Maxwell's Equations in English

Gauss's Law: $\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\epsilon_0}$

Gauss's Law for B: $\oint \vec{B} \cdot d\vec{A} = 0$

Faraday's Law: $\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$

Ampere-Maxwell Law: $\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{enc}} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$

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Left side is the field that is being produced.

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Right side is the source.

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Changing magnetic flux produces electric field.

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Changing electric flux produces magnetic field.

Self-Propagating Fields

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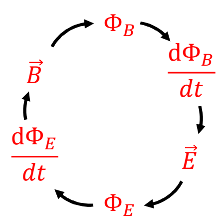
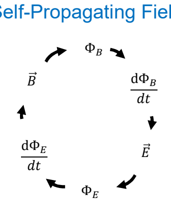


Self-Propagating Fields

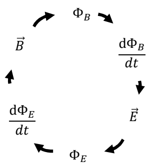


Leads to:

$$\frac{\partial^2 E_y}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_y}{\partial t^2} \quad \text{and} \quad \frac{\partial^2 B_z}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 B_z}{\partial t^2}$$

With $\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$

Self-Propagating Fields



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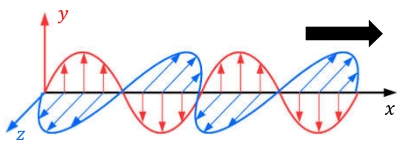
With $\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$

Equations describing waves.

Self-Propagating Fields – Electromagnetic Waves

Solutions:

$$\vec{E} = E_{\max} \sin(kx - \omega t)\,\hat{\jmath}$$
$$\vec{B} = B_{\max} \sin(kx - \omega t)\,\hat{k}$$



Electromagnetic Waves

Solutions:

$$\vec{E} = E_{\max} \sin(kx - \omega t)\,\hat{\jmath}$$
$$\vec{B} = B_{\max} \sin(kx - \omega t)\,\hat{k}$$

Wavelength, λ, distance for complete oscillation:

$$k\lambda = 2\pi$$

Wave number, $k = \frac{2\pi}{\lambda}$

Period, T, time for complete oscillation:

$$\omega T = 2\pi$$

Angular frequency, $\omega = \frac{2\pi}{T}$

Electromagnetic Waves

Solutions:

$$\vec{E} = E_{\text{max}} \sin(kx - \omega t)\,\hat{j}$$
$$\vec{B} = B_{\text{max}} \sin(kx - \omega t)\hat{k}$$

$$k = \frac{2\pi}{\lambda} \qquad \omega = \frac{2\pi}{T}$$

Frequency, f, oscillations per time:

$$f = \frac{1}{T}$$
$$f = \frac{\omega}{2\pi}$$

Electromagnetic Waves

Solutions:

$$\vec{E} = E_{\text{max}} \sin(kx - \omega t)\,\hat{j}$$
$$\vec{B} = B_{\text{max}} \sin(kx - \omega t)\hat{k}$$

$$k = \frac{2\pi}{\lambda} \qquad \omega = \frac{2\pi}{T} \qquad f = \frac{\omega}{2\pi}$$

Wave speed, c, distance per time:

$$c = \lambda f = \frac{\omega}{k} = \frac{E_{\text{max}}}{B_{\text{max}}} = \frac{E}{B} = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

Electromagnetic Waves

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In material, the speed of electromagnetic waves (light) may be slower.

$$v = \frac{1}{\sqrt{\mu \epsilon}}$$