Today’s agenda:

Induced Electric Fields.

You must understand how a changing magnetic flux induces an electric field, and be able to calculate induced electric fields.

Eddy Currents.

You must understand how induced electric fields give rise to circulating currents called “eddy currents.”

Displacement Current and Maxwell’s Equations.

Displacement currents explain how current can flow “through” a capacitor, and how a time-varying electric field can induce a magnetic field.

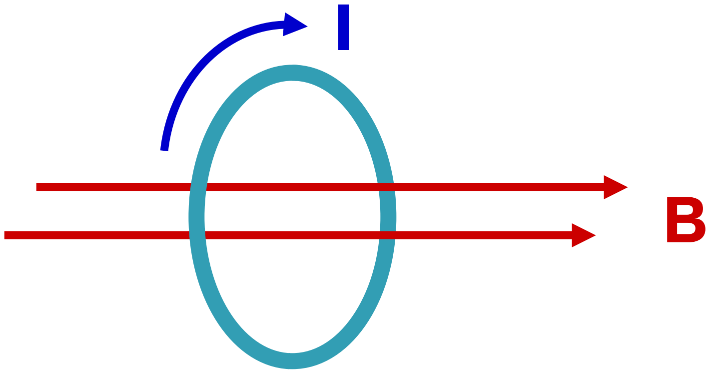
Back emf.

A current in a coil of wire produces an emf that opposes the original current.

Induced Electric Fields

Recall (lecture 19):

- changing magnetic flux through wire loop produces induced emf



Faraday's law $\varepsilon = -N \frac{d\Phi_B}{dt}$

Question:

What force makes the charges move around the loop?

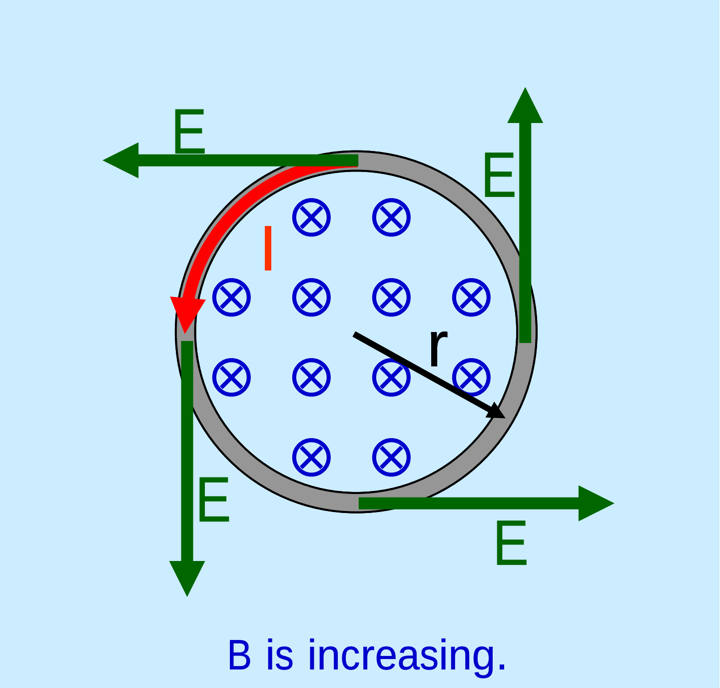
- cannot be magnetic force because
  (i) magnetic force does not accelerate particles
  (ii) wire loop does not even have to be in B-field

Induced Electric Fields

Solution:

- there must be a **tangential electric field** around the loop
- voltage is integral of electric field along path
- Faraday's law turns into

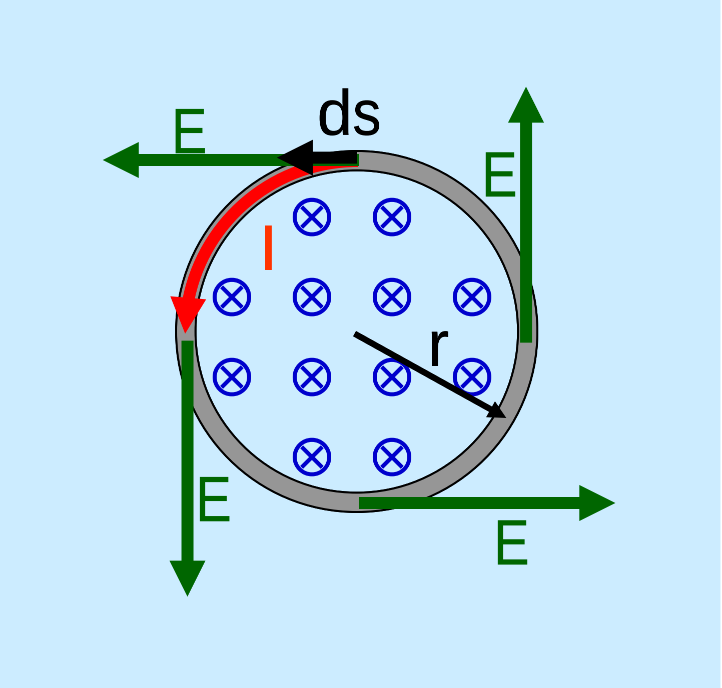
$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$



This electric field exits in space even if there is not an actual wire loop!

Work done by Induced Electric Fields

- induced electric field exerts force qE on charged particle
- instantaneous displacement is parallel to this force



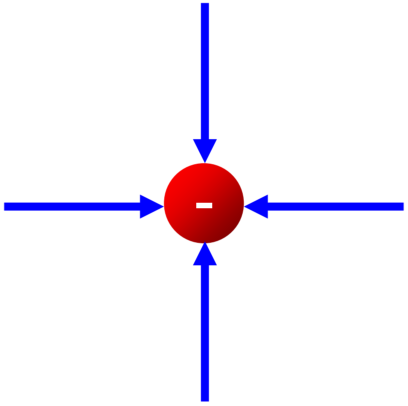
Work done by electric field in moving charge once around the loop:

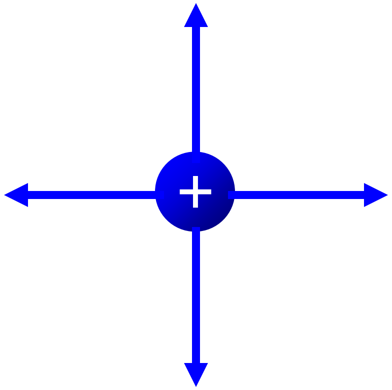
$$W = \oint \vec{F} \cdot d\vec{s} = q\oint \vec{E} \cdot d\vec{s} = qE\oint ds = qE(2\pi r) \neq 0$$

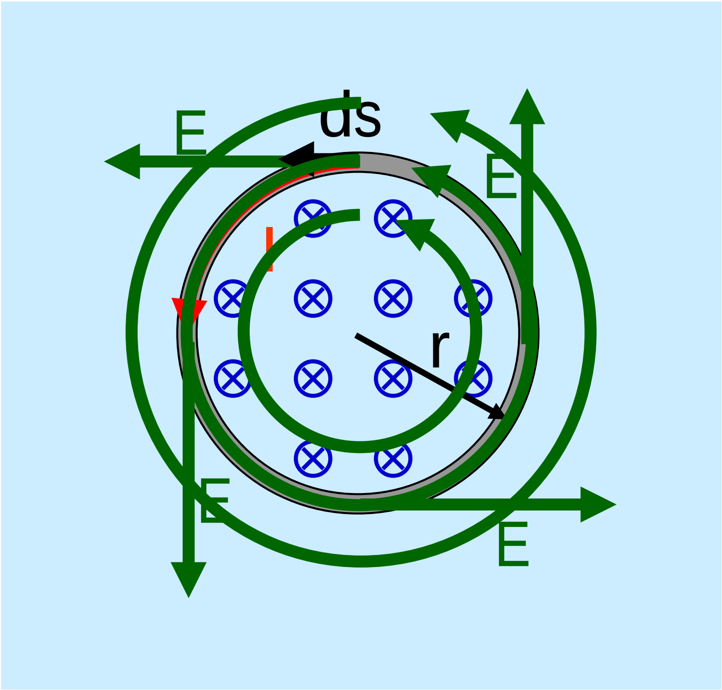
- work depends on path
- force of induced electric field is **not conservative**
- one **cannot** define potential energy for this force

Different character of Coulomb and induced electric fields reflected in **electric field line geometry:**

$$E = k\frac{|q|}{r^2}$$, away from +







Coulomb: field lines begin and end at source charges

Induced: field lines form continuous, closed loops.

Induced Electric Fields: summary of key ideas

A changing magnetic flux induces an electric field, as given by Faraday's Law:

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

This is a **different** manifestation of the electric field than the one you are familiar with; it is not the electrostatic field caused by the presence of stationary charged particles.

Unlike the electrostatic electric field, this "new" electric field is nonconservative.

$$\vec{E} \Leftrightarrow \vec{E}_C + \vec{E}_{NC}$$

"conservative," or "Coulomb" ($\vec{E}_C$) "nonconservative" ($\vec{E}_{NC}$)

It is better to say that there is an electric field, as described by Maxwell's equations. We saw in lecture 17 that what an observer measures for the magnetic field depends on the motion of the observer relative to the source of the field. We see here that the same is true for the electric field. There aren't really two different kinds of electric fields. There is just an electric field, which seems to "behave" differently depending on the relative motion of the observer and source of the field.

Direction of Induced Electric Fields

The direction of $\vec{E}$ is in the direction a positively charged particle would be accelerated by the changing flux.

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

Hint:

Imagine a wire loop through the point of interest.
Use Lenz's Law to determine the direction the changing magnetic flux would cause a current to flow.
That is the direction of $\vec{E}$.

Example—to be worked at the blackboard in lecture

A long thin solenoid has 500 turns per meter and a radius of 3.0 cm. The current is decreasing at a steady rate of 50 A/s. What is the magnitude of the induced electric field near the center of the solenoid 1.0 cm from the axis of the solenoid?

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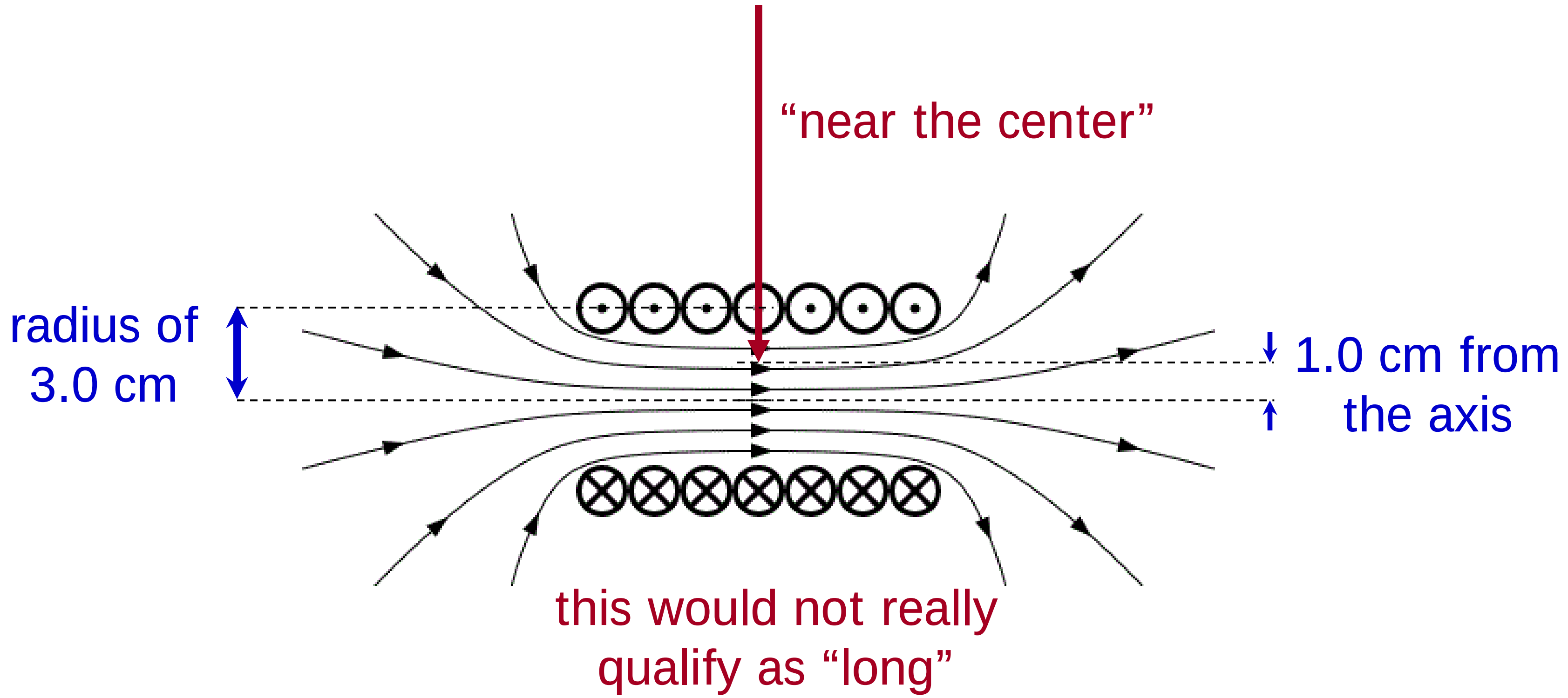

Image from: http://commons.wikimedia.org/wiki/User:Geek3/Gallery

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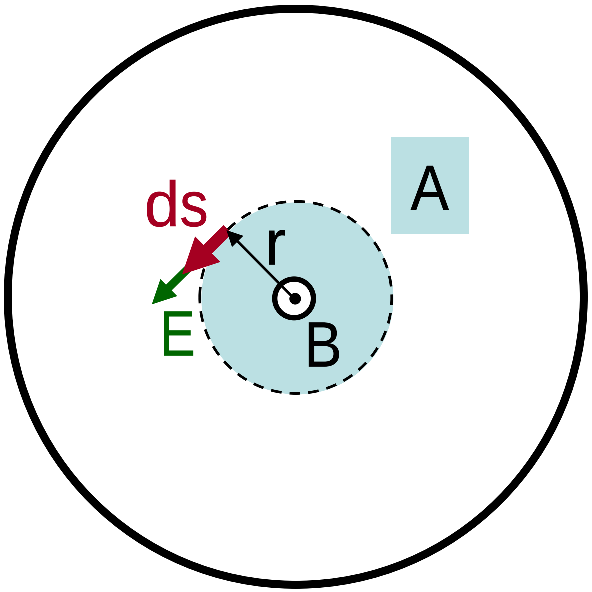


B is decreasing

$$\oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

$$E(2\pi r) = \left| -\frac{d\Phi_B}{dt} \right| = \left| \frac{d(BA)}{dt} \right| = A\left| \frac{dB}{dt} \right|$$

$$E(2\pi r) = \pi r^2 \left| \frac{d(\mu_0 n I)}{dt} \right| = \pi r^2 \mu_0 n \left| \frac{d(I)}{dt} \right|$$

$$E = \frac{r}{2}\mu_0 n \left| \frac{dI}{dt} \right|$$

$$E = 1.57 x 10^{-4}\ \frac{V}{m}$$

Some old and new applications of Faraday's Law

- Magnetic Tape Readers
- Phonograph Cartridges
- Electric Guitar Pickup Coils
- Ground Fault Interruptors
- Alternators
- Generators
- Transformers
- Electric Motors

Application of Faraday's Law (MAE Plasma Lab)

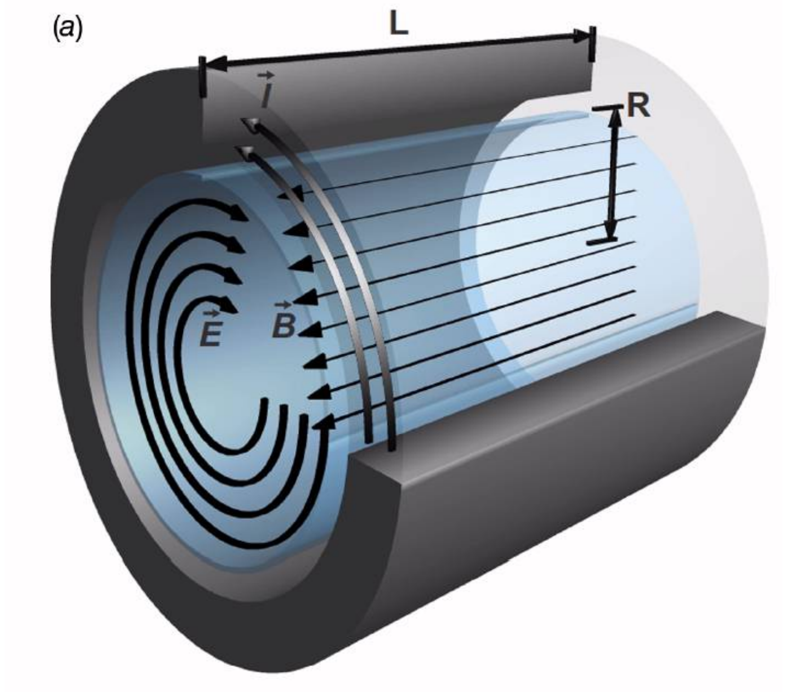
From Meeks and Rovey, Phys. Plasmas **19**, 052505 (2012); doi: 10.1063/1.4717731. Online at http://dx.doi.org/10.1063/1.4717731.T

"The theta-pinch concept is one of the most widely used inductive plasma source designs ever developed. It has established a workhorse reputation within many research circles, including thin films and material surface processing, fusion, high-power space propulsion, and academia, filling the role of not only a simply constructed plasma source but also that of a key component...

"Theta-pinch devices utilize relatively simple coil geometry to induce electromagnetic fields and create plasma...

"This process is illustrated in Figure 1(a), which shows a cut-away of typical theta-pinch operation during an initial current rise.

"FIG. 1. (a) Ideal theta-pinch field topology for an increasing current, I."



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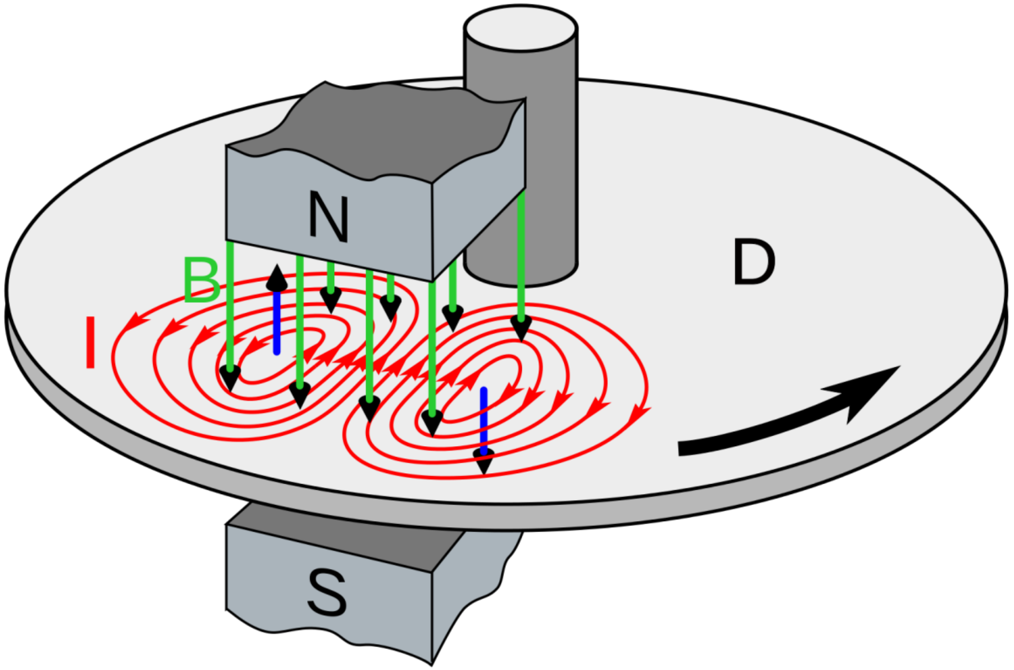
Eddy Currents

General idea of induction:

- relative motion between a conductor and a magnetic field leads to induced currents

- in large masses of metal these currents can circulate and are called “eddy currents”



Eddy Currents

Eddy currents give rise to magnetic fields that oppose the motion (Lentz's rule)

Applications:

- metal detectors
- coin recognition systems
- security scanners
- circular saw brakes
- roller coaster brakes

Eddy currents heat the material ($P = I^2R$)

- unwanted energy losses
- can be used in heating applications

Eddy Current Demos

cylinders falling through a tube

magnetic “guillotine”

hopping coil

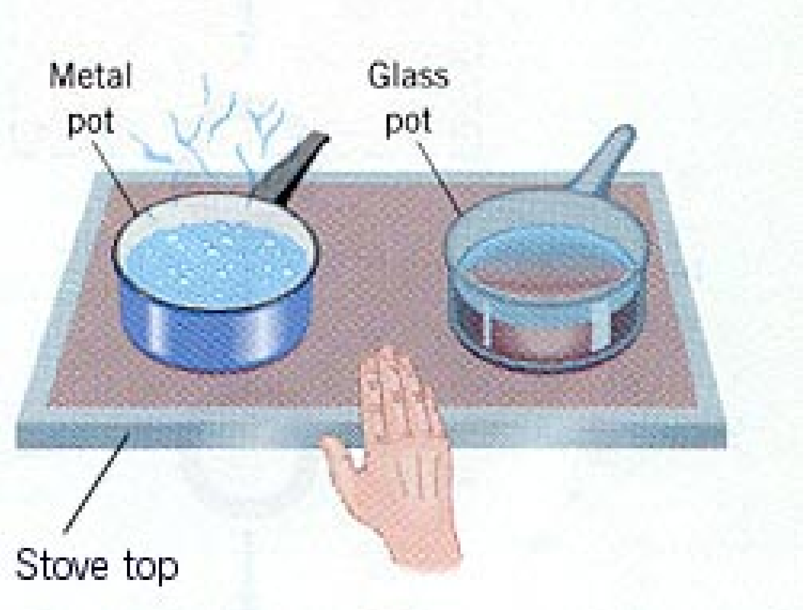
coil launcher

magnetic flasher

Example: Induction Stove

An ac current in a coil in the stove top produces a changing magnetic field at the bottom of a metal pan.

The changing magnetic field gives rise to a current in the bottom of the pan.



Because the pan has resistance, the current heats the pan. If the coil in the stove has low resistance it doesn't get hot but the pan does.

An insulator won't heat up on an induction stove.

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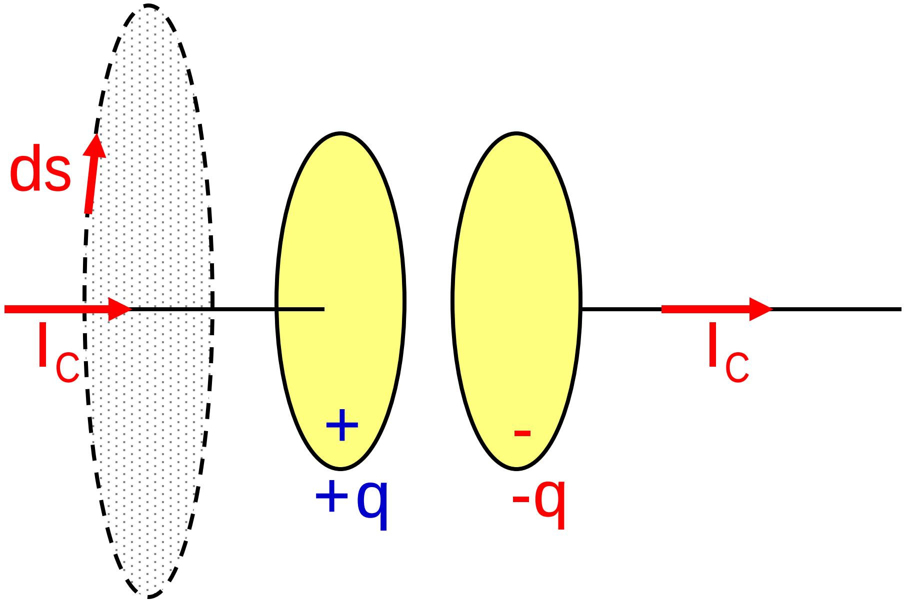
Displacement Current

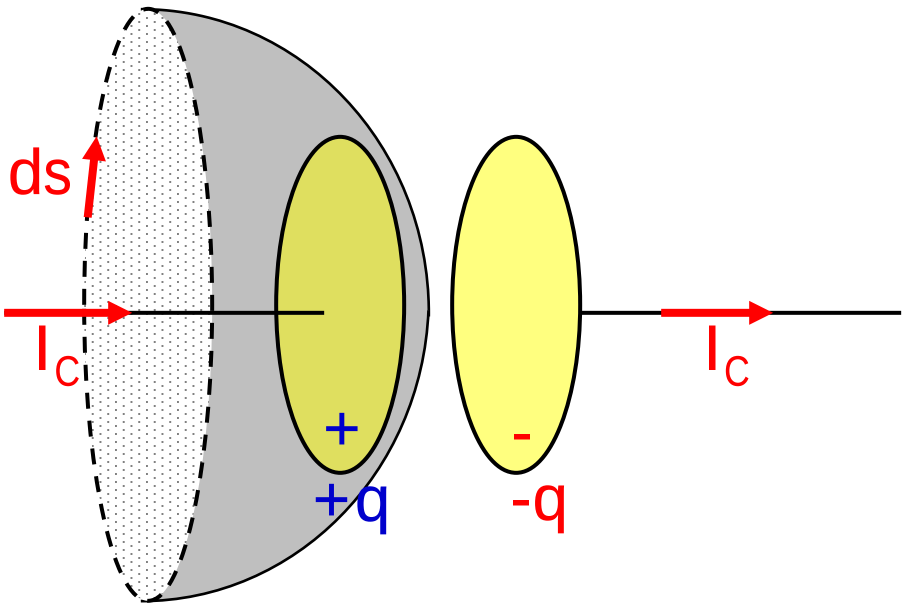
Apply Ampere's Law to a charging capacitor.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_C$$



Ampere's law is **universal**:
Shape of surface shouldn't matter, as long as "path" is the same

"Soup bowl" surface, with the + plate resting near the bottom of the bowl.



Ampere's law:

$$\oint \vec{B} \cdot d\vec{s} = I_{encl} = 0$$

two different surfaces give:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_C \qquad \oint \vec{B} \cdot d\vec{s} = 0$$

Contradiction! (The equation on the right is actually incorrect, and the equation on the left is incomplete.)

How to fix this?

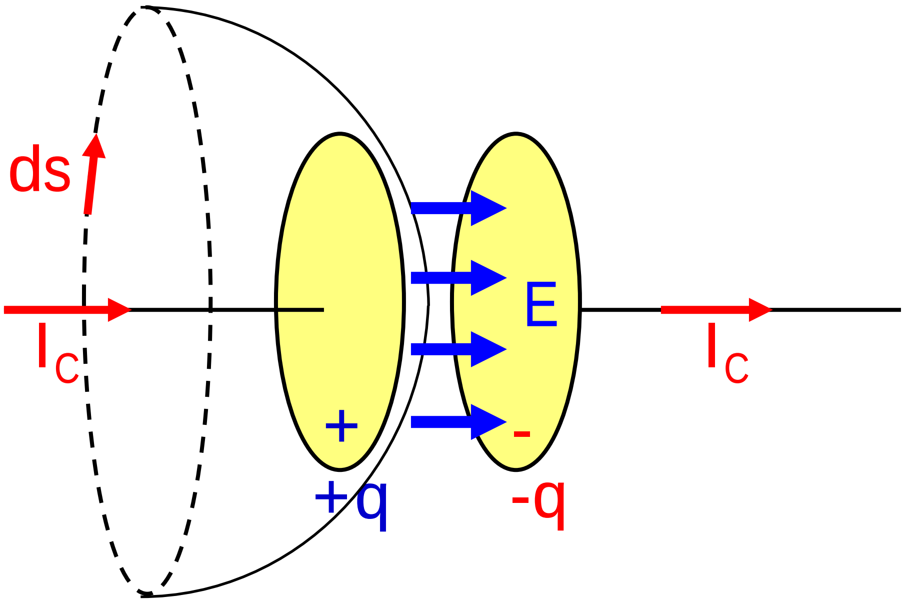
- Ampere's law (as used so far) must be incomplete
- charging capacitor produces changing electric flux between plates

$$q = C\Delta V = \left(\kappa\varepsilon_0 \frac{A}{d}\right)(Ed) = \kappa\varepsilon_0 EA = \kappa\varepsilon_0 \Phi_E$$

Changing electric flux acts like current

$$\frac{dq}{dt} = \frac{d}{dt}\left(\kappa\varepsilon_0 \Phi_E\right) = \underbrace{\kappa\varepsilon_0 \frac{d}{dt}\left(\Phi_E\right)}$$

This term has units of current.

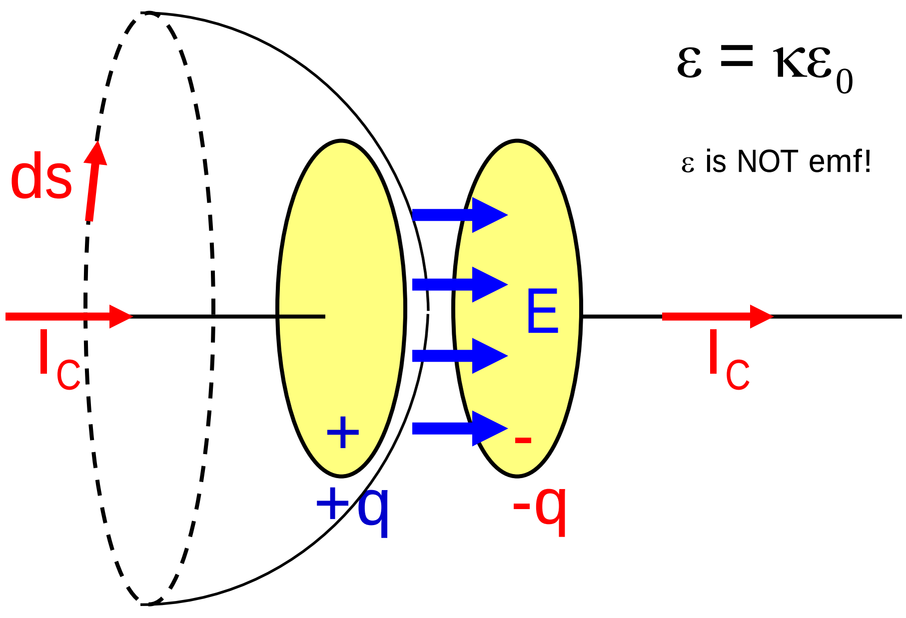


κ Is the dielectric constant of the medium in between the capacitor plates. In the diagram, with an air-filled capacitor, κ = 1.

Define a virtual current:
displacement current

$$I_D = \kappa\varepsilon_0 \frac{d}{dt}\left(\Phi_E\right).$$

changing electric flux through "bowl" surface is equivalent to current I_C through flat surface



$\varepsilon = \kappa\varepsilon_0$

ε is NOT emf!

- include displacement current in Ampere's law
- complete form of Ampere's law:

$$\oint \vec{B}\cdot d\vec{s} = \mu_0\left(I_C + I_D\right)_{encl} = \mu_0 I_{encl} + \mu_0\varepsilon\frac{d\Phi_E}{dt}.$$

Magnetic fields are produced by both conduction currents and time varying electric fields.

The Big Picture

Gauss's Law for both electricity and magnetism,

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o} \qquad \oint \vec{B} \cdot d\vec{A} = 0$$

Faraday's Law of Induction, and Ampere's Law:

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \qquad \oint \vec{B} \cdot d\vec{s} = \mu_0 I_{encl} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}.$$

- these four equations are famous **Maxwell equations** of electromagnetism
- govern all of electromagnetism

The Big Picture

Maxwell equations can also be written in differential form:

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$$

$$\vec{\nabla} \times \vec{B} = \frac{1}{c^2}\frac{d\vec{E}}{dt} + \mu_0 \vec{J}$$

Missouri S&T Society of Physics Students T-Shirt!

$$\vec{\nabla} \bullet \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla} \bullet \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

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This will not be tested on the exam.

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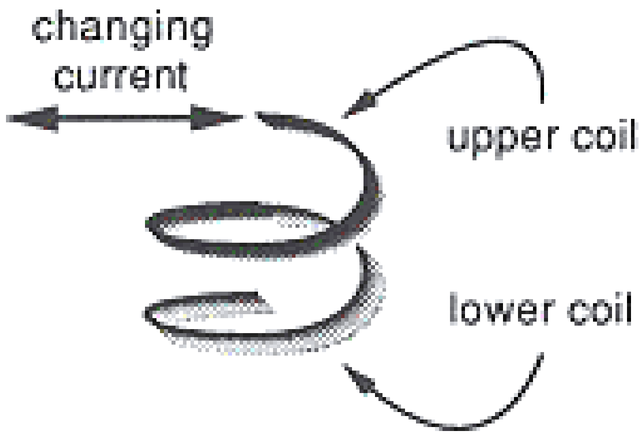
back emf (also known as “counter emf”) (if time permits)

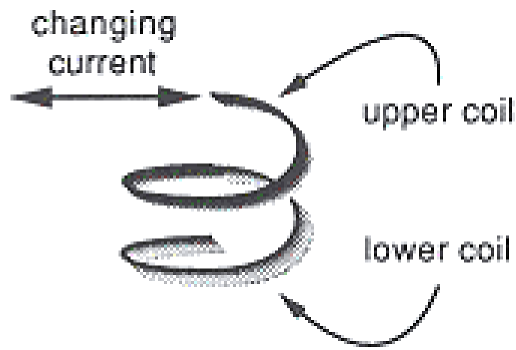
- changing magnetic field in wire produces a current
- changing current produces magnetic field, which by Lenz’s law, opposes the change in flux which produced it



Changing current in upper coil creates a changing magnetic field that induces a counter EMF in the lower coil in a direction to oppose the change of current.

http://campus.murraystate.edu/tsm/tsm118/Ch7/Ch7_4/Ch7_4.htm

The effect is “like” that of friction.



Changing current in upper coil creates a changing magnetic field that induces a counter EMF in the lower coil in a direction to oppose the change of current.

The counter emf is “like” friction that opposes the original change of current.

Motors have many coils of wire, and thus generate a large counter emf when they are running.

Good—keeps the motor from “running away.” Bad—”robs” you of energy.

If your house lights dim when an appliance starts up, that's because the appliance is drawing lots of current and not producing a counter emf.

When the appliance reaches operating speed, the counter emf reduces the current flow and the lights "undim."

Motors have design speeds their engineers expect them to run at. If the motor runs at a lower speed, there is less-than-expected counter emf, and the motor can draw more-than-expected current.

If a motor is jammed or overloaded and slows or stops, it can draw enough current to melt the windings and burn out. Or even burn up.