Today's agenda:

Review

You must understand the similarities and differences between electric forces and magnetic forces on charged particles.

Magnetic forces on currents and current-carrying wires.

You must be able to calculate the magnetic force on currents.

Magnetic forces and torques on current loops.

You must be able to calculate the torque and magnetic moment for a current-carrying wire in a uniform magnetic field.

Applications: galvanometers, electric motors, rail guns.

You must be able to use your understanding of magnetic forces and magnetic fields to describe how electromagnetic devices operate.

Magnetic and Electric Forces

Electric force acts **in the direction of the electric field.**

$$\vec{F}_E = q\vec{E}$$

Electric force is nonzero even if v=0.

Magnetic force acts **perpendicular to the magnetic field.**

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

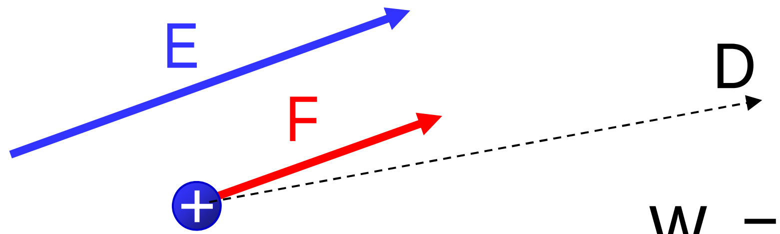
Magnetic force is zero if v=0.

$$\vec{F}_B (v = 0) = (q)\left(\vec{0}\right) \times \vec{B} = 0$$

Magnetic and Electric Forces

Electric force does work in displacing a charged particle.

$$\vec{F}_E = q\vec{E}$$

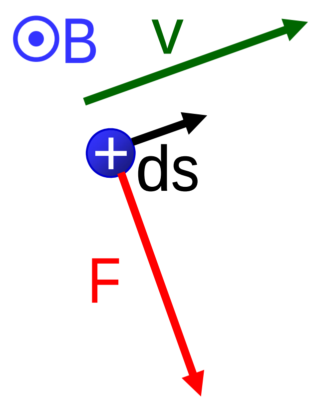


$$W_F = \vec{F}\cdot\vec{D} = FD = qED$$

Electric force can be used to accelerate particles.

Magnetic force does not do work in displacing a charged particle!

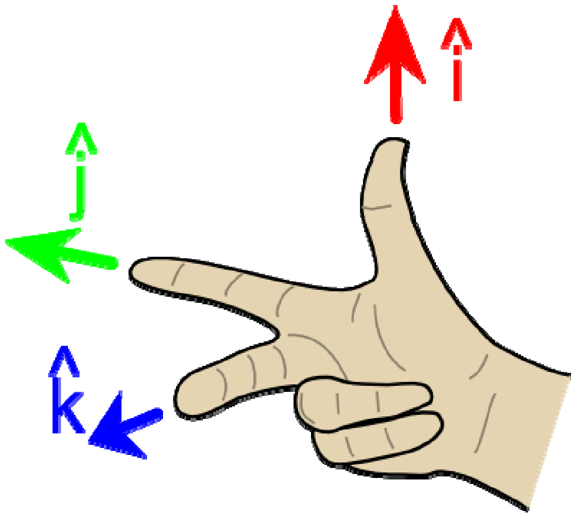
$$\vec{F}_B = q\vec{v}\times\vec{B}$$



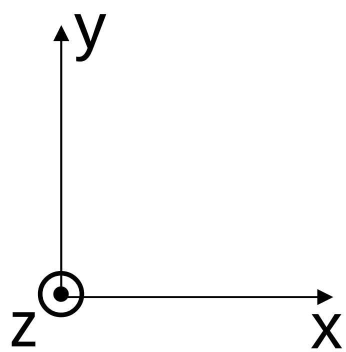
$$W_F = \vec{F}\cdot d\vec{s} = 0$$

Reminder: left- and right-hand coordinate systems

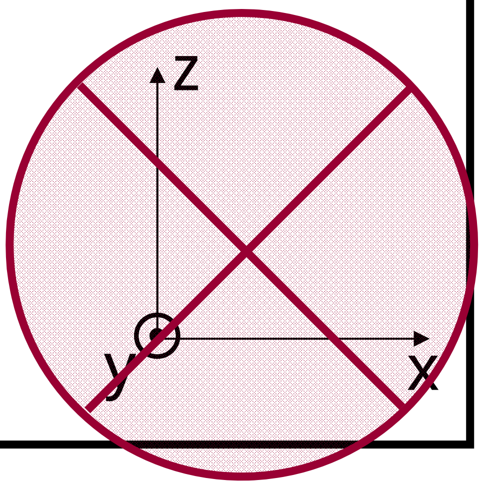
- use a right-handed coordinate system!
  (otherwise cross products may come out incorrect)
- unit vectors $\hat{i}, \hat{j}, \hat{k}$ (in this order) must form right handed triple



This is a right-handed coordinate system:



This is not:



Reminder: signs

$$\vec{F} = q\vec{v} \times \vec{B}$$

Include the sign on q, properly account for the directions of any two of the vectors, and the direction of the third vector is calculated “automatically.”

$$F = |q| vB \sin\theta$$

If you determine the direction “by hand,” use the magnitude of the charge.

Everything in this equation is a magnitude.
The sign of r had better be +!

$$r = \frac{mv}{|q|B}$$

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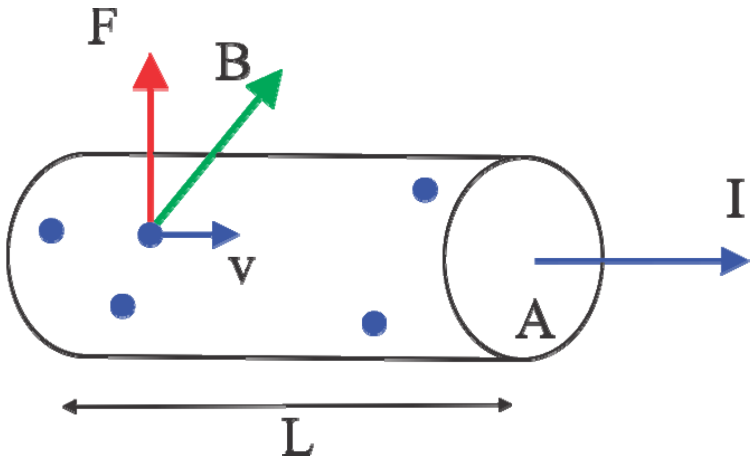
Magnetic Forces on Currents

- magnetic force on single charged particle: $\vec{F} = q\vec{v} \times \vec{B}$

Current-carrying wire:



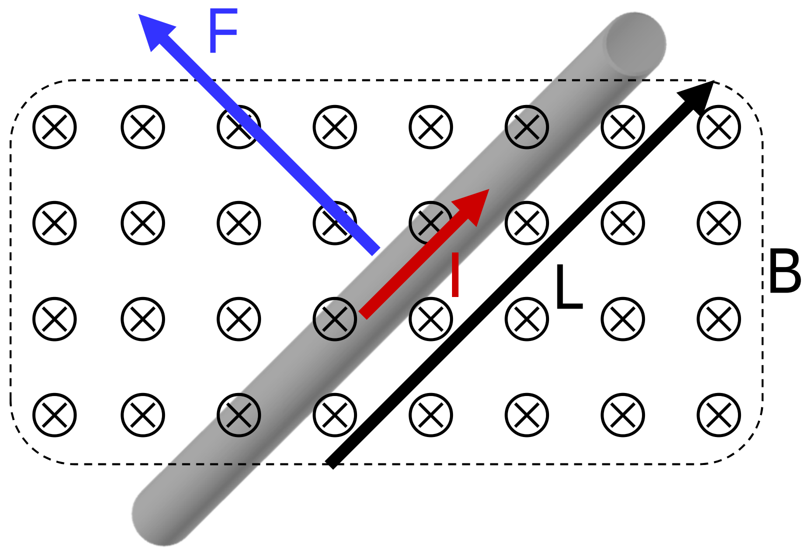
- number of charges: LAn
- total force on all carriers:

$$\vec{F}_{tot} = LA\,n\,q\vec{v} \times \vec{B} = LA\;\vec{j} \times \vec{B} = I\vec{L} \times \vec{B}$$

Magnetic force on current-carrying wire:

$$\boxed{\vec{F} = I\vec{L} \times \vec{B}}$$

vector $\vec{L}$ is in direction of current

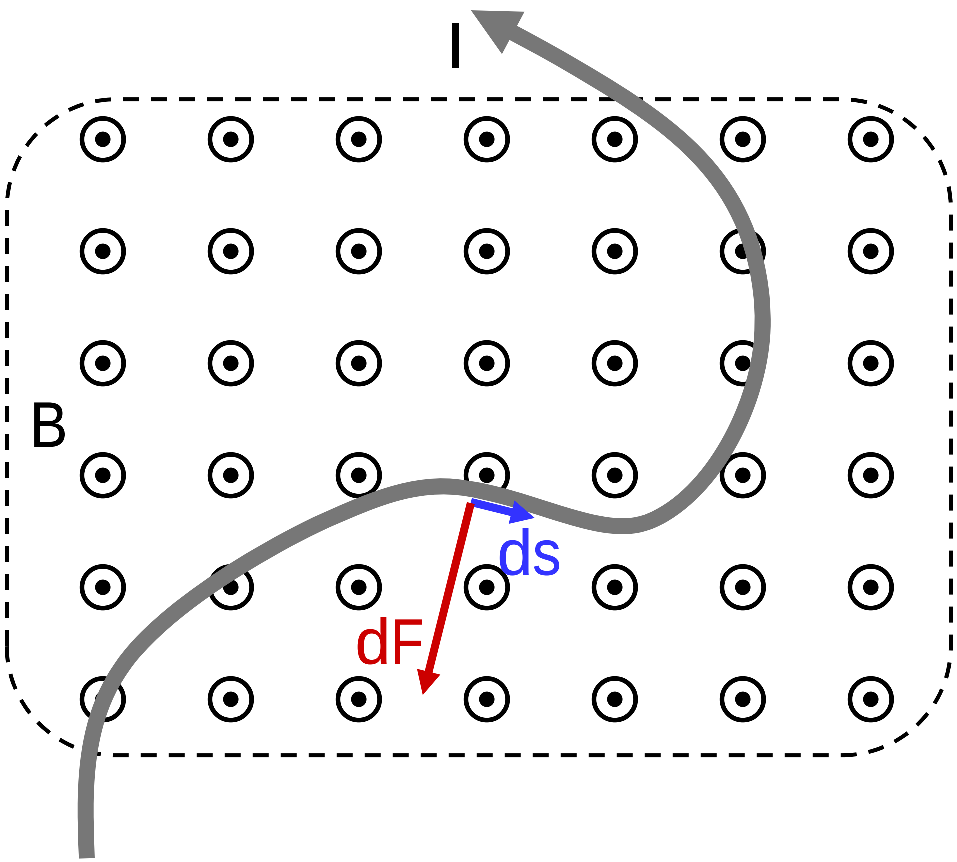


$$\vec{F} = I\vec{L} \times \vec{B}$$

Valid for straight wire, length L inside region of magnetic field, constant magnetic field, constant current I

You could apply this equation to a beam of charged particles moving through space, even if the charged particles are not confined to a wire.

What if the wire is not straight?



$$d\vec{F} = I\, d\vec{s} \times \vec{B}$$

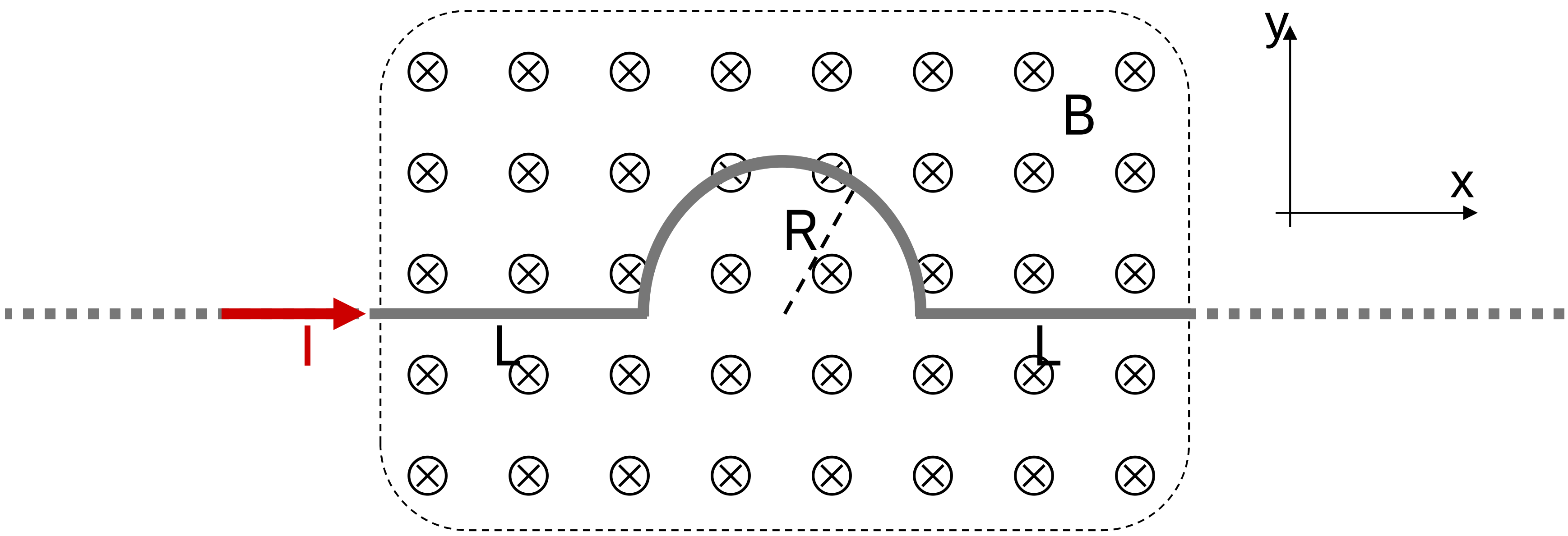
$$\vec{F} = \int d\vec{F}$$

$$\vec{F} = I \int \left(d\vec{s} \times \vec{B} \right)$$

Integrate over the part of the wire that is in the magnetic field region.

Homework Hint: if you have a tiny piece of a wire, just calculate dF; no need to integrate.

Example: a wire carrying current I consists of a semicircle of radius R and two horizontal straight portions each of length L. It is in a region of constant magnetic field as shown. What is the net magnetic force on the wire?



There is no magnetic force on the portions of the wire outside the magnetic field region.

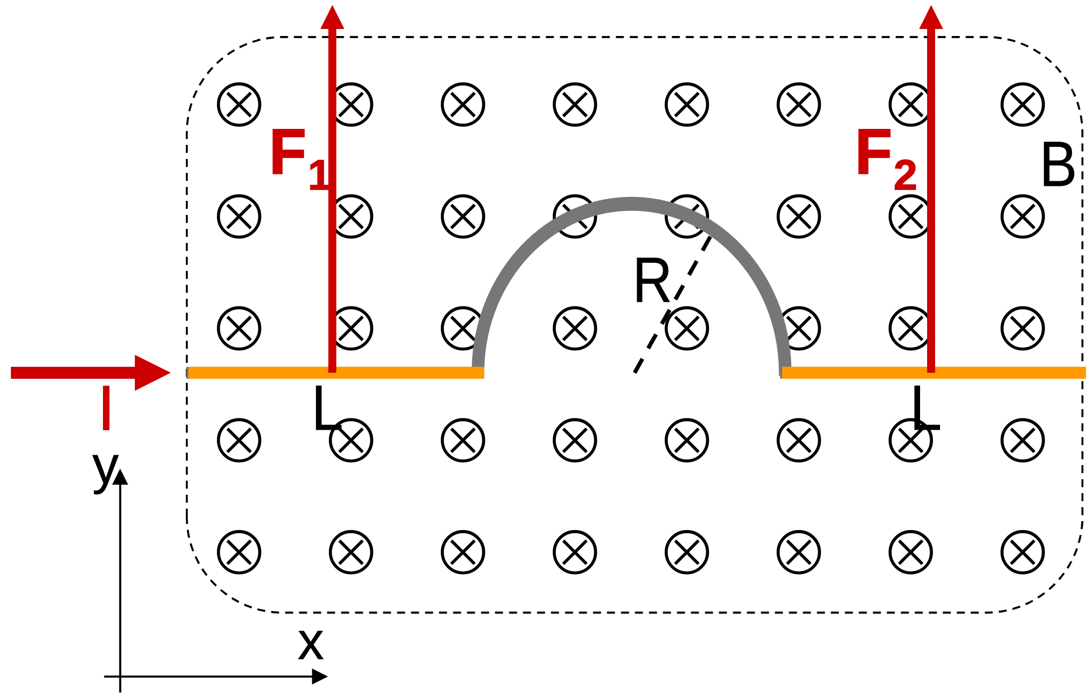
Straight sections:

$$\vec{F} = I\vec{L} \times \vec{B}$$

$\vec{L} \perp \vec{B}$, so

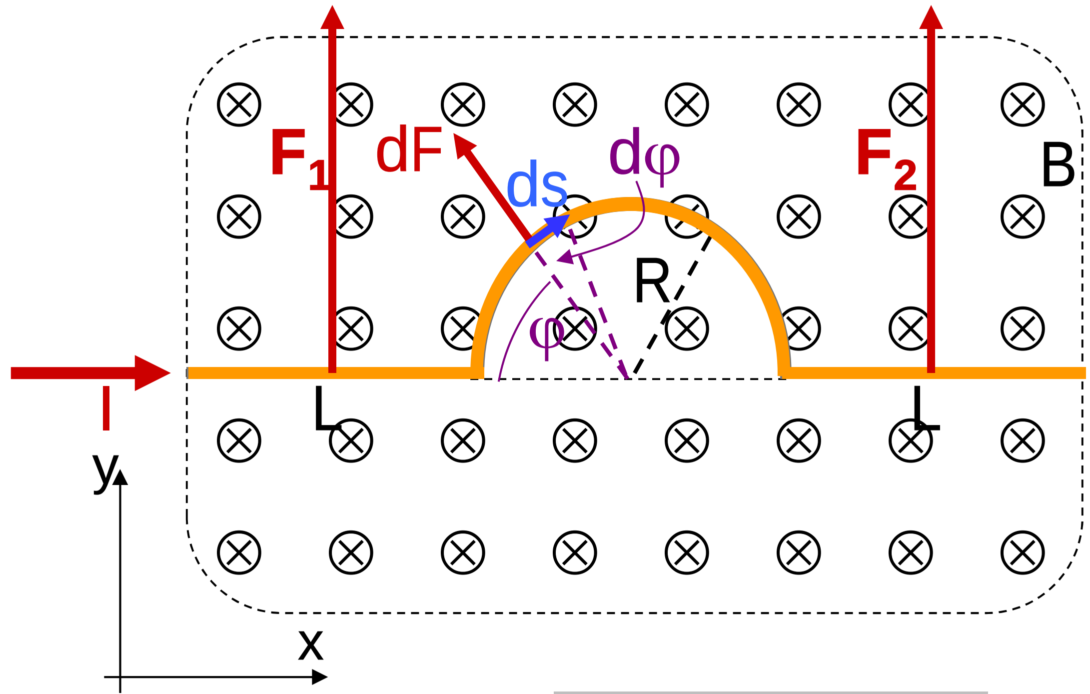
$$F_1 = F_2 = ILB$$

in positive y-direction



Semicircular section:

infinitesimal force $\vec{dF}$ on an infinitesimal $\vec{ds}$ of current-carrying wire



$\vec{ds}$ subtends angle from φ to $\varphi+d\varphi$.

infinitesimal force is $\vec{dF} = I\, \vec{ds} \times \vec{B}$.

$\vec{ds} \perp \vec{B}$, so $dF = I\, ds\, B$.

Arc length $ds = R\, d\varphi$.

Finally, $dF = I\, R\, d\varphi\, B$.

Why did I call that angle φ instead of θ?

Because we usually use θ for the angle in the cross product.

y-component of F:

$$dF_y = I\,R\,d\varphi\,B\sin\varphi$$

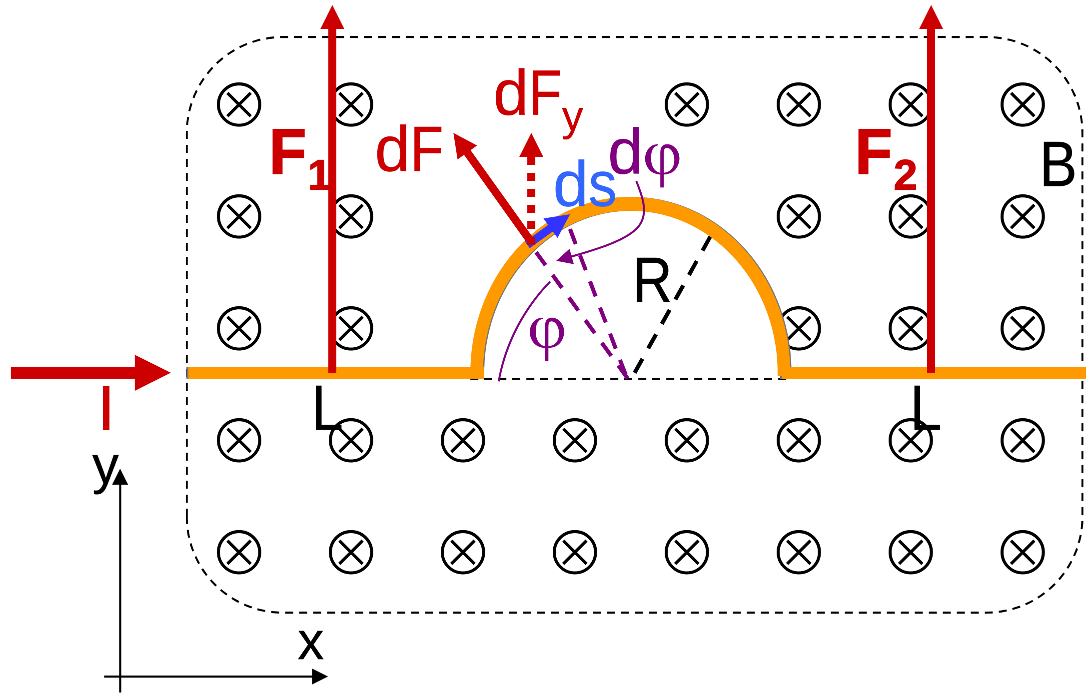
$$F_y = \int_0^{\pi} dF_y$$

$$F_y = \int_0^{\pi} I\,R\,d\varphi\,B\sin\varphi$$

$$F_y = I\,R\,B\int_0^{\pi}\sin\varphi\,d\varphi$$

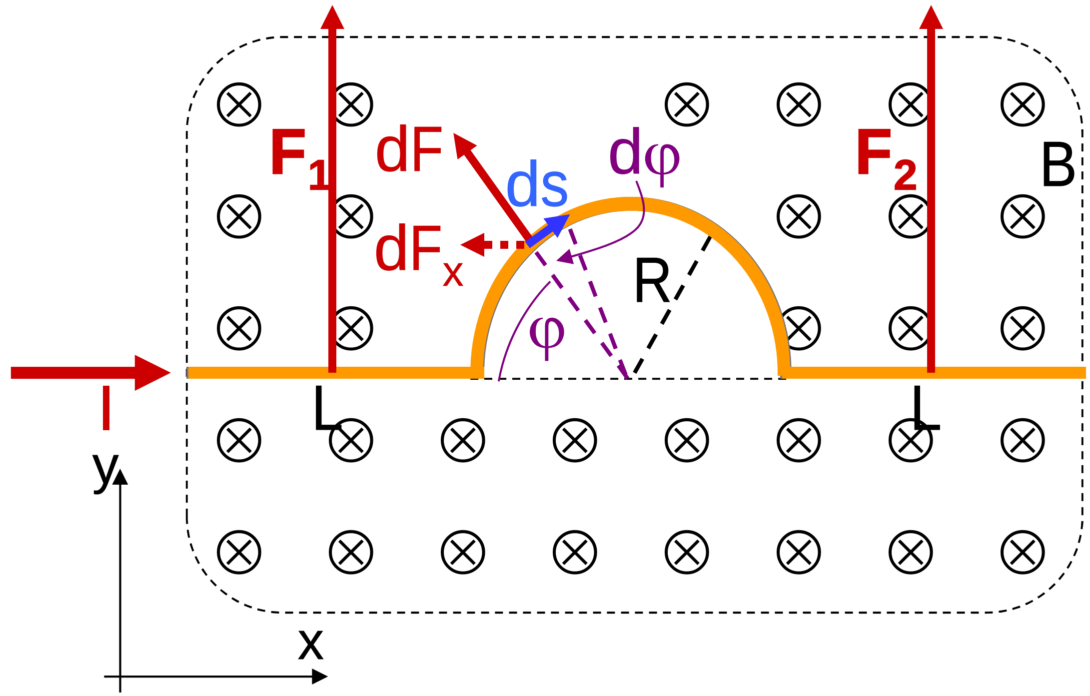
$$F_y = \left(-I\,R\,B\cos\varphi\right)\Big|_0^{\pi}$$

$$F_y = 2\,I\,R\,B$$



Interesting—just the force on a straight horizontal wire of length 2R.

Does symmetry give you F_x immediately?



x-component of F:

$$dF_x = -I\,R\,d\varphi\,B\cos\varphi$$

$$F_x = -\int_0^{\pi} I\,R\,d\varphi\,B\cos\varphi$$

$$F_x = -I\,R\,B\int_0^{\pi}\cos\varphi\,d\varphi$$

$$F_x = -\left(I\,R\,B\sin\varphi\right)\Big|_0^{\pi}$$

$$F_x = 0$$

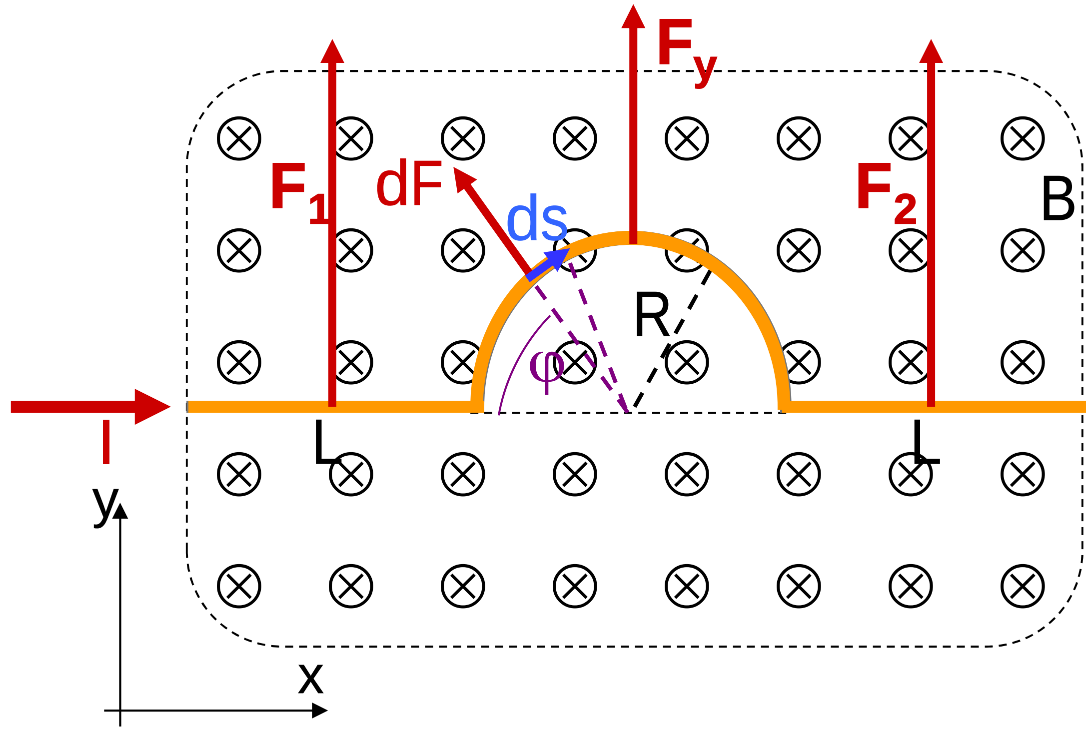
Total force:

$$F_{y,tot} = F_1 + F_2 + F_{y,arc}$$

$$F_{y,tot} = ILB + ILB + 2IRB$$

$$F_{y,tot} = 2IB(L + R)$$

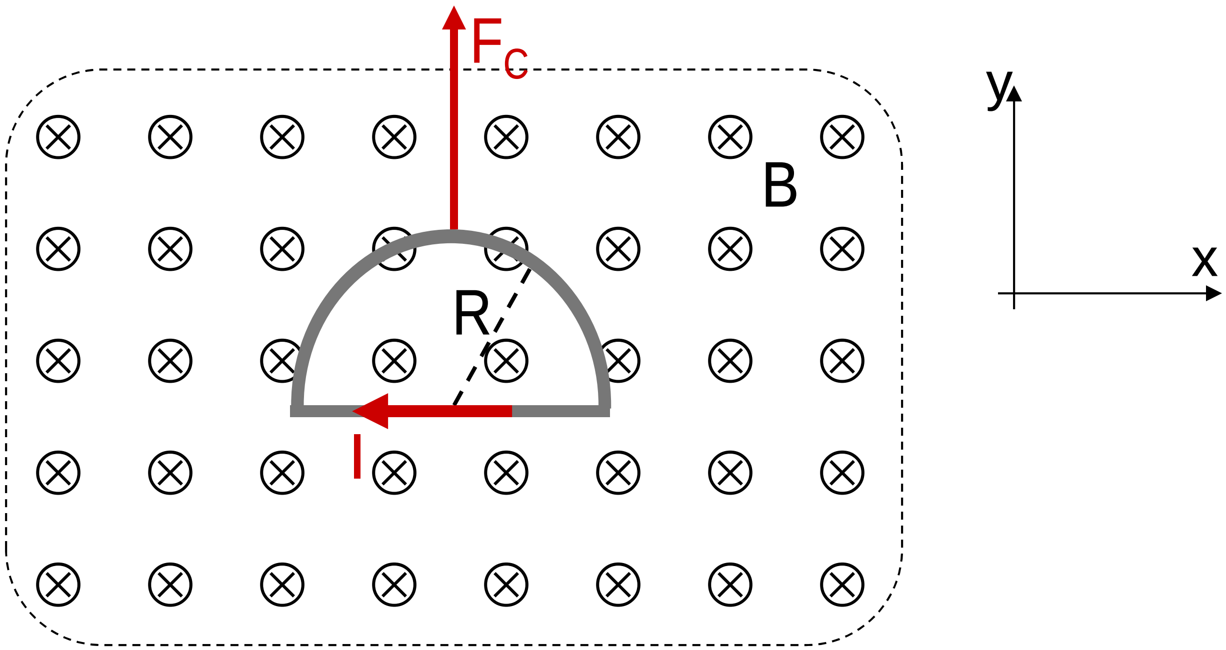
$$\vec{F} = 2IB(L + R)\,\hat{j}$$



We probably should write the force in vector form.

Possible homework hint: how would the result differ if the magnetic field were directed along the +x direction? If you have difficulty visualizing the direction of the force using the right hand rule, pick a $\vec{ds}$ along each different segment of the wire, express it in unit vector notation, and calculate the cross product.

Example: a semicircular closed loop of radius R carries current I. It is in a region of constant magnetic field as shown. What is the net magnetic force on the loop of wire?



We calculated the force on the semicircular part in the previous example (current is flowing in the same direction there as before).

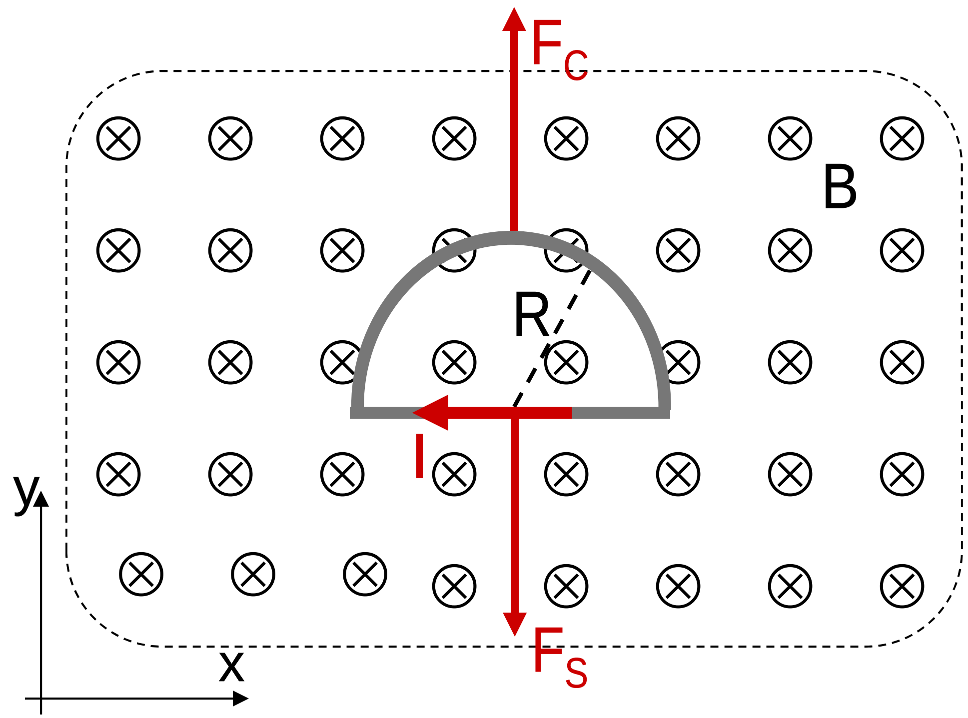
$$F_C = 2\, I\, R\, B$$

Next look at the straight section.

$$\vec{F}_S = I\vec{L}\times\vec{B}$$

$\vec{L} \perp \vec{B}$, and L=2R so

$$F_S = 2IRB$$



$\vec{F}_s$ is directed in the –y direction (right hand rule).

$$\vec{F}_{net} = \vec{F}_S + \vec{F}_c = -2IRB\,\hat{j} + 2IRB\,\hat{j} = 0$$

The net force on the closed loop is zero!

This is true in general for closed loops in a uniform magnetic field.

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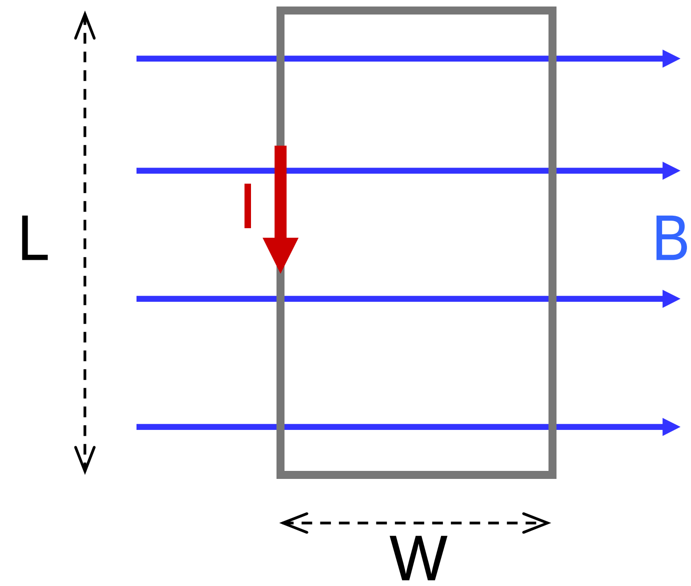
Magnetic Forces and Torques on Current Loops

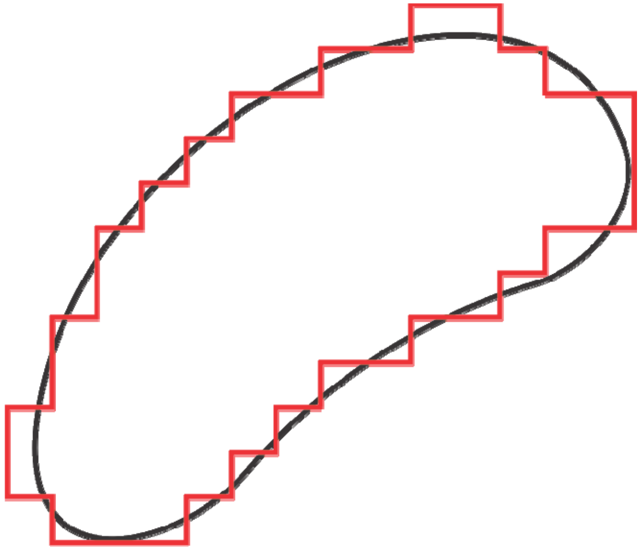
Rectangular loop:

- currents in left and right wires are equal and opposite
- magnetic forces on left and right wires are equal and opposite
- same for top and bottom wires
- **net magnetic force on loop is zero**



General loop:

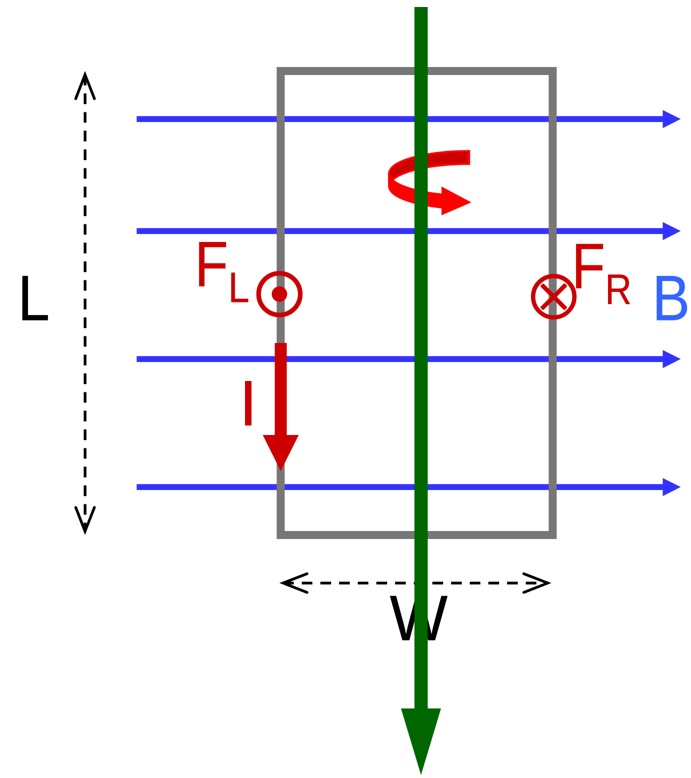
- can be decomposed into rectangular pieces



Net force on any closed current loop is zero

Torque on current loop:

- no force on top and bottom segments because current and magnetic field are parallel
- left vertical segment feels force "out of the page"
- right vertical segment feels force "into the page"



The two forces have the same magnitude: $F_L = F_R = I\,L\,B$.

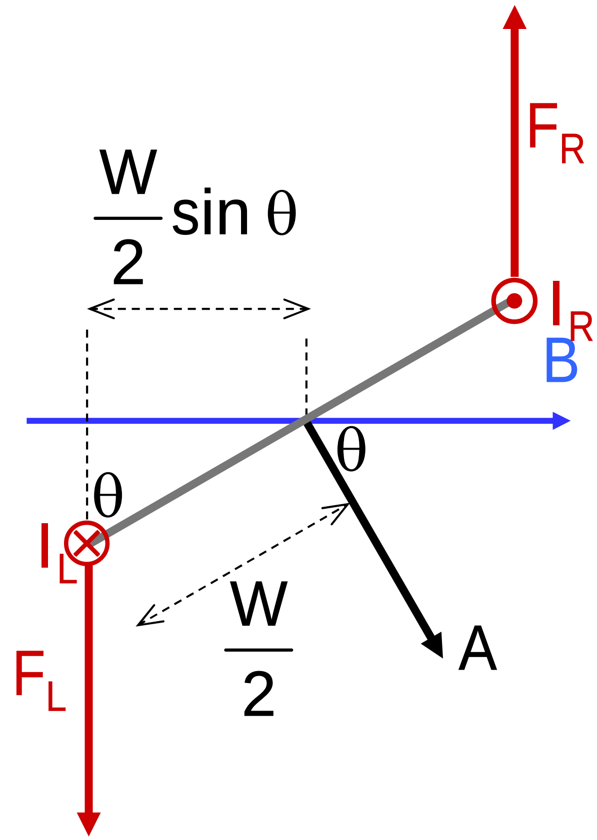
Because $\vec{F}_L$ and $\vec{F}_R$ are in opposite directions, there is no net force on the current loop, but **there is a net torque**.

Top view:

$$\tau_R = \frac{W}{2} F_R \sin\theta = \frac{1}{2} WILB \sin\theta$$

$$\tau_L = \frac{W}{2} F_L \sin\theta = \frac{1}{2} WILB \sin\theta$$

$$\tau_{net} = \tau_R + \tau_L = WILB \sin\theta = IAB \sin\theta$$



In vector form:

$$\vec{\tau} = I\vec{A} \times \vec{B}$$

Direction of $\vec{A}$: (right-hand rule)

- curl your fingers around the loop in the direction of the current; thumb points in direction of $\vec{A}$.

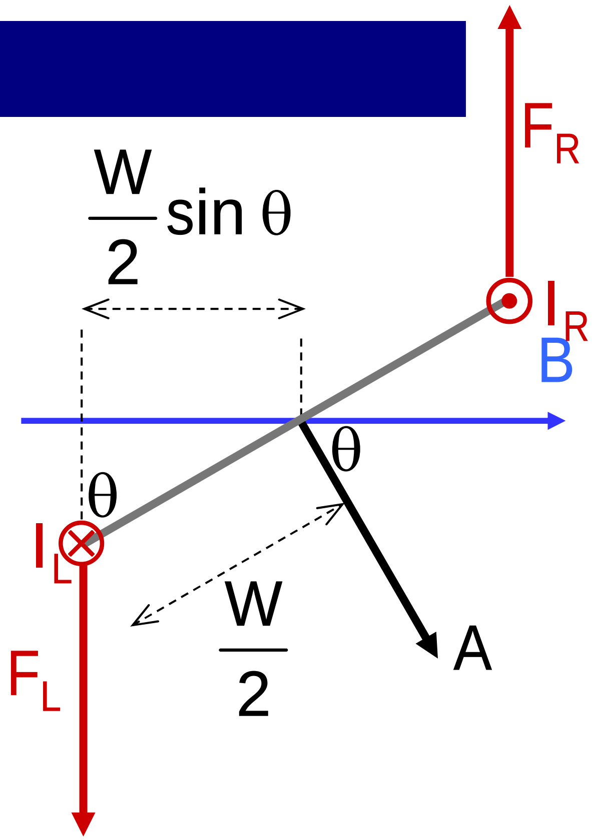
Magnetic Moment of a Current Loop

$$\vec{\tau} = I\vec{A} \times \vec{B}$$

magnetic moment of loop

$$\vec{\mu} = I\vec{A}$$

$$\boxed{\vec{\tau} = \vec{\mu} \times \vec{B}}$$



Your starting equation sheet has:

$$\boxed{\vec{\mu} = N\,I\,\vec{A} \qquad (N = 1 \text{ for a single loop})}$$

Magnetic dipoles

- current loop acts as **magnetic dipole** similar to bar magnet

You already know this:

Electric Dipole

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$$U = -\vec{p} \cdot \vec{E}$$

Today:

Magnetic Dipole

Homework Hint → $$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Homework Hint → $$U = -\vec{\mu} \cdot \vec{B}$$

Example: a magnetic dipole of moment $\vec{\mu}$ is in a uniform magnetic field $\vec{B}$. Under what conditions is the dipole's potential energy zero? Minimum? Under what conditions is the magnitude of the torque on the dipole minimum? Maximum?

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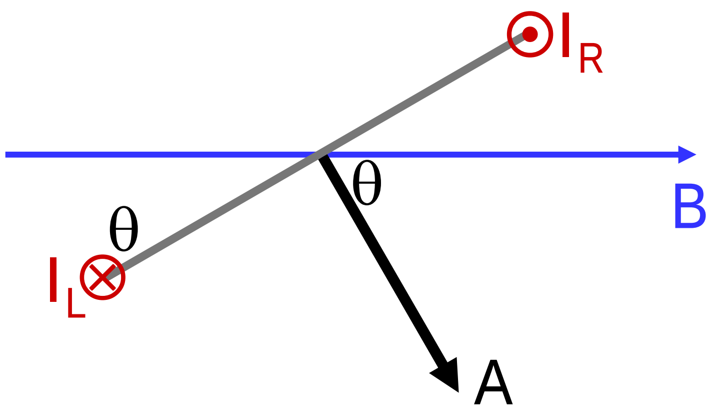
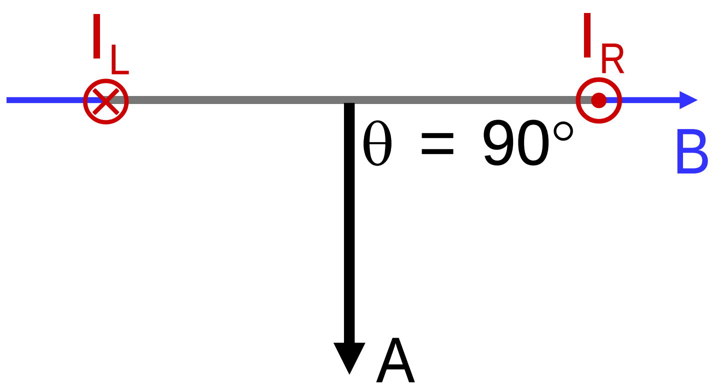
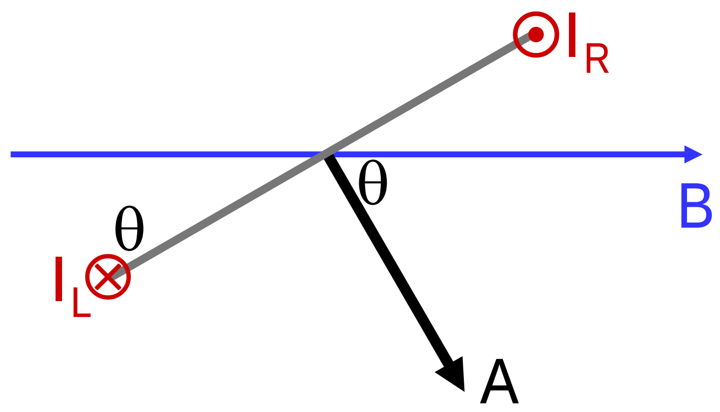


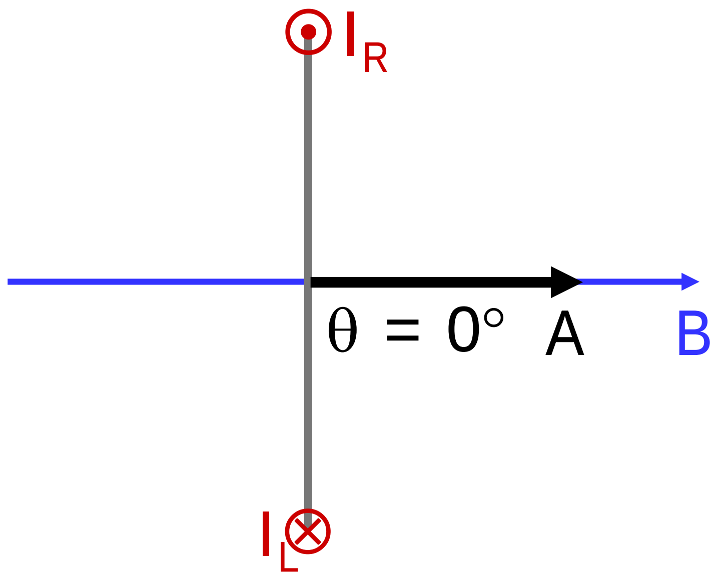


$$U = -I\vec{A}\cdot\vec{B} = -IAB\cos\theta$$

Potential energy is zero when $\cos\theta = 0$, or when $\theta = 90°$ or $270°$.

Example: a magnetic dipole of moment $\vec{\mu}$ is in a uniform magnetic field $\vec{B}$. Under what conditions is the dipole's potential energy zero? Minimum? Under what conditions is the magnitude of the torque on the dipole minimum? Maximum?

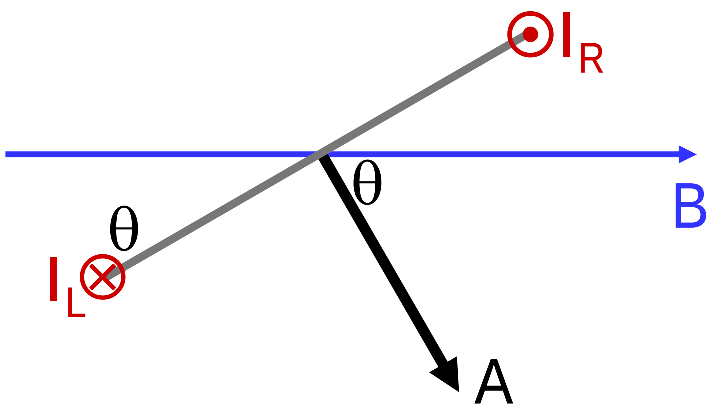


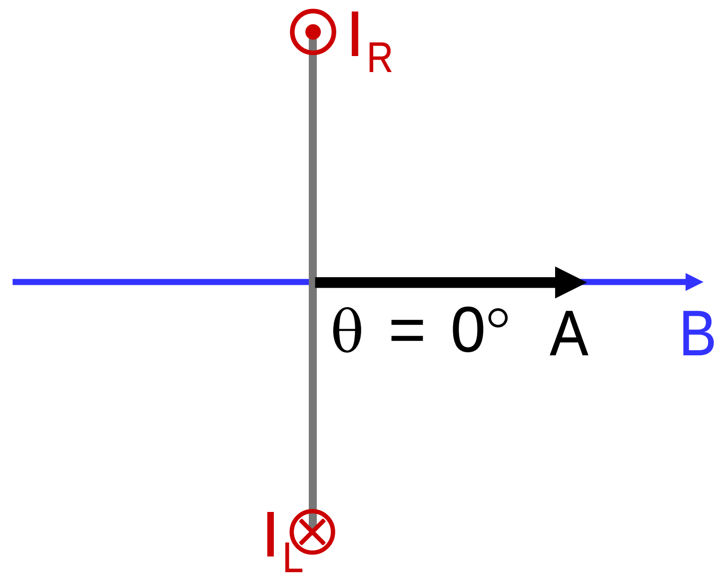


$$U = - I\vec{A}\cdot\vec{B} = - IAB\cos\theta$$

Potential energy is minimum when $\cos\theta = 1$, or when $\theta = 0°$ or 180°.

Example: a magnetic dipole of moment $\vec{\mu}$ is in a uniform magnetic field $\vec{B}$. Under what conditions is the dipole's potential energy zero? Minimum? Under what conditions is the magnitude of the torque on the dipole minimum? Maximum?

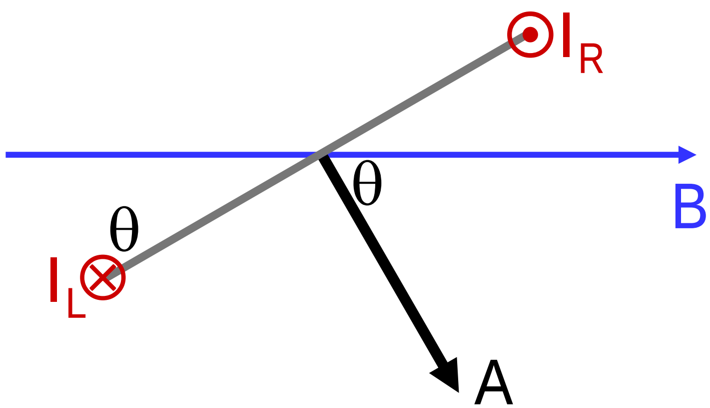


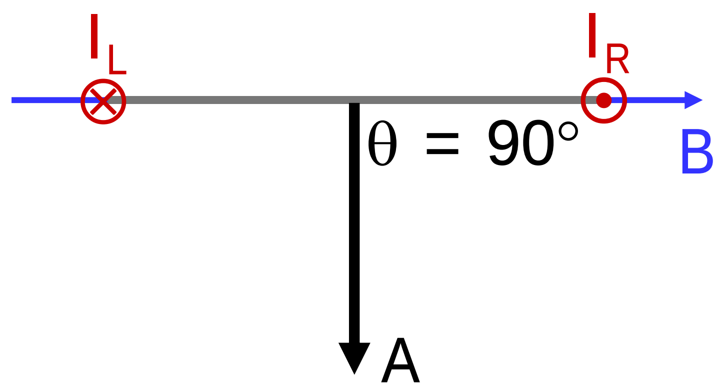


$$\left|\vec{\tau}\right| = \left|I\vec{A}\times\vec{B}\right| = IAB\left|\sin\theta\right|$$

Torque magnitude is minimum when $\sin\theta = 0$, or when $\theta = 0°$ or $180°$.

Example: a magnetic dipole of moment $\vec{\mu}$ is in a uniform magnetic field $\vec{B}$. Under what conditions is the dipole's potential energy zero? Minimum? Under what conditions is the magnitude of the torque on the dipole minimum? Maximum?

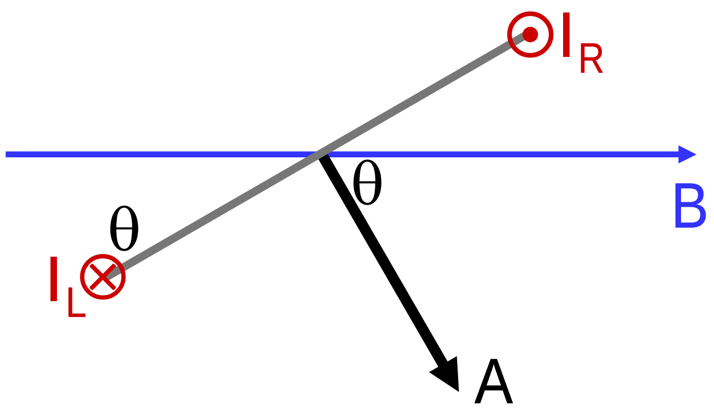




$$\left|\vec{\tau}\right| = \left|I\vec{A} \times \vec{B}\right| = IAB\left|\sin\theta\right|$$

Torque magnitude is maximum when $|\sin\theta| = 1$, or when $\theta = 90°$ or $270°$.

Example: a magnetic dipole of moment $\vec{\mu}$ is in a uniform magnetic field $\vec{B}$. Under what conditions is the dipole's potential energy maximum?



$$U = -I\vec{A}\cdot\vec{B} = -IAB\cos\theta$$

I left this for you to figure out.

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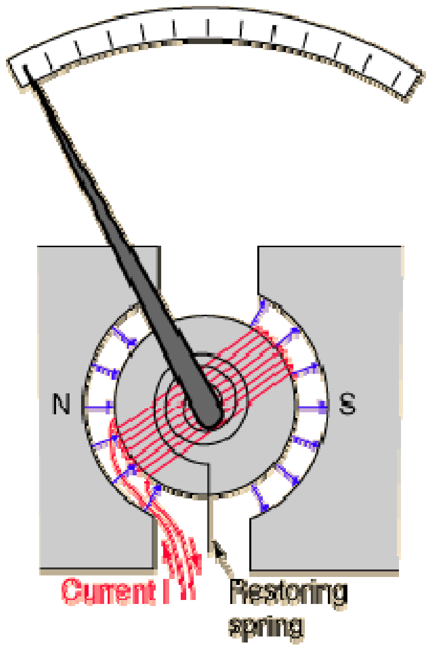
The Galvanometer

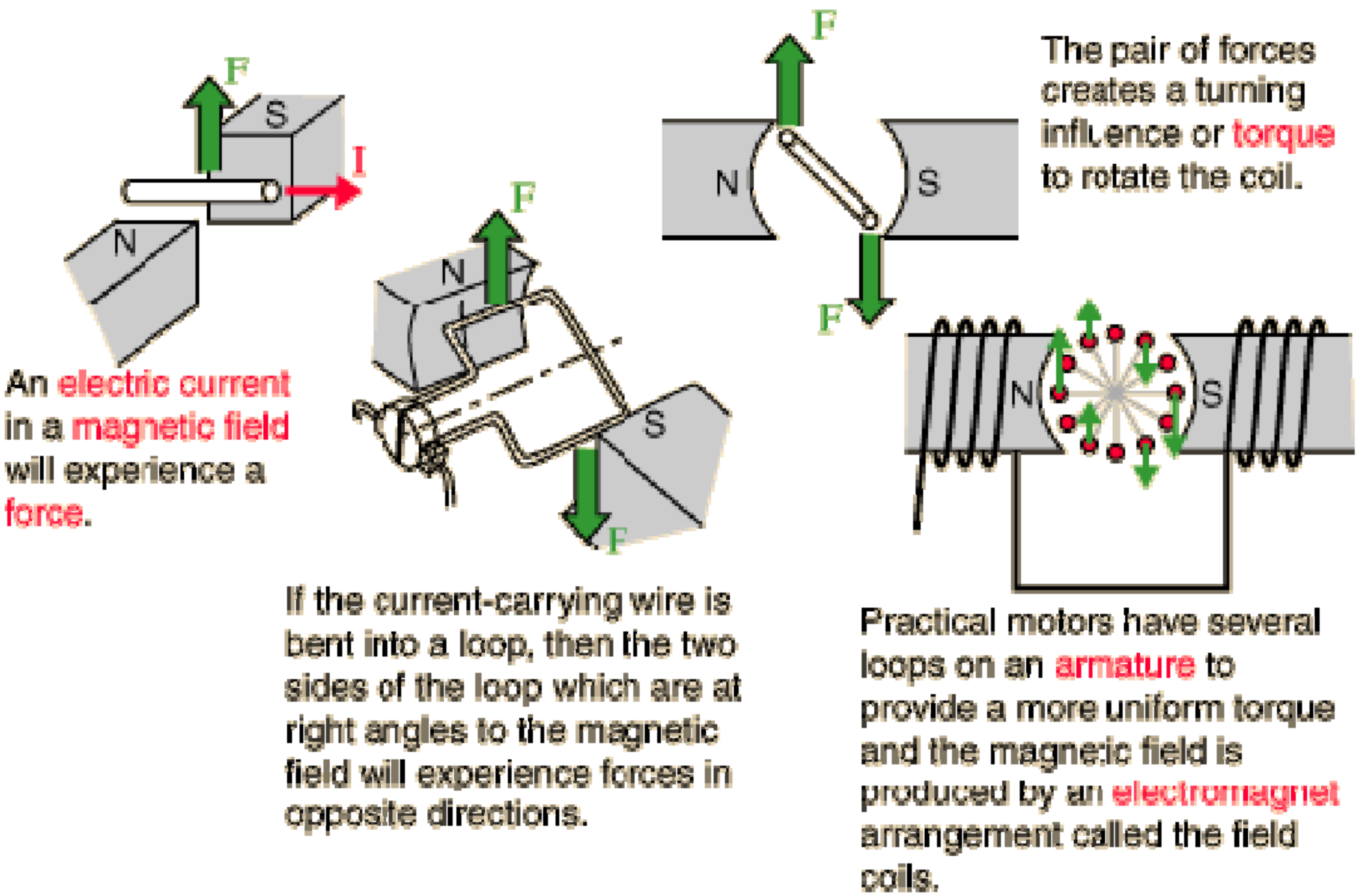
Now you can understand how a galvanometer works...

When a current is passed through a coil connected to a needle, the coil experiences a torque and deflects. See the link below for more details.



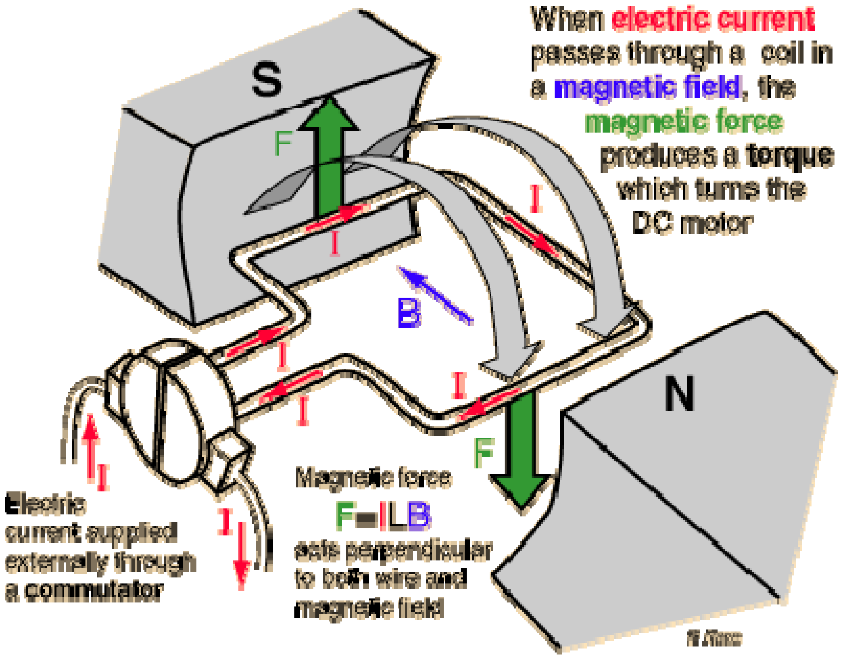
http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/galvan.html#c1

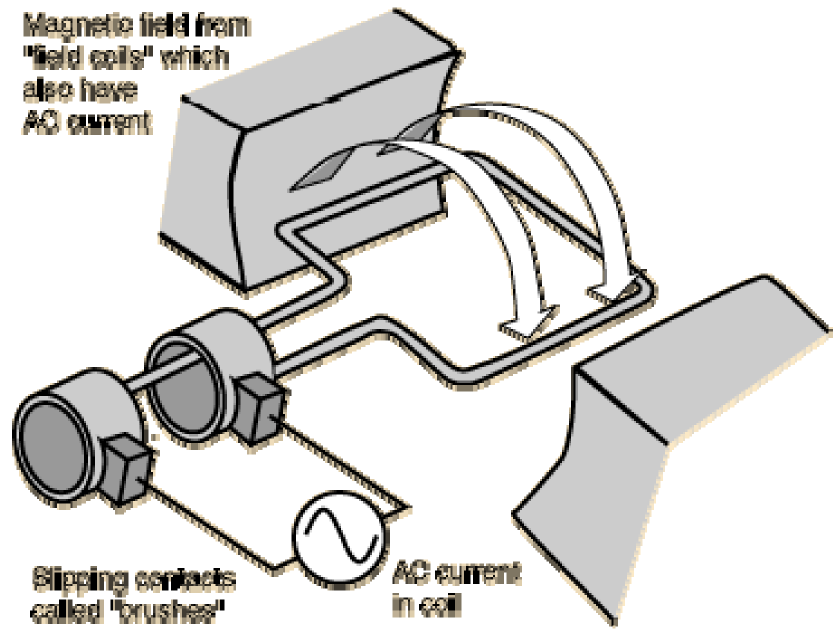
Electric Motors



http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/mothow.html#c1

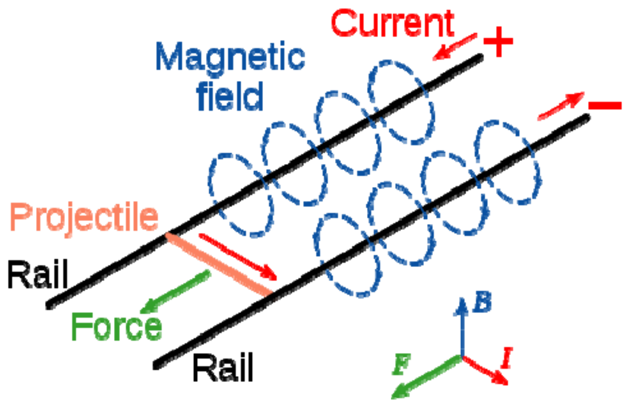
Hyperphysics has nice interactive graphics showing how dc and ac motors work.





No lecture on magnetic forces would be complete without...

...the rail gun!



Current in the rails (perhaps millions of amps) gives rise to magnetic field (we will study this after exam 2). Projectile is a conductor making contact with both rails. Magnetic field of rails exerts force on current-carrying projectile.

The 10 meter rail gun at the University of Texas.