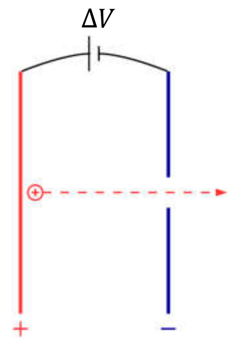
Applications of Electromagnetism (Optional Lecture)

- Particle Accelerators
  - Linear Accelerators
  - Cyclotrons
- Beamline Magnets
  - Steering Magnets
  - Focusing Magnets
- Velocity Selectors
- Detectors
  - Spectrometers
  - Drift Chambers

Linear Accelerator

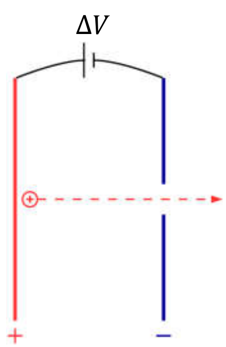
Charged particles are accelerated across a potential difference.



$$\vec{F} = q\vec{E}$$

Linear Accelerator

Charged particles are accelerated across a potential difference.



$$U_0 + K_0 = U_f + K_f$$

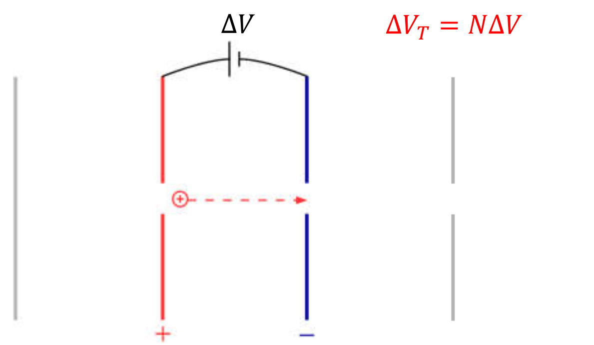
$$-\Delta U = K_f$$

$$-q\Delta V = \frac{1}{2}mv_f^2$$

$$\sqrt{\frac{2|q\Delta V|}{m}} = v_f$$

Linear Accelerator

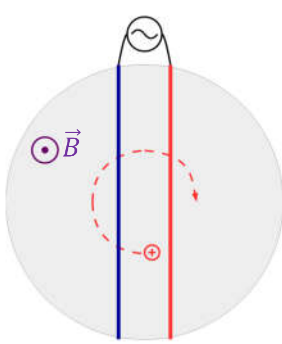
Potential difference across plates when particle is between plates. (N regions between plates)



$$\Delta V_T = N\Delta V$$

Cyclotron

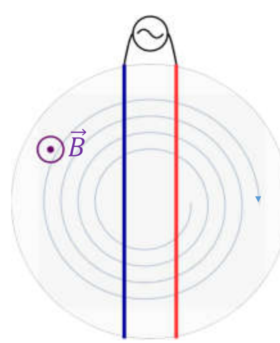
If you want to accelerate particles and are limited on space - keep sending them through the same two plates.



$$\vec{F} = q\vec{v} \times \vec{B}$$
$$\vec{F} = q\vec{E}$$

Cyclotron

Particles spiral out as they accelerate, increasing speed and radius proportionally.



$$|q\vec{v} \times \vec{B}| = \frac{mv^2}{r}$$

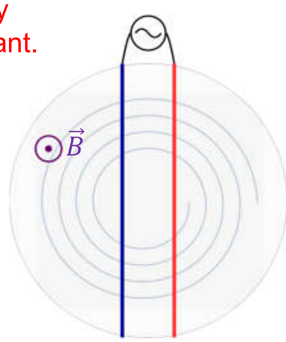
$$\mathrm{qvB} = \frac{\mathrm{mv}^2}{\mathrm{r}}$$

$$\frac{qB}{m} = \frac{v}{r}$$

Cyclotron

Particles spiral out as they accelerate, increasing speed and radius proportionally so that the period and frequency remain constant.

$$\frac{|q|B}{m} = \frac{v}{r}$$



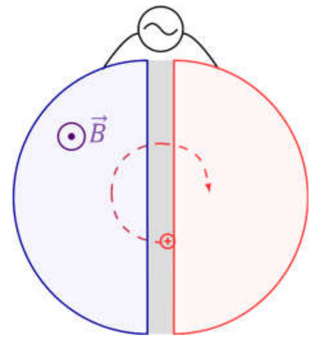
$$T = \frac{2\pi r}{v}$$

$$T = \frac{2\pi m}{|q|B}$$

$$f = \frac{|q|B}{2\pi r}$$

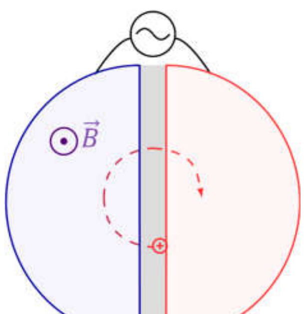
Cyclotron

Instead of plates, use "*D*'s", hollow *D*-shaped conductors with a top, a bottom and an outside edge.



Cyclotron

Instead of plates, use "*D*'s", hollow *D*-shaped conductors with a top, a bottom and an outside edge.

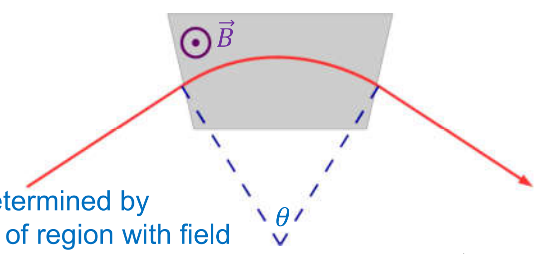


Modern cyclotrons use complex electromagnetic fields created without *D*'s.

Steering Magnets

Beamlines are turned with steering magnets.

$$r = \frac{mv}{|q|B}$$

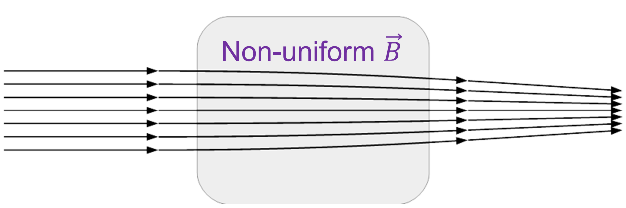


θ determined by size of region with field relative to r.

$$\vec{F} = q\vec{v} \times \vec{B}$$

Focusing Magnets

Beamlines are focused with special magnets.



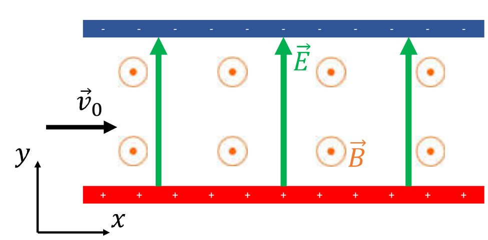
$$\vec{F} = q\vec{v} \times \vec{B}$$

(Unfortunately, a magnet can only focus in one transverse dimension and at one location along the beam. Beamlines may have many such focusing magnets.)

Velocity Selector
(See previous lecture.)

If $v_0 = \frac{E}{B}$, then $\vec{F} = 0$ and the trajectory is a straight line.

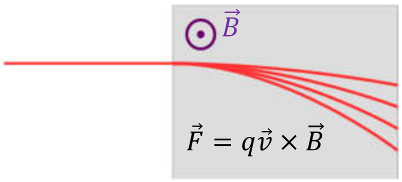
$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$



Spectrometers

Consider a bending magnet with a back wall that registers interactions with charged particles. The particles will separate according to their radius of curvature.

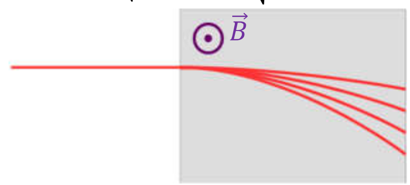
$$r = \frac{mv}{|q|B}$$



Spectrometers

Consider nuclei that have been accelerated across a potential difference ΔV yielding a radius of curvature in the magnetic field

$$r = \frac{1}{B}\sqrt{\frac{2m\Delta V}{q}} \sim \sqrt{\frac{m}{q}} \sim \sqrt{\frac{A}{Z}}$$

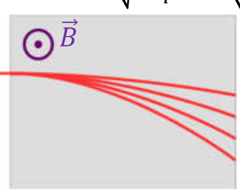


Spectrometers

Isotope	$\sqrt{A/Z}$
${}^{13}_{7}\mathrm{N}$	$\sqrt{13/7}$
${}^{14}_{7}\mathrm{N}$	$\sqrt{2}$
${}^{15}_{7}\mathrm{N}$	$\sqrt{15/7}$
${}^{16}_{8}\mathrm{O}$	$\sqrt{2}$
${}^{17}_{8}\mathrm{O}$	$\sqrt{17/8}$
${}^{18}_{8}\mathrm{O}$	$\sqrt{9/4}$

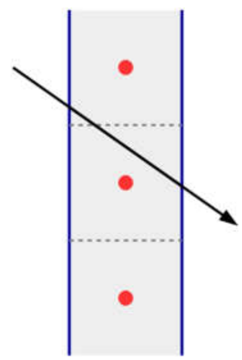
Separation of fully ionized isotopes by mass to charge ratio.

$$r = \frac{1}{B}\sqrt{\frac{2m\Delta V}{q}} \sim \sqrt{\frac{m}{q}} \sim \sqrt{\frac{A}{Z}}$$



Drift Chambers

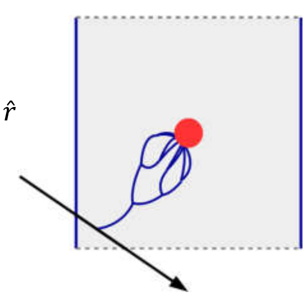
Drift chambers signal the passage of ionizing particles.



Drift Chambers

- Passing particle ionizes atoms in gas.
- Free electrons accelerate towards cathode.
- Electrons ionize more atoms on way to cathode.
- Momentary measureable current created.

$$\vec{F} = k\frac{q_1 q_2}{r^2}\hat{r}$$



$$\vec{F} = q\vec{E}$$

Accelerator Physics

Examples from today's lecture:

- Particles accelerated with electric fields.
- Particle beams steered with magnetic fields.
- Particle beams focused with magnetic fields.
- Particle velocities selected with crossed electric and magnetic fields.
- Particles identified with magnetic fields.
- Particle location and direction determined with electric fields.