdot d\vec{A} = 0$$

Faraday's Law: $$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

Ampere-Maxwell Law: $$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{enc}} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

Maxwell's Equations in English

Left side is the field that is being produced.

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Changing electric flux produces magnetic field.

Self-Propagating Fields

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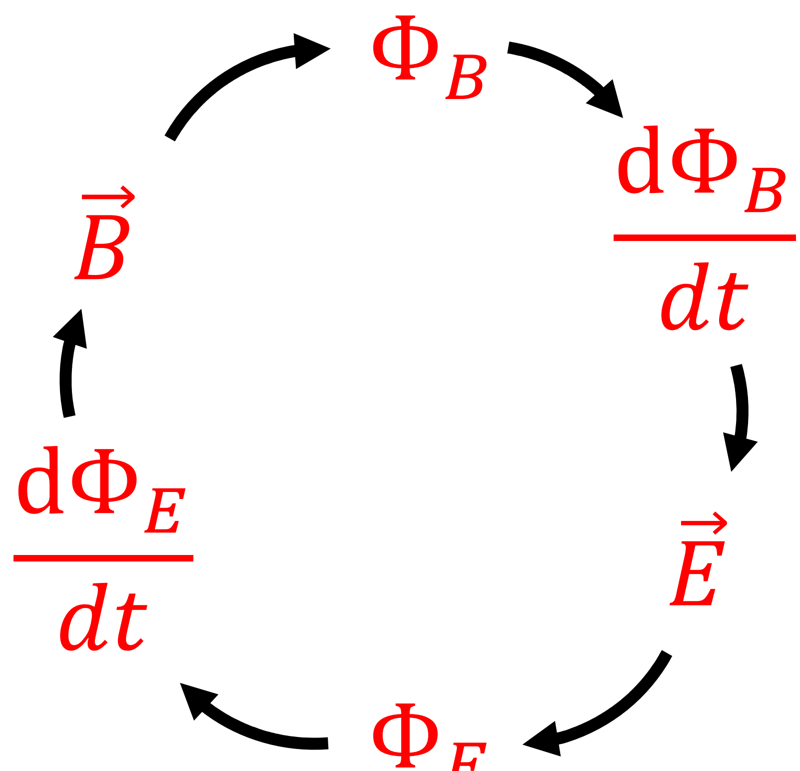
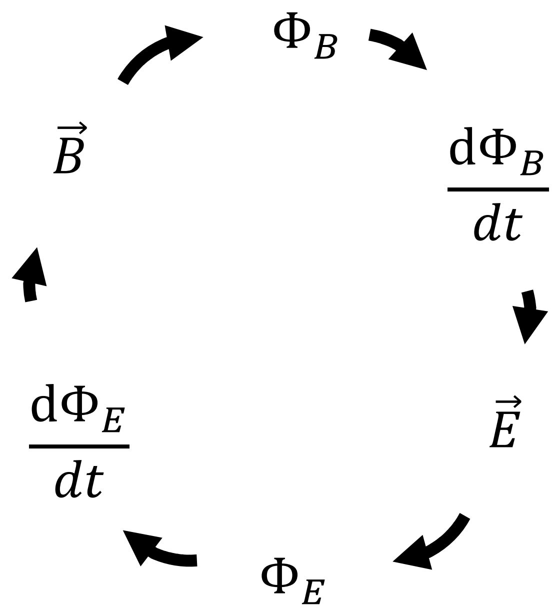


Self-Propagating Fields



Leads to:

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_y}{\partial t^2} \quad \text{and} \quad \frac{\partial^2 B_z}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 B_z}{\partial t^2}$$

With $\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$

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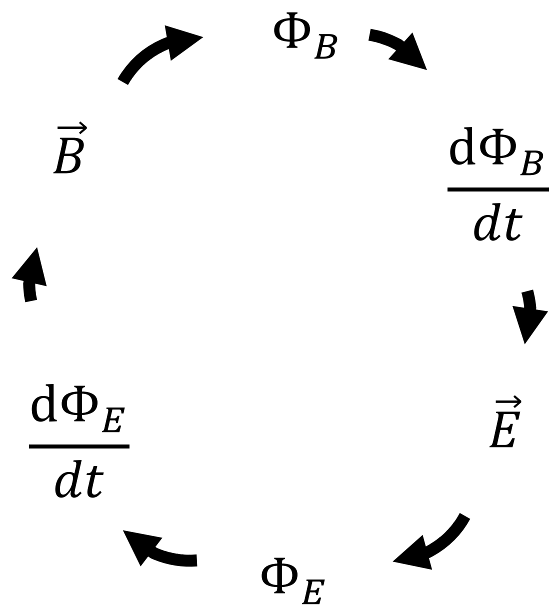

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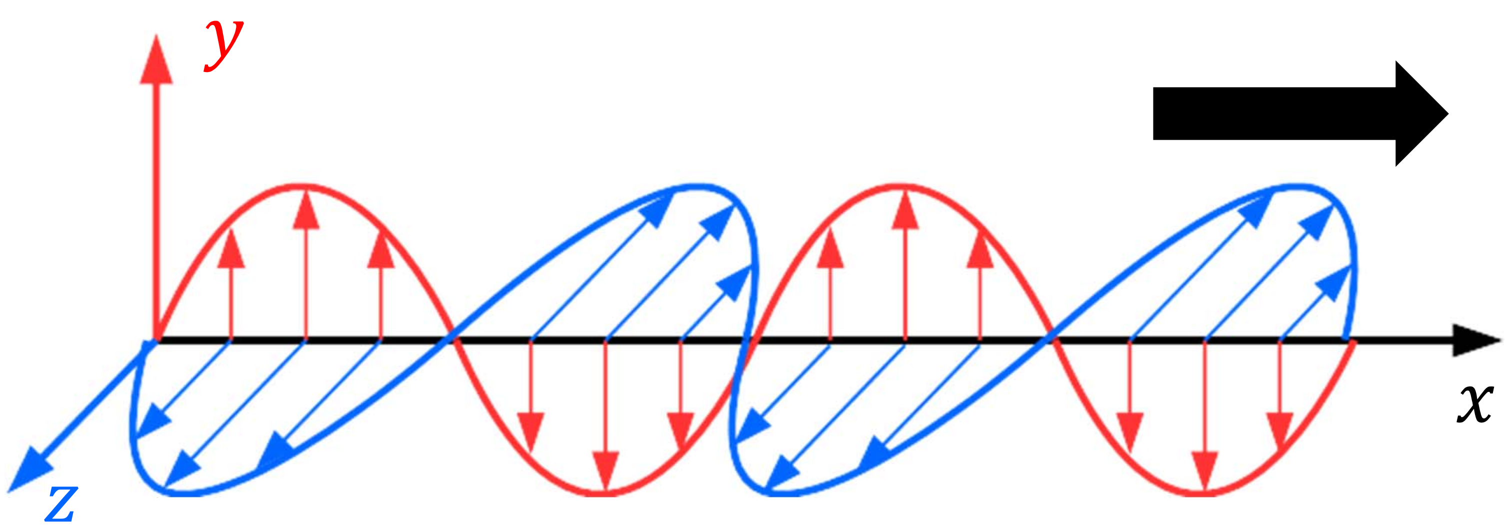
Equations describing waves.

Self-Propagating Fields – Electromagnetic Waves

Solutions:

$$\vec{E} = E_{\max} \sin(kx - \omega t)\,\hat{j}$$
$$\vec{B} = B_{\max} \sin(kx - \omega t)\,\hat{k}$$



Electromagnetic Waves

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$$\vec{E} = E_{\max} \sin(kx - \omega t)\,\hat{j}$$
$$\vec{B} = B_{\max} \sin(kx - \omega t)\hat{k}$$

Wavelength, λ, distance for complete oscillation:

$$k\lambda = 2\pi$$

Wave number, $k = \frac{2\pi}{\lambda}$

Period, T, time for complete oscillation:

$$\omega T = 2\pi$$

Angular frequency, $\omega = \frac{2\pi}{T}$

Electromagnetic Waves

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$$\vec{B} = B_{\max} \sin(kx - \omega t)\hat{k}$$

$$k = \frac{2\pi}{\lambda} \qquad \omega = \frac{2\pi}{T}$$

Frequency, f, oscillations per time:

$$f = \frac{1}{T}$$
$$f = \frac{\omega}{2\pi}$$

Electromagnetic Waves

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Wave speed, c, distance per time:

$$c = \lambda f = \frac{\omega}{k} = \frac{E_{\text{max}}}{B_{\text{max}}} = \frac{E}{B} = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

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In material, the speed of electromagnetic waves (light) may be slower. $v = \frac{1}{\sqrt{\mu\epsilon}}$

Energy Carried by Electromagnetic Waves

Poynting Vector, $\vec{S}$

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

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- Energy current density
- (Energy per area) per time
- Power per area
- Units are $\left[\frac{\mathrm{J}}{\mathrm{m^2 s}}\right] = \left[\frac{\mathrm{W}}{\mathrm{m^2}}\right]$.
- Direction of $\vec{S}$ is direction of wave propagation.

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There is energy in a region with electric and magnetic fields. (Recall: energy stored in a capacitor may be interpreted as being stored in the electric field.)

Energy Carried by Electromagnetic Waves

Poynting Vector, $\vec{S}$

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

The average of $\vec{S}$ over an integer number of cycles is called wave intensity, I.

$$I = \langle S \rangle = \left\langle \frac{1}{\mu_0} E_{\text{max}} B_{\text{max}} \sin^2(kx - \omega t) \right\rangle$$

$$I = \frac{E_{\text{max}} B_{\text{max}}}{2\mu_0} = \frac{1}{2} c \epsilon_0 E_{\text{max}}^2 = \frac{1}{2} \frac{c B_{\text{max}}^2}{\mu_0}$$

Energy Density

$$\langle u \rangle \equiv \frac{U}{V} = \frac{U}{Ad} = \frac{U}{A(ct)} = \frac{I}{c}$$

$$\langle u \rangle = \frac{\epsilon_0 E_{\max}^2}{2} = \frac{B_{\max}^2}{2\mu_0}$$

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Radiation Pressure

IF radiation is completely absorbed,

$$\langle P_{\text{rad}} \rangle = \frac{F}{A} = \frac{U}{V} = \langle u \rangle = \frac{I}{c}$$

IF radiation is completely reflected,

$$\langle P_{\text{rad}} \rangle = \frac{2I}{c}$$

Why is there a difference of a factor of 2?

Example: A radio station broadcasts with a total average power of 50kW. (a) Determine the electric and magnetic fields measured at a distance of 20km from the transmitter. (b) How much energy is absorbed in a day by an antenna of area, $0.25\ \text{m}^2$ facing the transmitter 20km from the transmitter? (c) Determine the force exerted on the antenna by the electromagnetic wave.