Lorentz Force

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

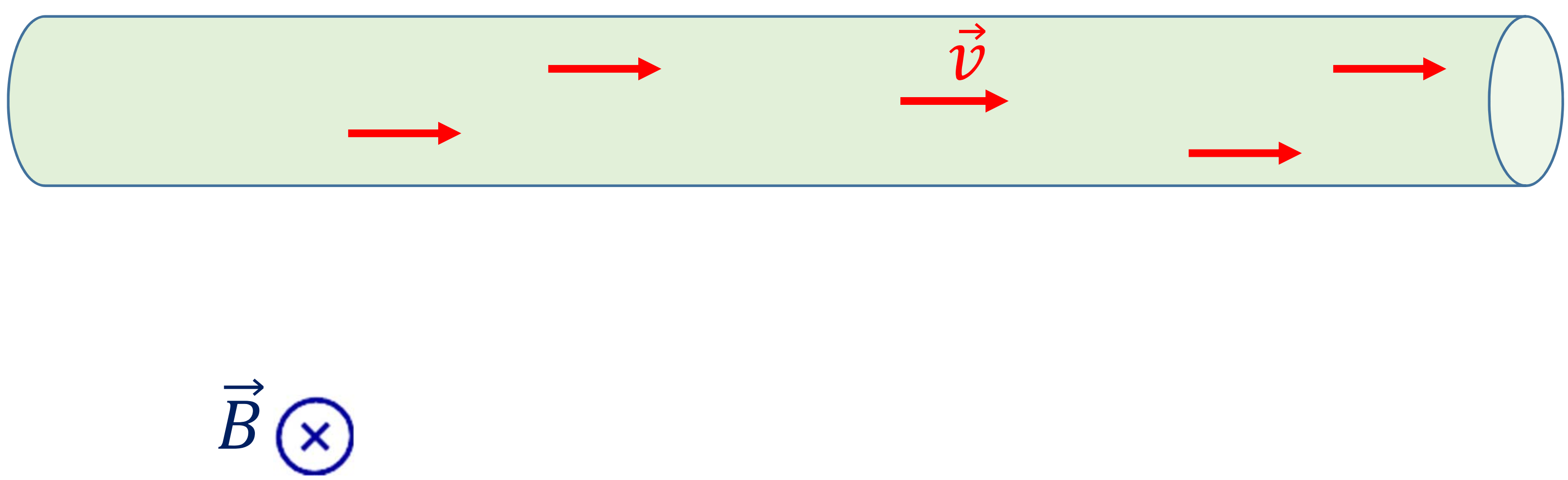
- Electric force can accelerate objects by changing their speed and/or direction.
- Electric force can do work, $W_E = \int q\vec{E} \cdot d\vec{s}$

- Magnetic force can only accelerate objects by changing their direction. (Acts perpendicular to velocity.)
- Magnetic force does not do work,

$$W_B = \int\left(q\vec{v} \times \vec{B}\right) \cdot \vec{v}dt = 0 \text{ (where } \vec{v} = \frac{d\vec{s}}{dt}\text{)}$$

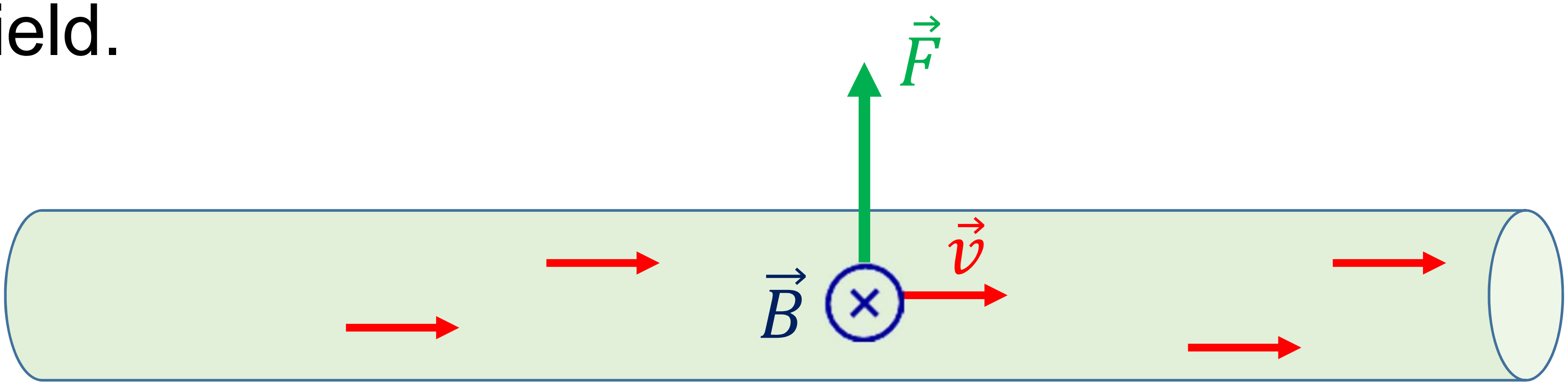
Magnetic Forces on Currents

Consider charges moving in a wire in a magnetic field.

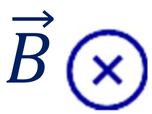


Magnetic Forces on Currents

Consider charges moving in a wire in a magnetic field.



$$\vec{F} = q\vec{v} \times \vec{B}$$



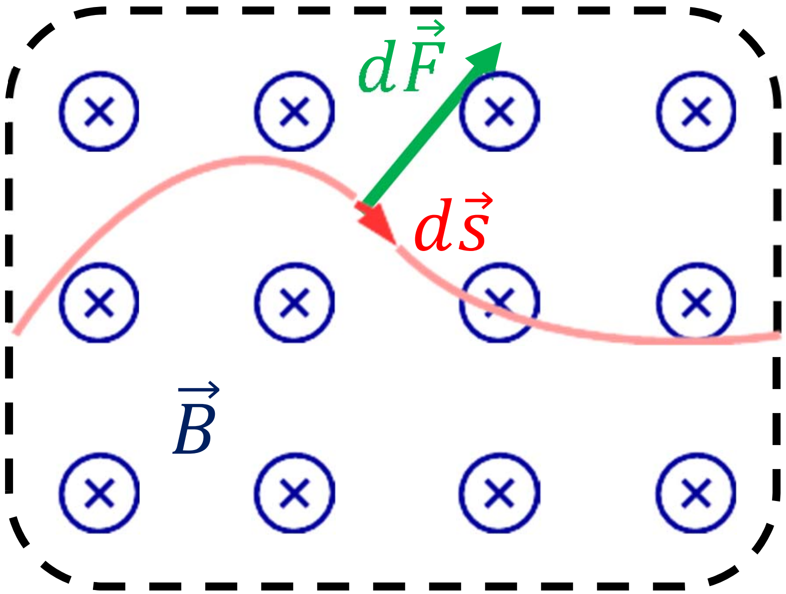
$$\vec{F} = q\frac{\Delta\vec{s}}{\Delta t} \times \vec{B}$$

$$\vec{F} = I\vec{L} \times \vec{B}$$

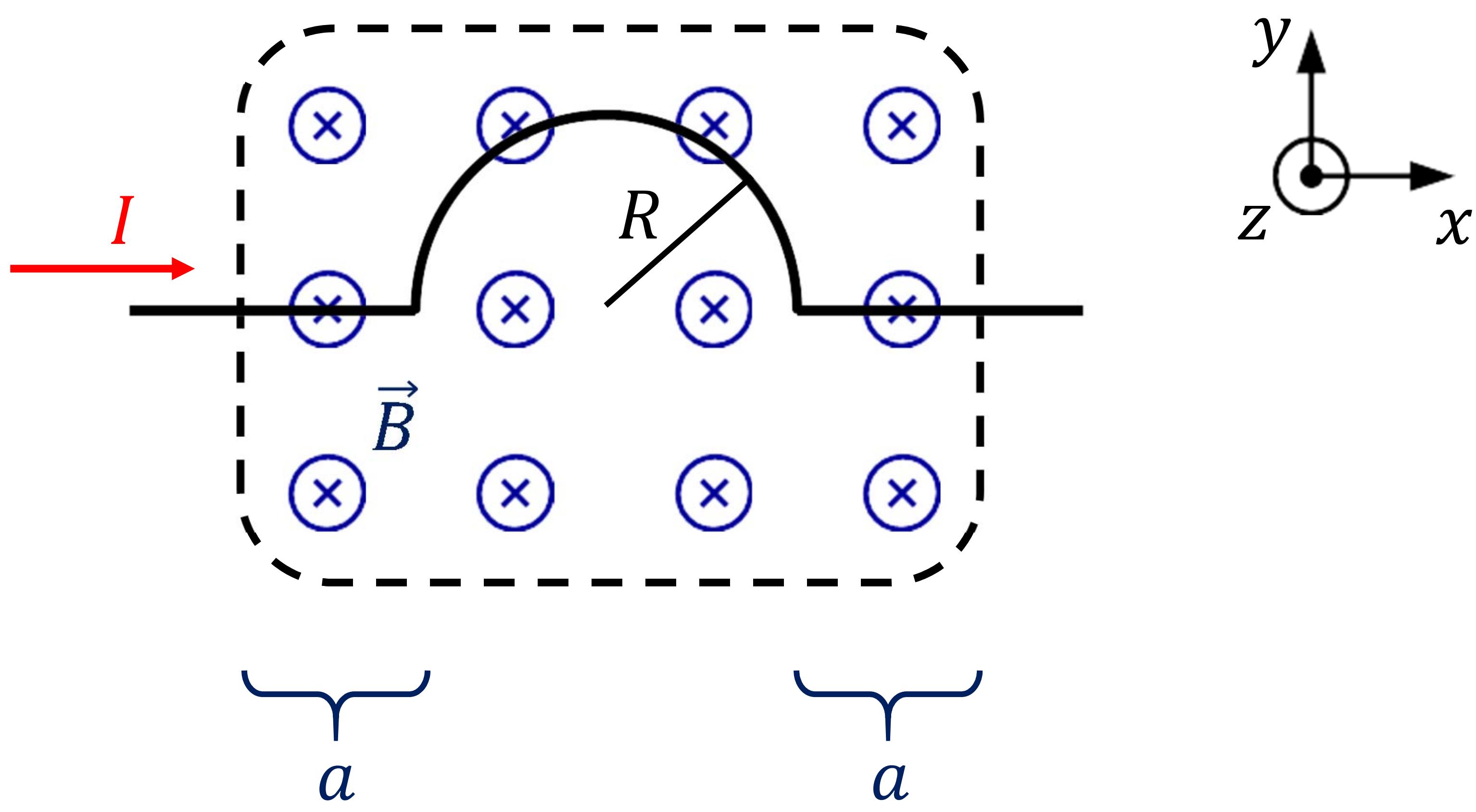
Magnetic Forces on Currents
Curved wires

$$\vec{F} = \int d\vec{F}$$

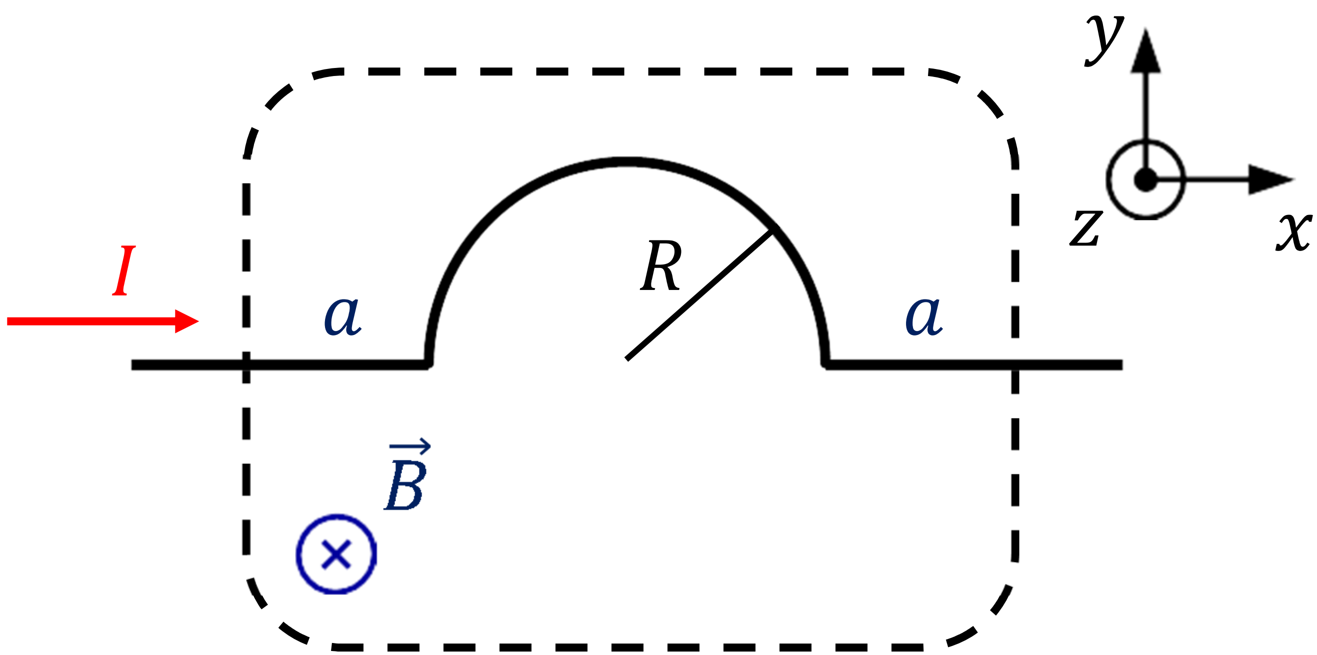
$$\vec{F} = I \int d\vec{s} \times \vec{B}$$



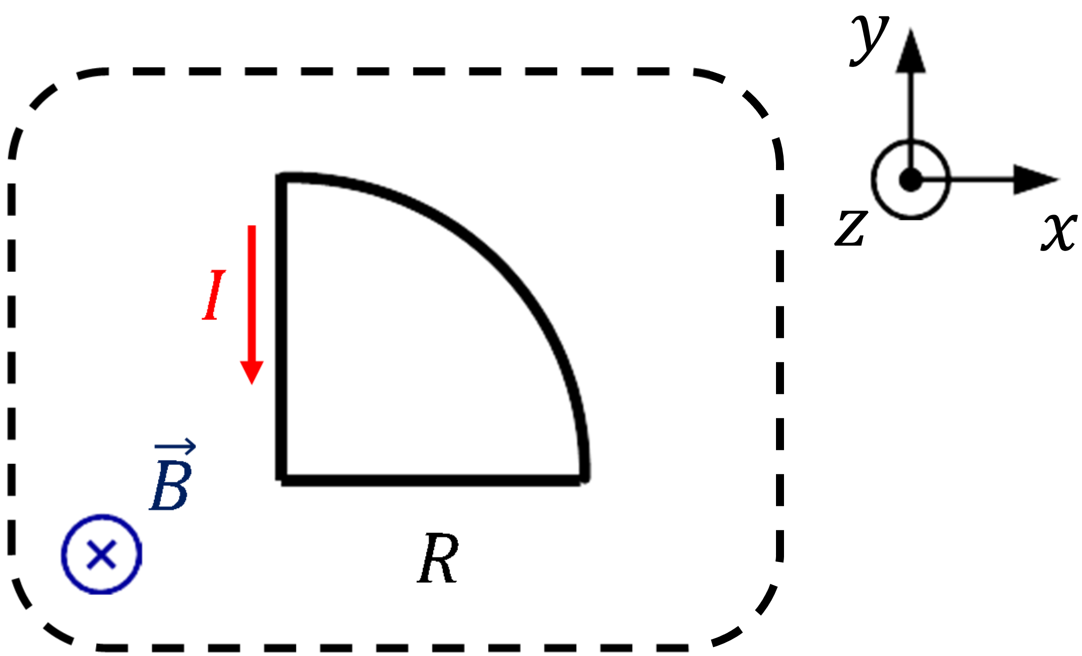
Example: A section of current-carrying wire is in a uniform magnetic field, as illustrated. Determine the net force on the wire.



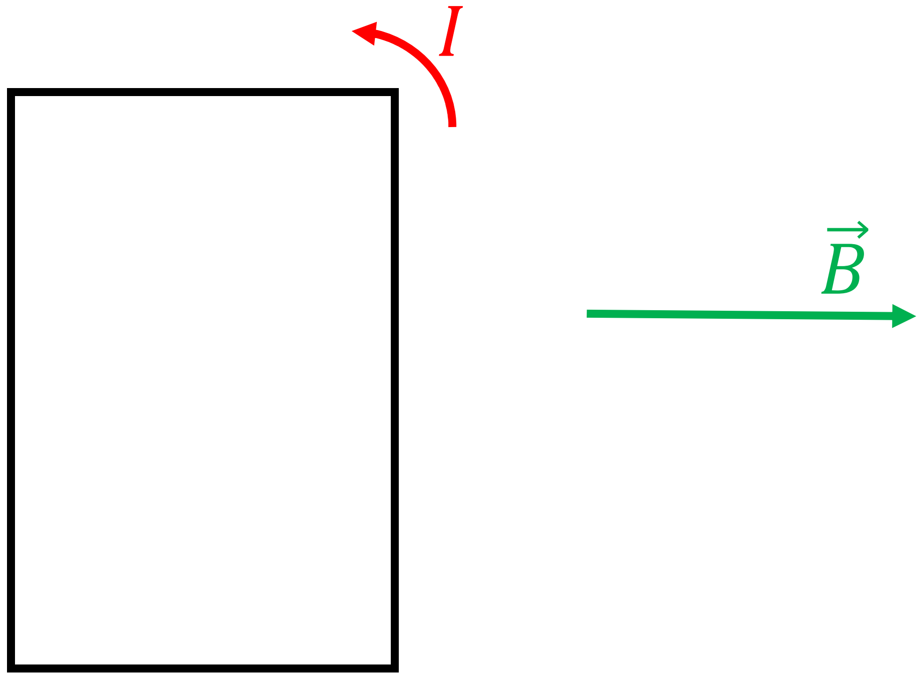
Example: A section of current-carrying wire is in a uniform magnetic field, as illustrated. Determine the net force on the wire.



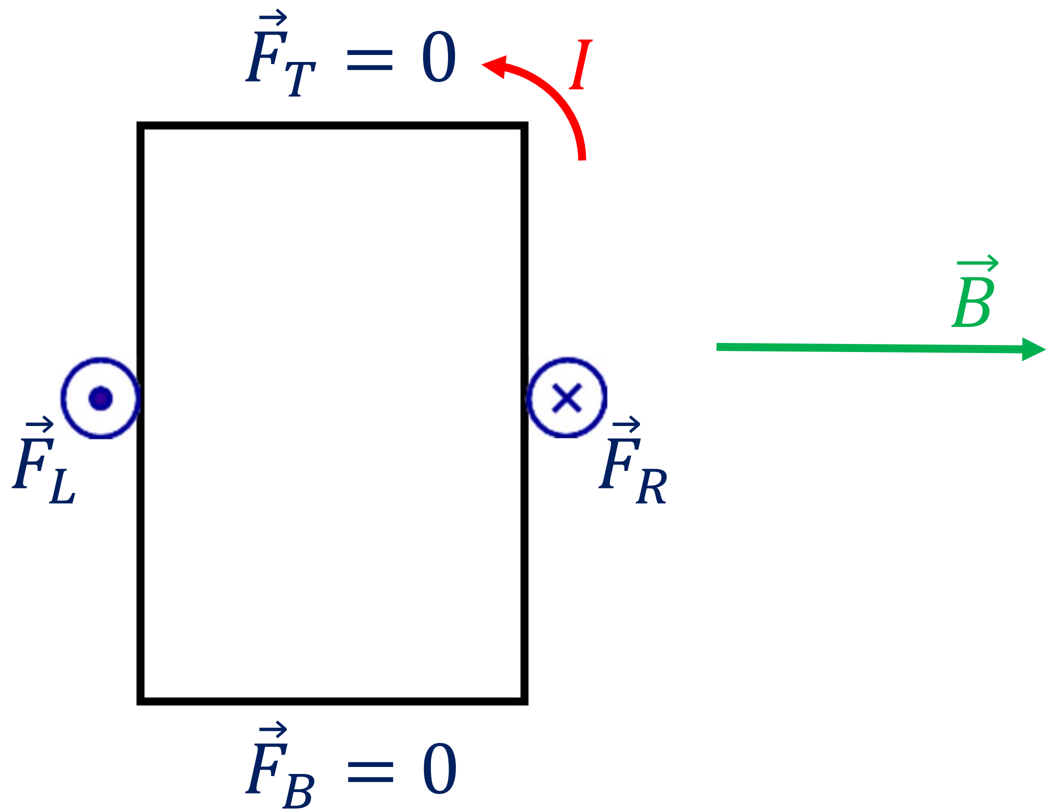
Example: A quarter of a circular loop of current-carrying wire is in a uniform magnetic field, as illustrated. Determine the net force on the wire.



Magnetic Forces on Current Loops
Rectangular Loops

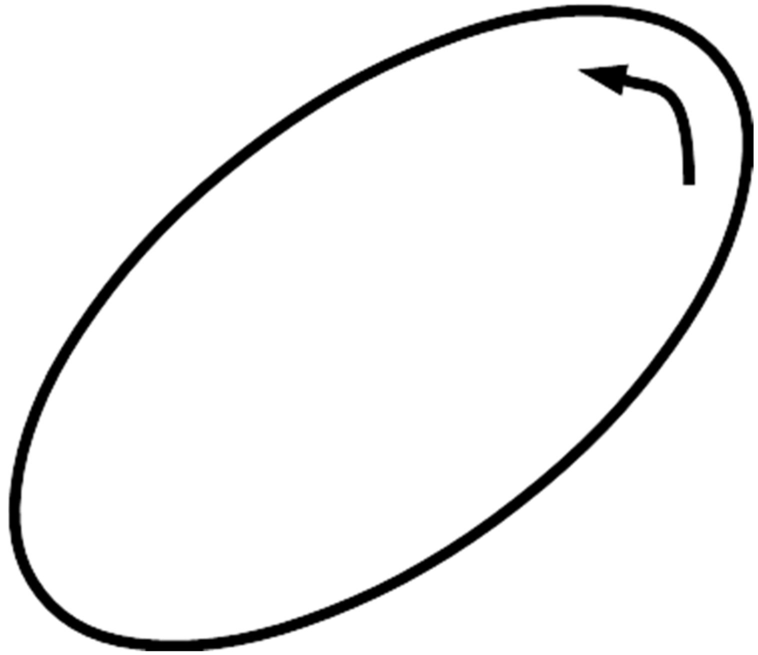


Magnetic Forces on Current Loops
Rectangular Loops

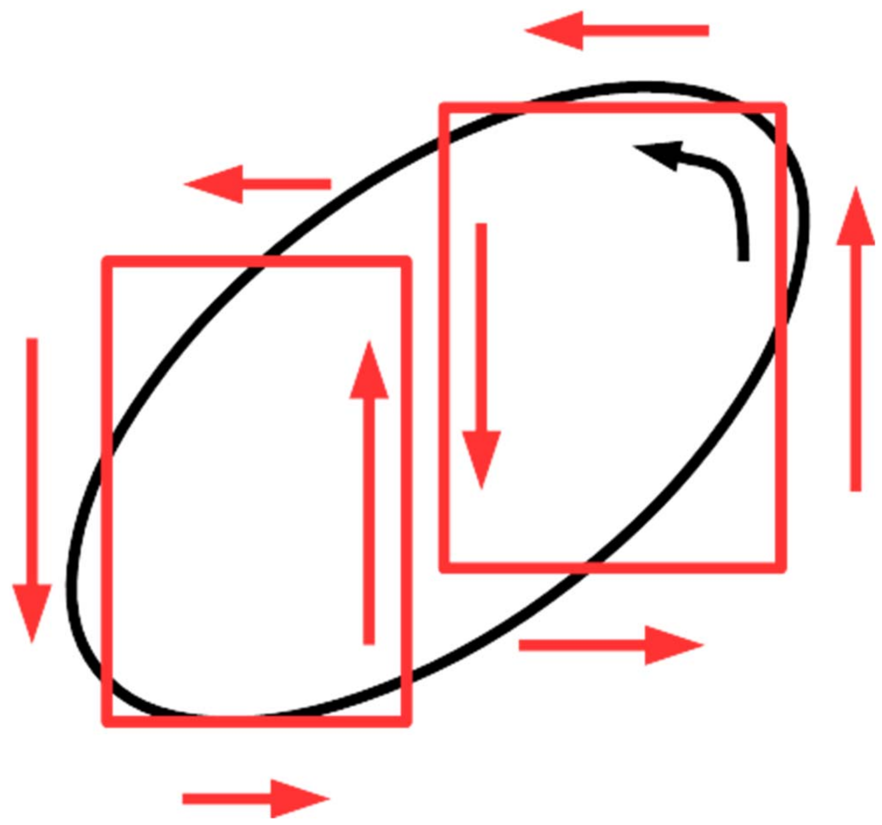


$$\vec{F}_{\text{Total}} = 0$$

Magnetic Forces on Current Loops
General Loops

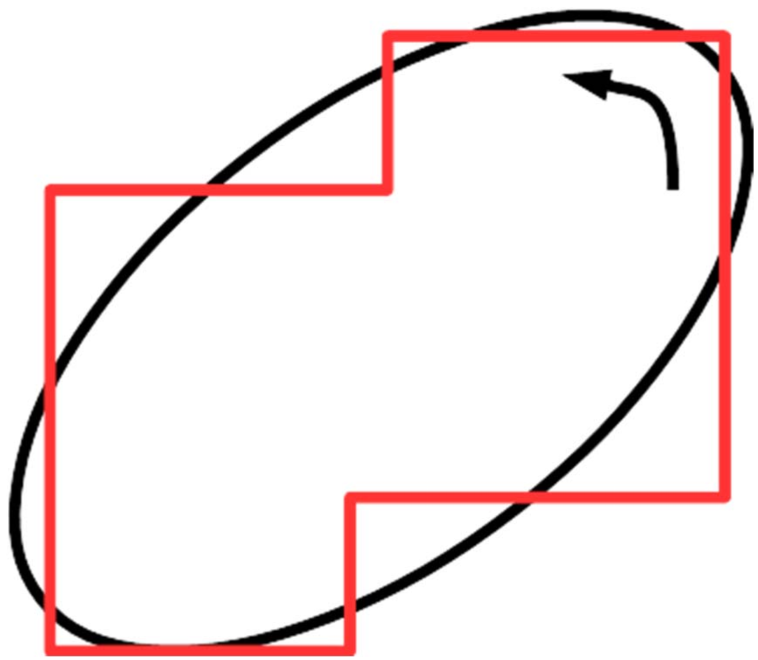


Magnetic Forces on Current Loops
General Loops



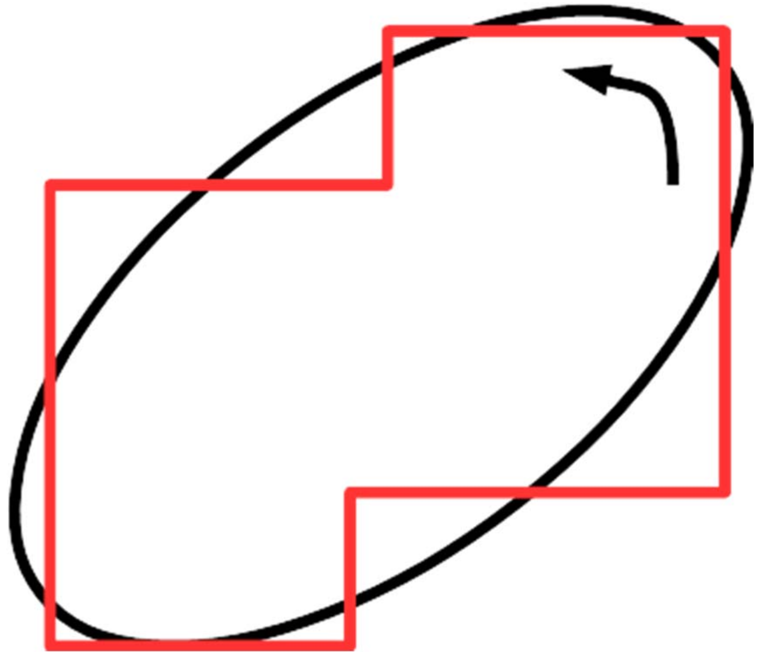
Approximate loop by set of rectangular loops.

Magnetic Forces on Current Loops
General Loops



Approximate loop by set of rectangular loops.
Interior sections cancel.

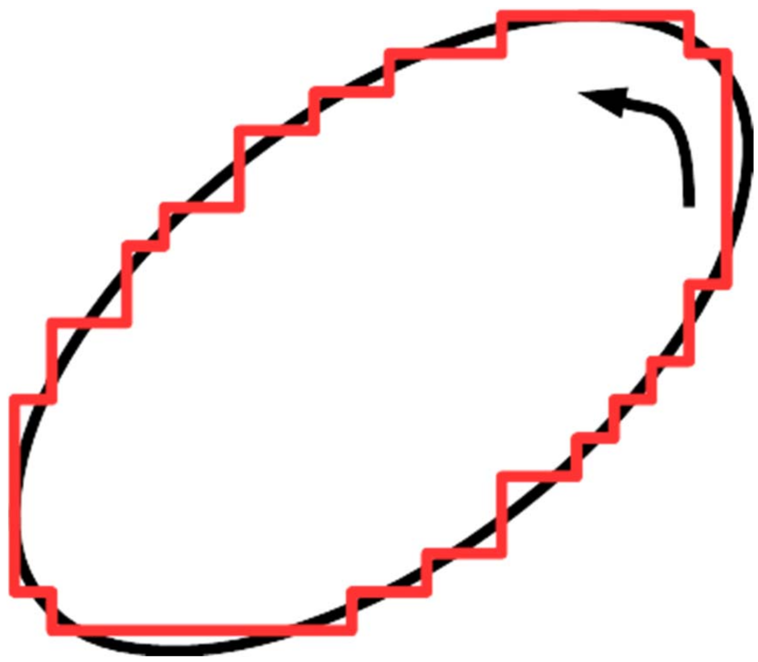
Magnetic Forces on Current Loops
General Loops



Approximate loop by set of rectangular loops.

Interior sections cancel. $\vec{F}_{\text{Total}} = 0$

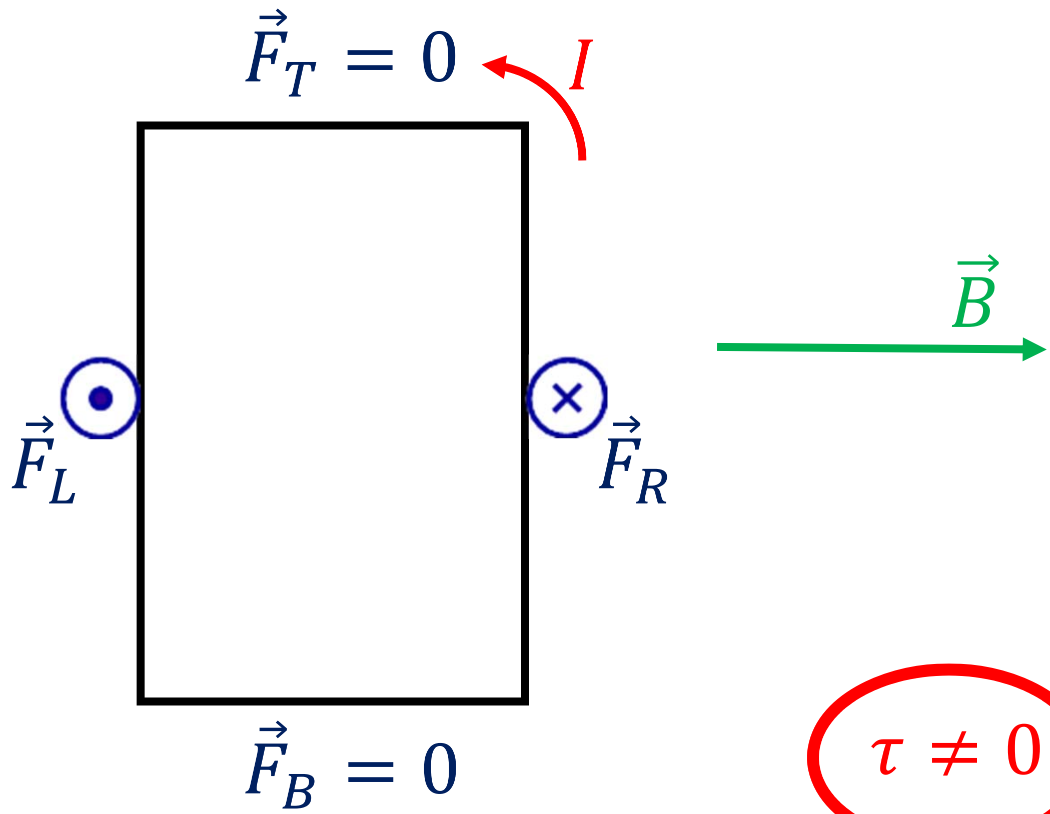
Magnetic Forces on Current Loops
General Loops



Approximate loop by set of rectangular loops.

Interior sections cancel. $\vec{F}_{\mathrm{Total}} = 0$

Magnetic Torques on Current Loops
Rectangular Loops



$\vec{F}_{\text{Total}} = 0$

$\tau \neq 0$

Magnetic Torques on Current Loops
Rectangular Loops

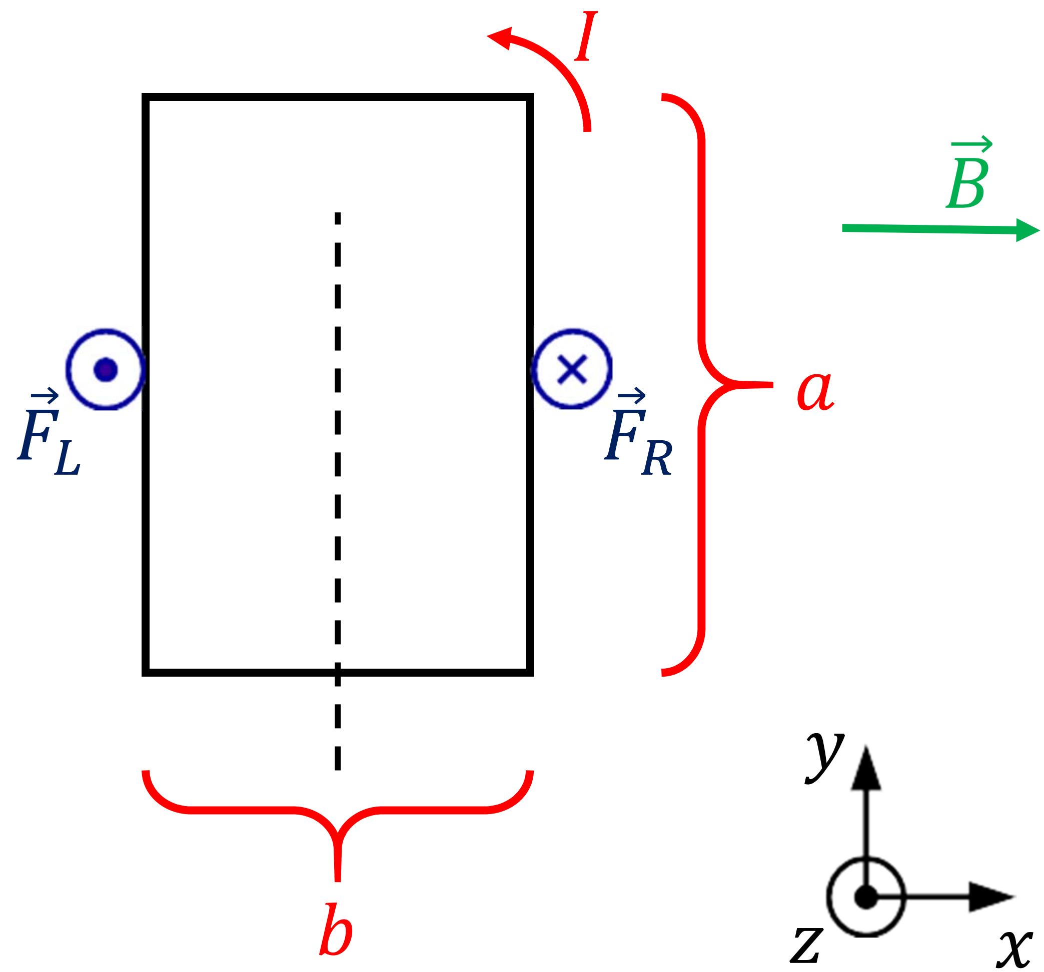
$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{\tau}_L = \frac{b}{2}(-\hat{\imath}) \times IaB\hat{k}$$

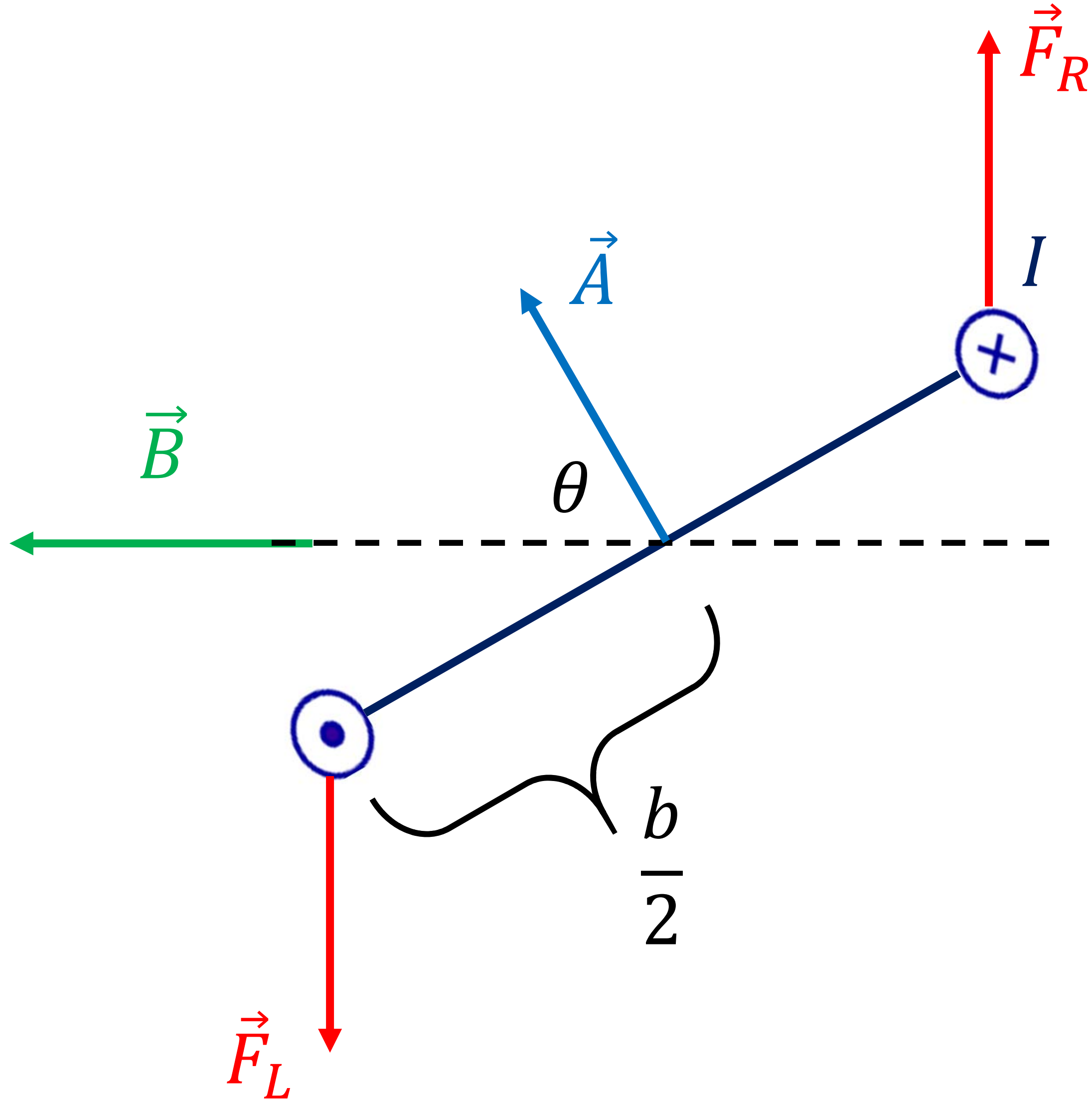
$$\vec{\tau}_L = \frac{1}{2} IabB\hat{\jmath}$$

$$\vec{\tau}_R = \frac{1}{2} IabB\hat{\jmath}$$

$$\vec{\tau}_{\text{Total}} = \text{IAB}\hat{\jmath}$$



Magnetic Torques on Current Loops
Field not necessarily in plane of loop



$$\tau_L = \frac{b}{2} \sin\theta \; IaB$$

$$\tau_R = \frac{b}{2} \sin\theta \; IaB$$

$$\tau_{\text{Total}} = IabB \sin\theta$$

$$\vec{\tau} = I\vec{A} \times \vec{B}$$

Magnetic Dipole Moment

$$\vec{\tau} = I\vec{A} \times \vec{B}$$

Can be written as

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Where

$$\vec{\mu} = NI\vec{A}$$

(N is the number of wire loops.)

Magnetic Dipole Moment

$$\vec{\mu} = NI\vec{A}$$

Torque:

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Energy:

$$U = -\vec{\mu} \cdot \vec{B}$$

Magnetic Dipole Moment

$$\vec{\mu} = NI\vec{A}$$

Torque:

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Energy:

$$U = -\vec{\mu} \cdot \vec{B}$$

Recall electric dipoles.

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$U = -\vec{p} \cdot \vec{E}$$

Example: A magnetic dipole is in a uniform magnetic field. Under what conditions is (a) the torque a minimum, (b) the torque zero, (c) the potential energy a minimum, (d) the potential energy zero?

$$\vec{\mu} = NI\vec{A}$$

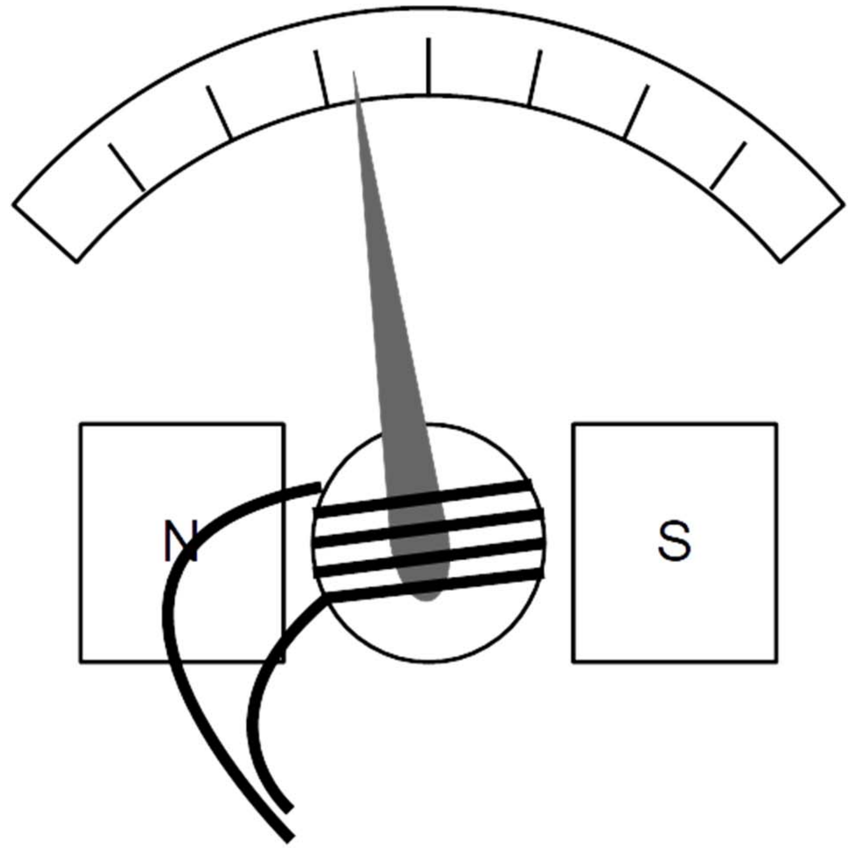
Torque:

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Energy:

$$U = -\vec{\mu} \cdot \vec{B}$$

Galvanometer



What causes the forces on the needle?

Hyperphysics has nice interactive graphics showing how [dc]() and [ac]() motors work.