Magnetism

Magnetic “charges”

- Called poles
- Two types, North and South
- Like poles repel each other
- Opposite poles attract each other
- Found only in North/South pairs (Dipoles)

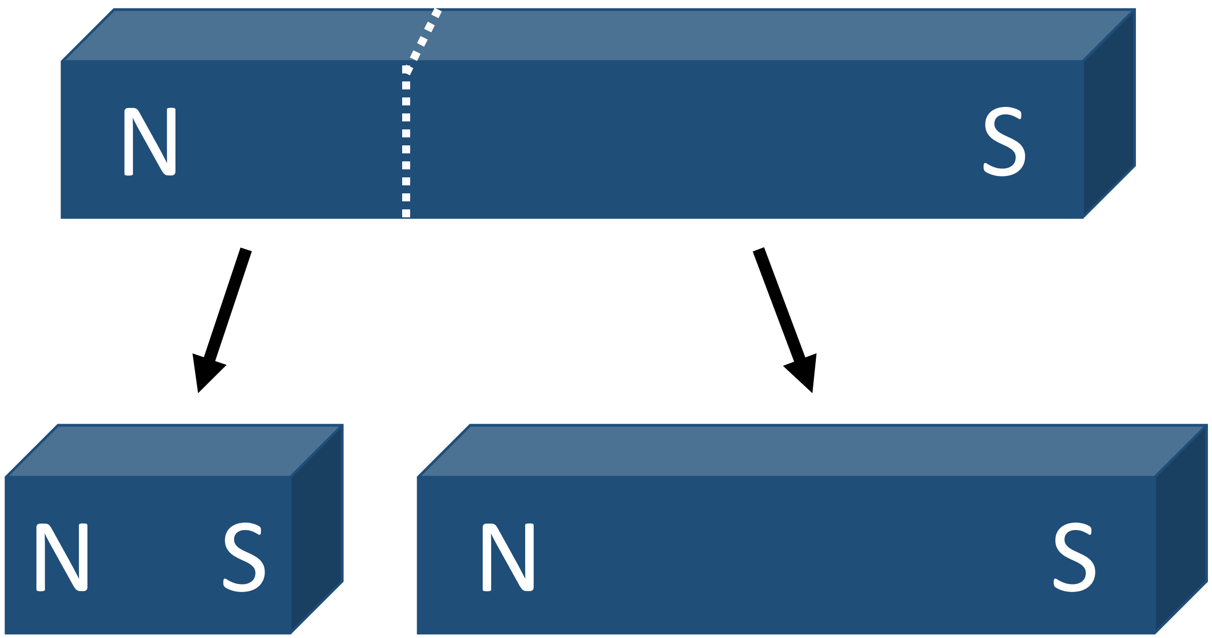
Magnetism

Magnetic poles

- Found only in North/South pairs

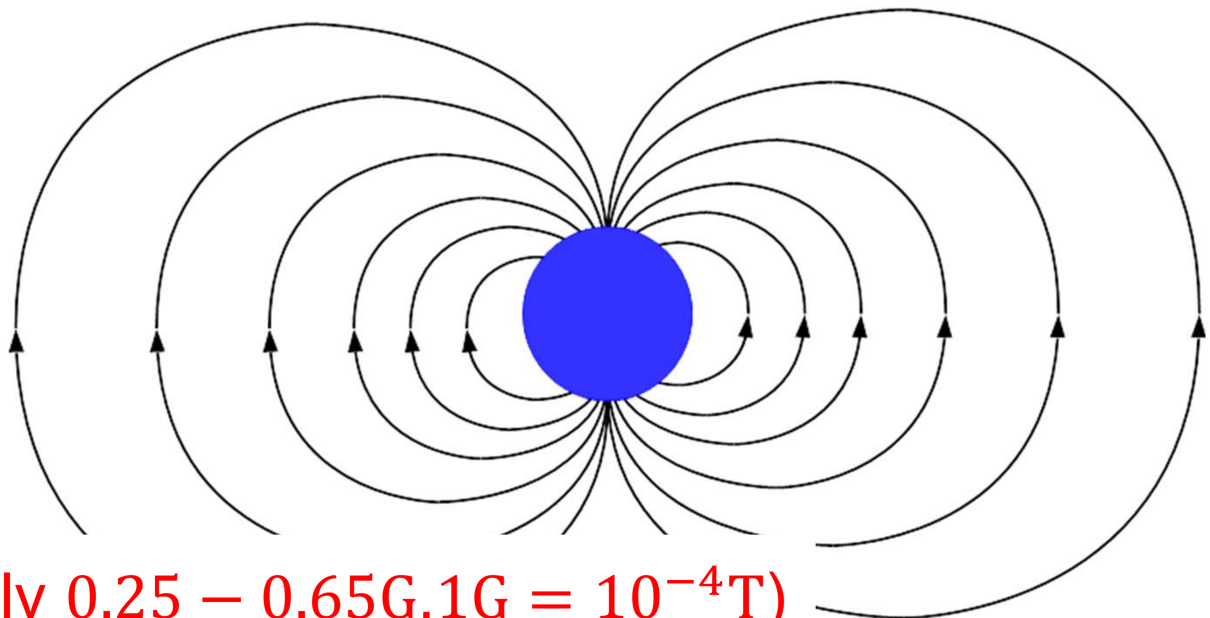
Cutting a magnet in two will not isolate the poles.

Cutting a magnet in two produces two magnets.



Earth as a Magnet

The strength and orientation of the earth's magnetic field varies over time and location.

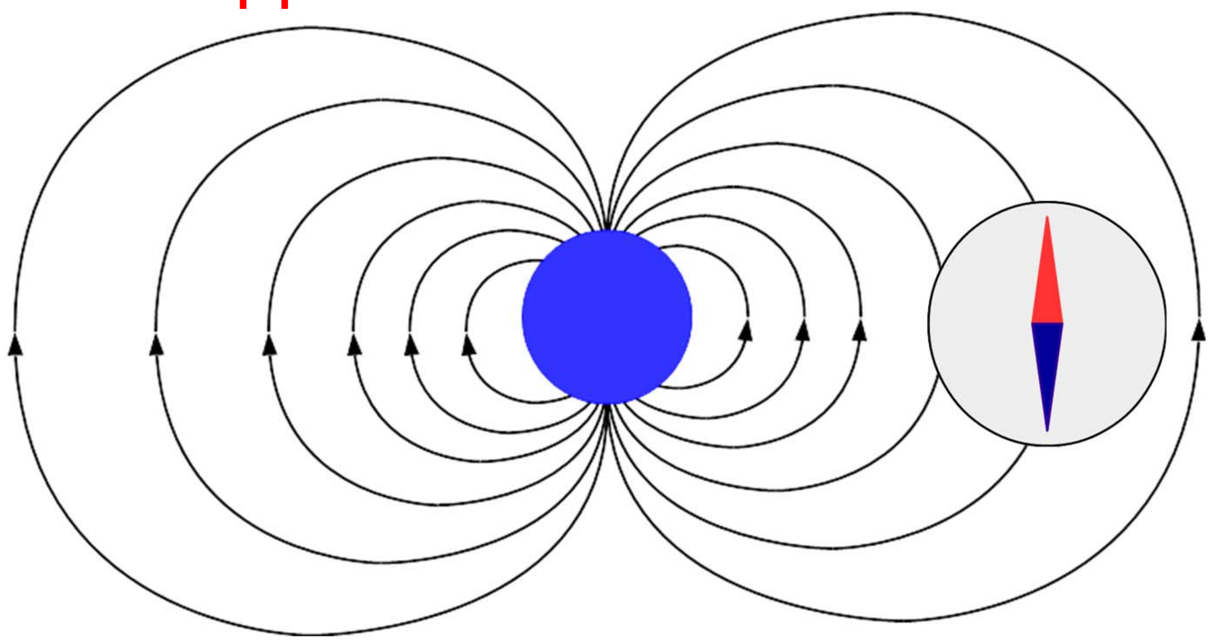


(Currently $0.25 - 0.65\mathrm{G}, 1\mathrm{G} = 10^{-4}\mathrm{T}$)

The earth's magnetic poles are "near" the geographic poles (where the axis of rotation intersects the earth's surface).

Earth as a Magnet

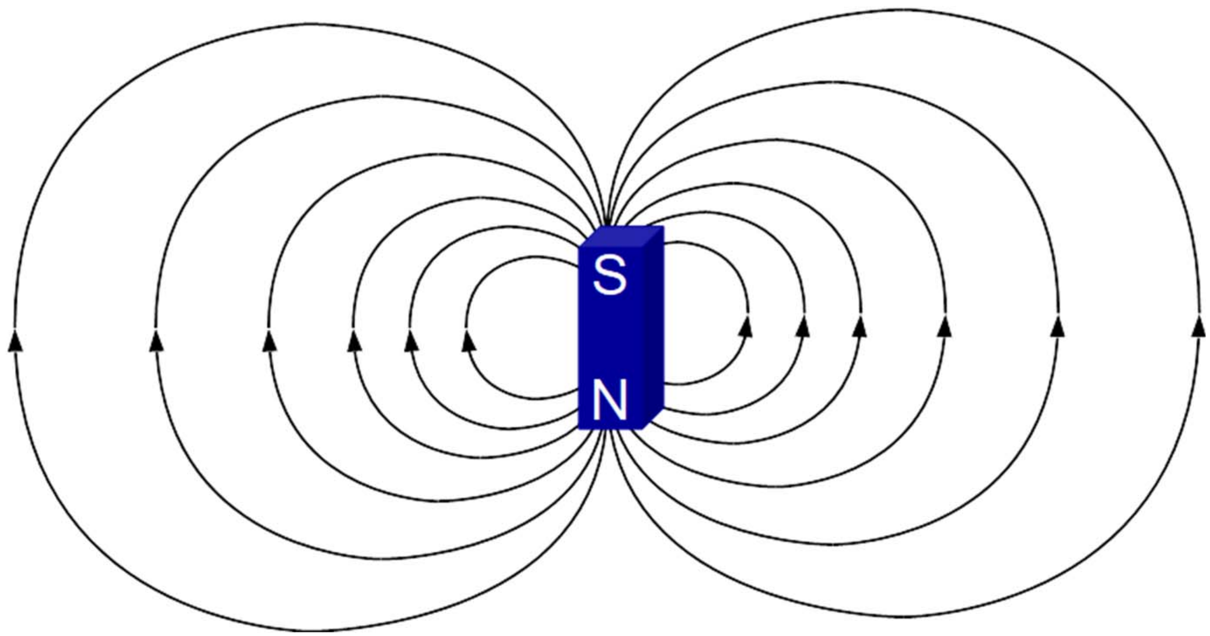
North Pole of compass points towards Geographic North Pole. Opposites attract.



Geographic North Pole is (near) Magnetic South Pole and Geographic South Pole is (near) Magnetic North Pole.

Magnetic Field Lines

Originate at North Poles
Terminate at South Poles



Magnetic Forces

Magnetic fields can produce forces

$$\vec{F} = q\vec{v} \times \vec{B}$$

- No forces on particles at rest
- No forces on particles moving parallel to field
- Force is perpendicular to field
- Force is perpendicular to particle's velocity

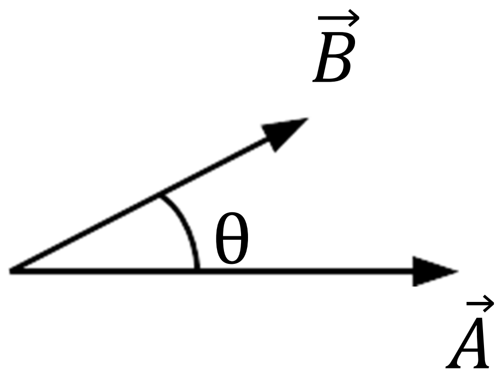
Cross Product Review

$$\vec{A} \times \vec{B}$$

Magnitude of cross product is the product of perpendicular components.

$$\left|\vec{A} \times \vec{B}\right| = AB \sin\theta$$

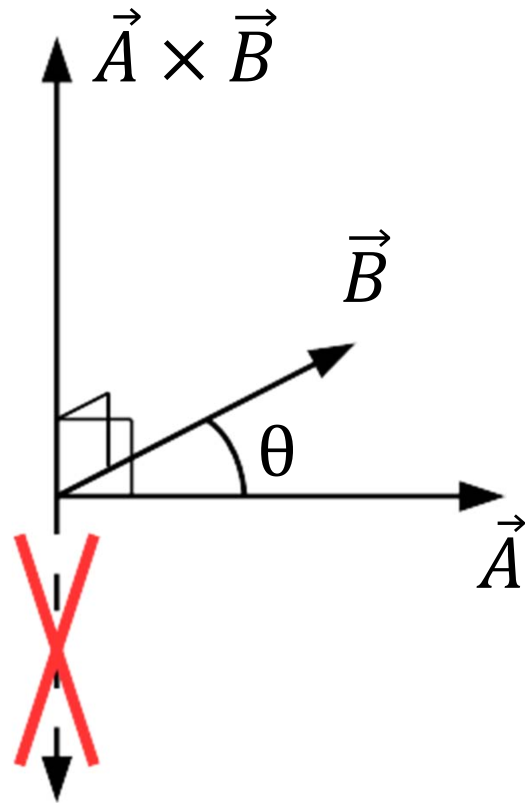
where θ is the angle between $\vec{A}$ and $\vec{B}$.



Cross Product Review

$$\vec{A} \times \vec{B}$$

Direction of cross product is the direction perpendicular to both $\vec{A}$ and $\vec{B}$ with the ambiguity removed by application of the "Right Hand Rule".

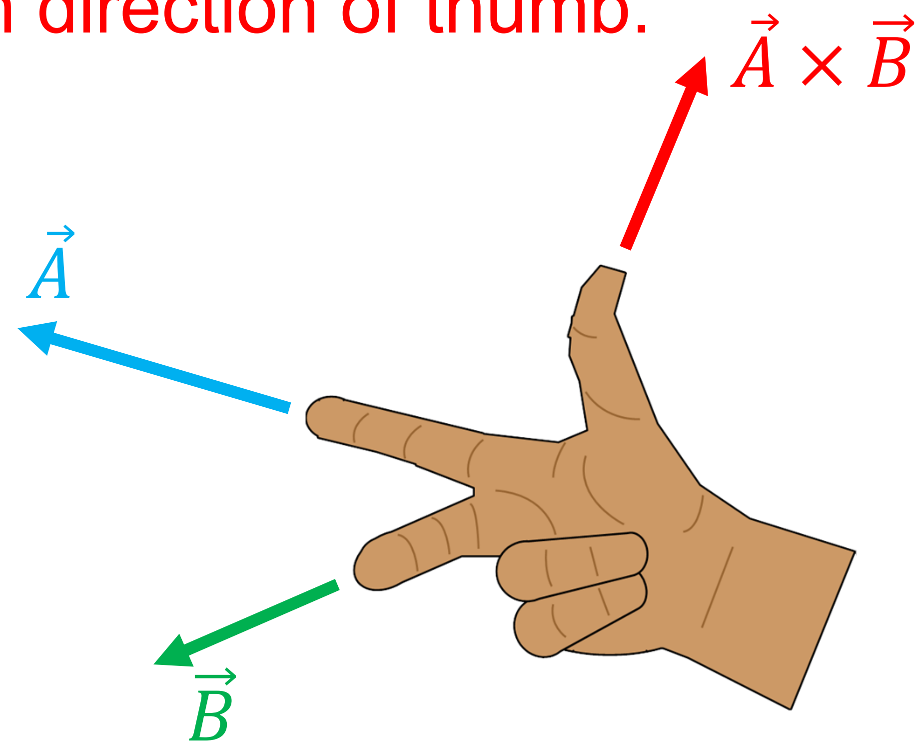


Cross Product Review
Right Hand Rule

Point 1st finger in direction of $\vec{A}$.

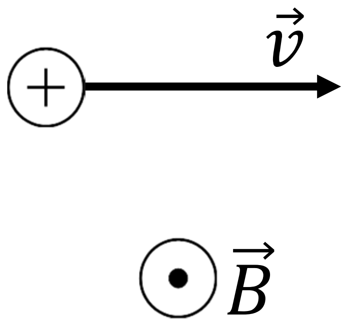
Point 2nd finger in direction of $\vec{B}$.

Cross product is in direction of thumb.

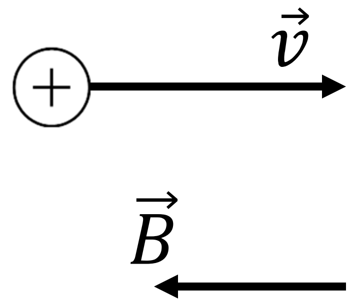


Examples: Determine the direction of the force for each combination of charge, velocity and magnetic field.

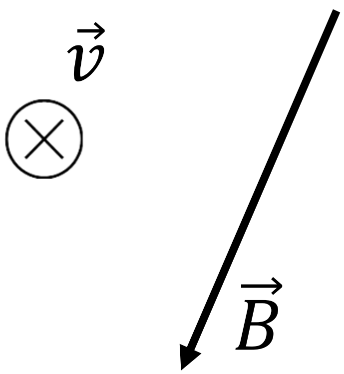
Proton



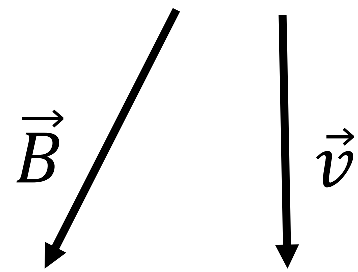
Proton



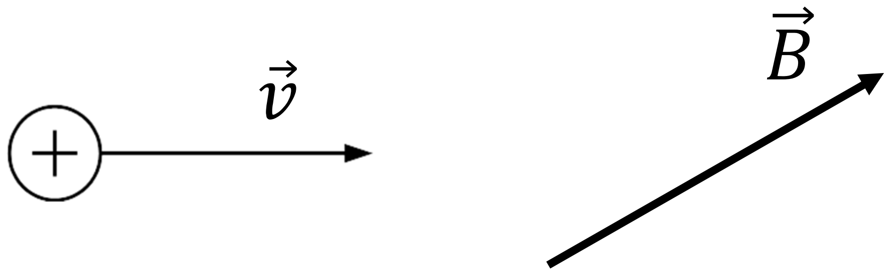
Electron



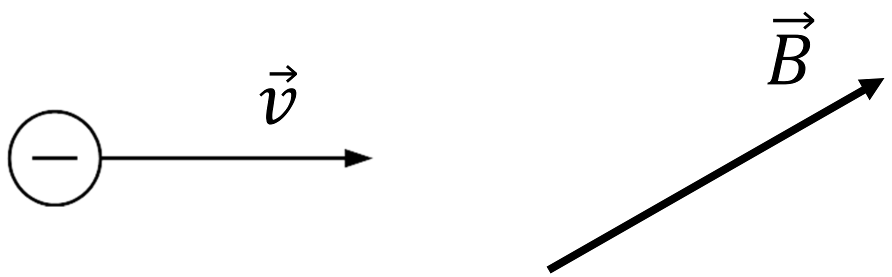
Proton



Example: Determine the force acting on a proton moving with speed $v_0 = 200\text{m}/s$ at an angle of $30°$ relative to a magnetic field, $B = 0.04\text{T}$.



Example: Determine the force acting on an electron moving with speed $v_0 = 200\text{m}/s$ at an angle of $30°$ relative to a magnetic field, $B = 0.04\text{T}$.



Cross Product Review
Multiplying Components

$$\vec{A} \times \vec{B} = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ \hat{\imath} & \hat{\jmath} & \hat{k} \end{vmatrix}$$

$$= \left(A_y B_z - A_z B_y\right)\hat{\imath} + (A_z B_x - A_x B_z)\hat{\jmath} + \left(A_x B_y - A_y B_x\right)\hat{k}$$

Cross Product Review
Multiplying Components

$$\vec{A} \times \vec{B} = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ \hat{\imath} & \hat{\jmath} & \hat{k} \end{vmatrix}$$

$$= \left(A_y B_z - A_z B_y\right)\hat{\imath} + (A_z B_x - A_x B_z)\hat{\jmath} + \left(A_x B_y - A_y B_x\right)\hat{k}$$

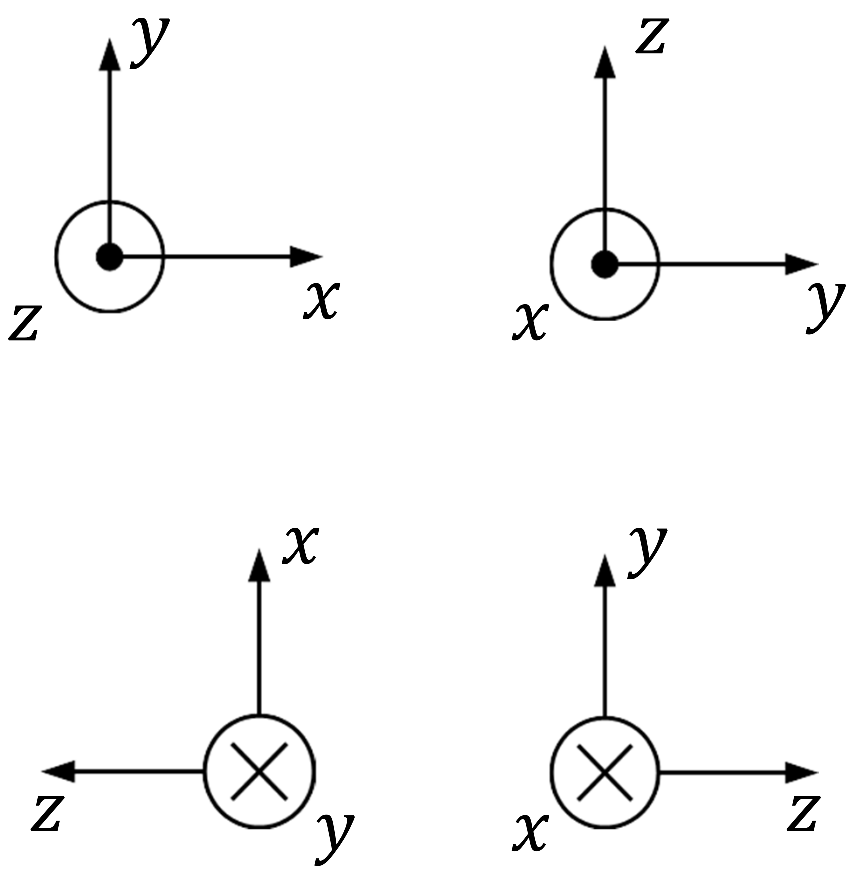
Note that

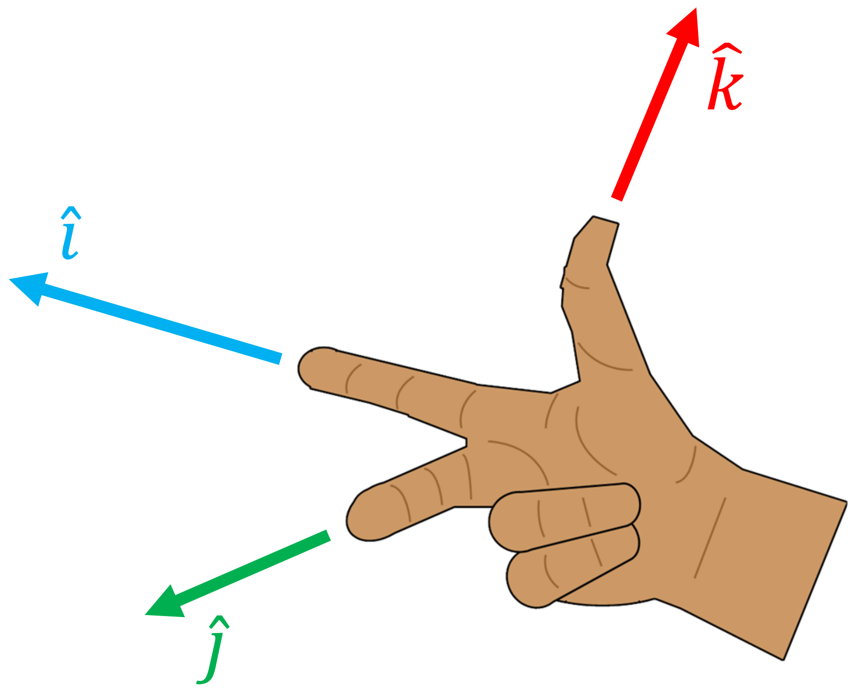
$\hat{\imath} \times \hat{\jmath} = \hat{k}$ $\quad$ $\hat{\jmath} \times \hat{k} = \hat{\imath}$ $\quad$ $\hat{k} \times \hat{\imath} = \hat{\jmath}$

$\hat{\jmath} \times \hat{\imath} = -\hat{k}$ $\quad$ $\hat{k} \times \hat{\jmath} = -\hat{\imath}$ $\quad$ $\hat{\imath} \times \hat{k} = -\hat{\jmath}$

Cross Product Review

All versions of determining the cross product assume a right-handed coordinate system.





Example: An object with charge, $q = 5\mathrm{C}$, is moving with initial velocity, $\vec{v}_0 = 2(\mathrm{m/s})\hat{\imath} - 3(\mathrm{m/s})\hat{\jmath}$, in a region with a uniform magnetic field, $\vec{B} = -4\mathrm{T}\hat{\imath} + 4\mathrm{T}\hat{\jmath} + 5\mathrm{T}\hat{k}$. Determine the initial force on the object.

Example: An electron entering a region of uniform magnetic field, $\vec{B} = 0.50\mathrm{T}\hat{\jmath}$, experiences a force, $\vec{F} = 3.28 \times 10^{-13}\mathrm{N}\hat{k}$. Determine the initial velocity of the electron.

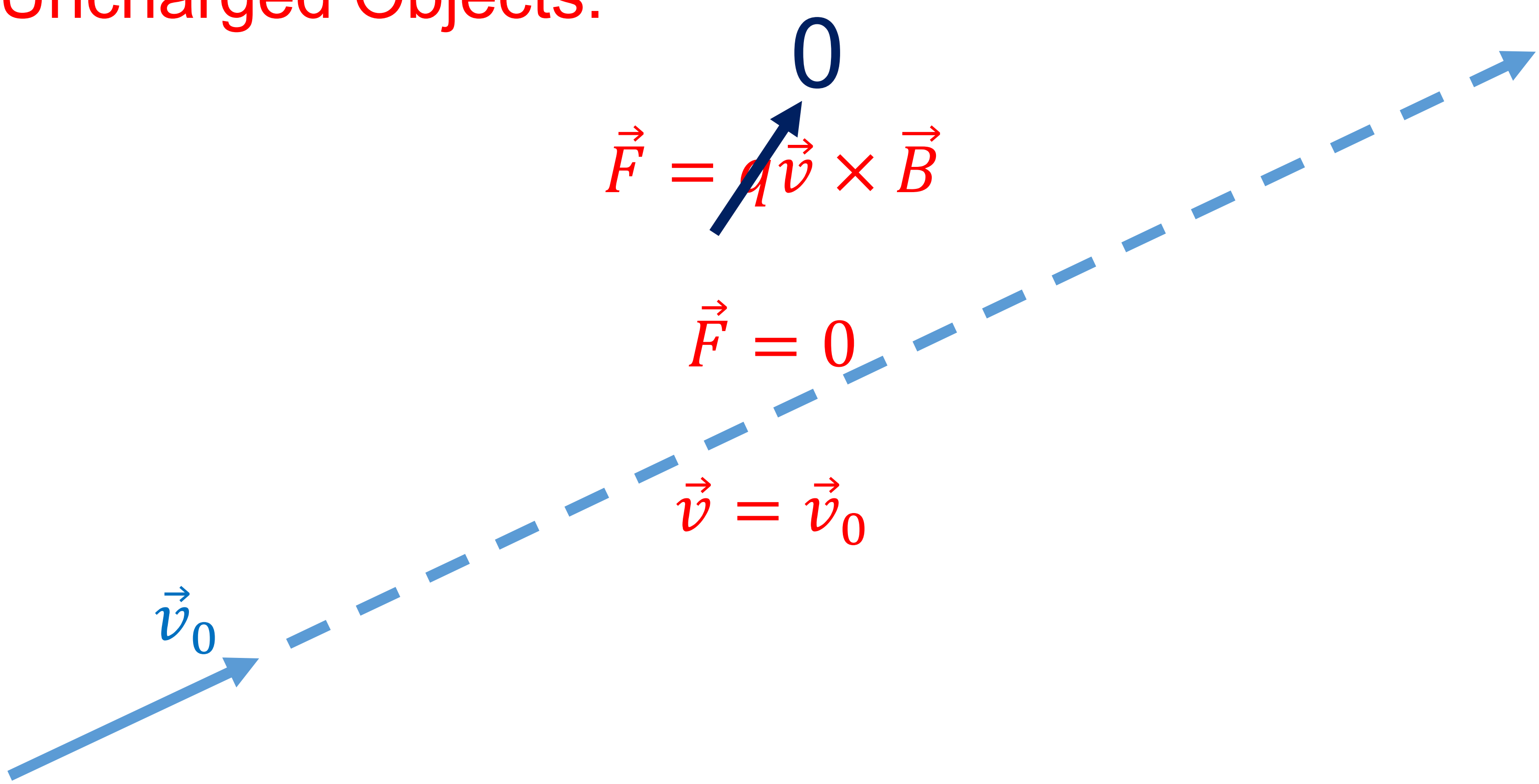
Motion in a Magnetic Field

Uncharged Objects:

$$\vec{F} = q\vec{v} \times \vec{B}$$

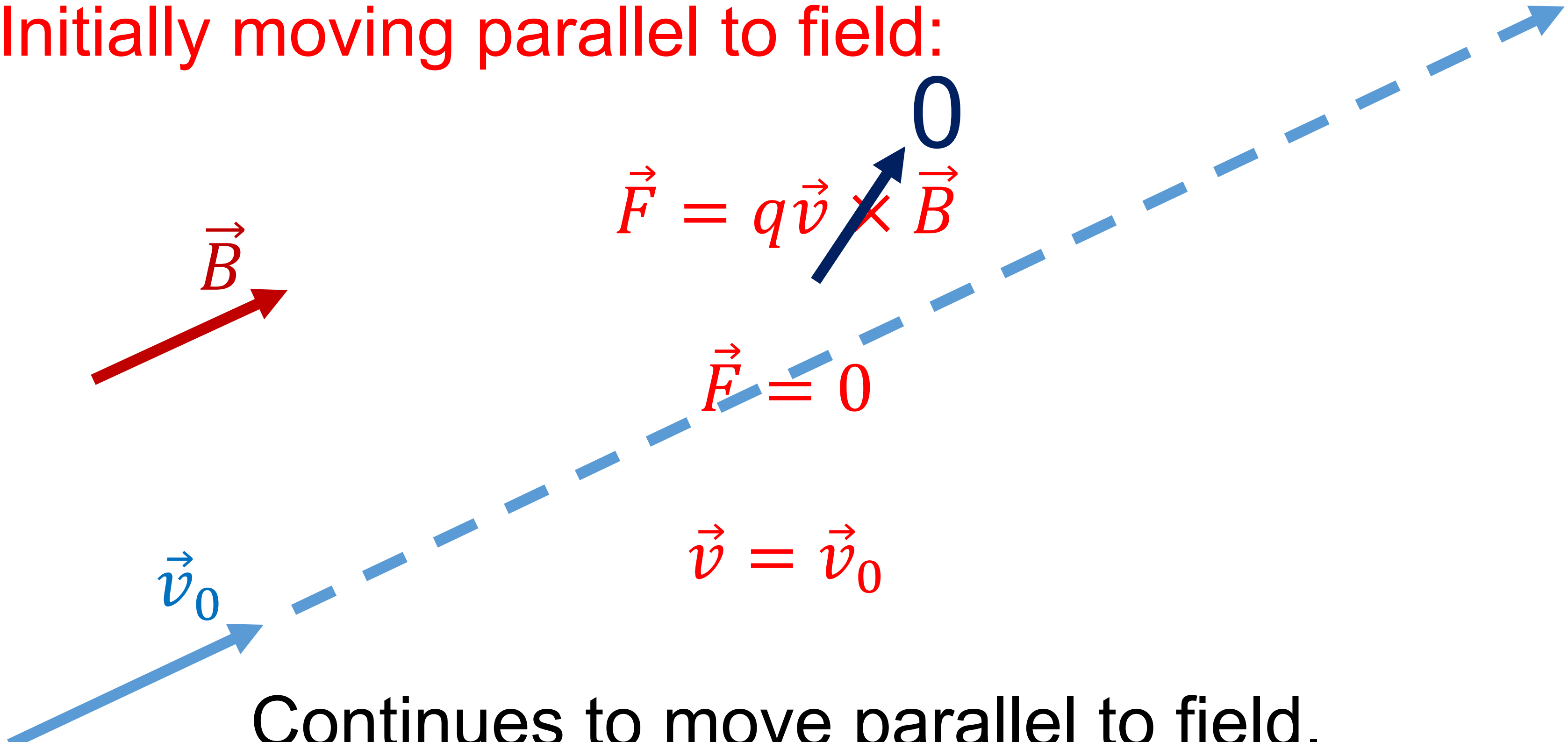
$$\vec{F} = 0$$

$$\vec{v} = \vec{v}_0$$



Motion of Charged Object in a Uniform Magnetic Field

Initially moving parallel to field:



$$\vec{F} = q\vec{v} \times \vec{B}$$

$$\vec{F} = 0$$

$$\vec{v} = \vec{v}_0$$

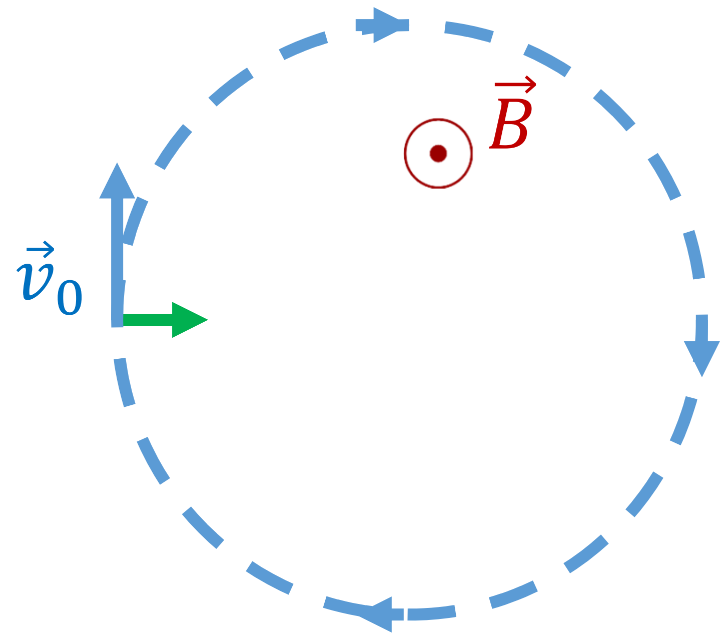
Continues to move parallel to field.

Motion of Charged Object in a Uniform Magnetic Field

Initially moving perpendicular to field:

$\vec{F} = q\vec{v} \times \vec{B}$

Circular motion:



Motion of Charged Object in a Uniform Magnetic Field

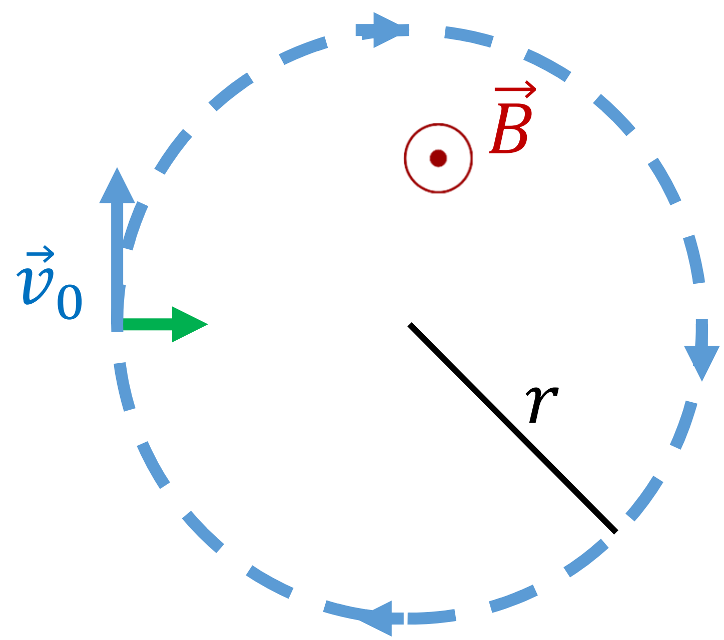
Initially moving perpendicular to field:

$\vec{F} = q\vec{v} \times \vec{B}$

Force, $F_r = -|q|vB$

Circular motion:

$$F_r = -\frac{mv_t^2}{r}$$



Motion of Charged Object in a Uniform Magnetic Field

Initially moving perpendicular to field:

$\vec{F} = q\vec{v} \times \vec{B}$

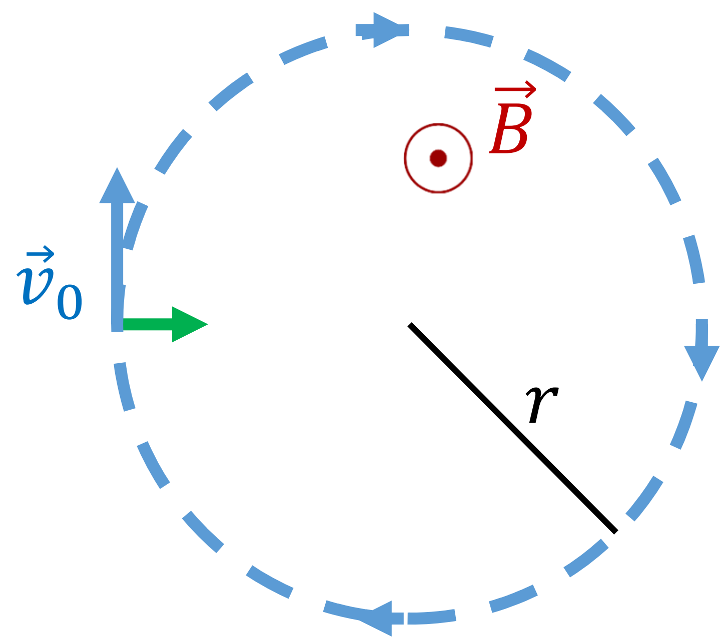
Force, $F_r = -|q|vB$

Circular motion:

$$F_r = -\frac{mv_t^2}{r}$$

$$-|q|v_0B = -\frac{mv_0^2}{r}$$

$$r = \frac{mv_0}{|q|B}$$



Motion of Charged Object in a Uniform Magnetic Field

Initially moving perpendicular to field:

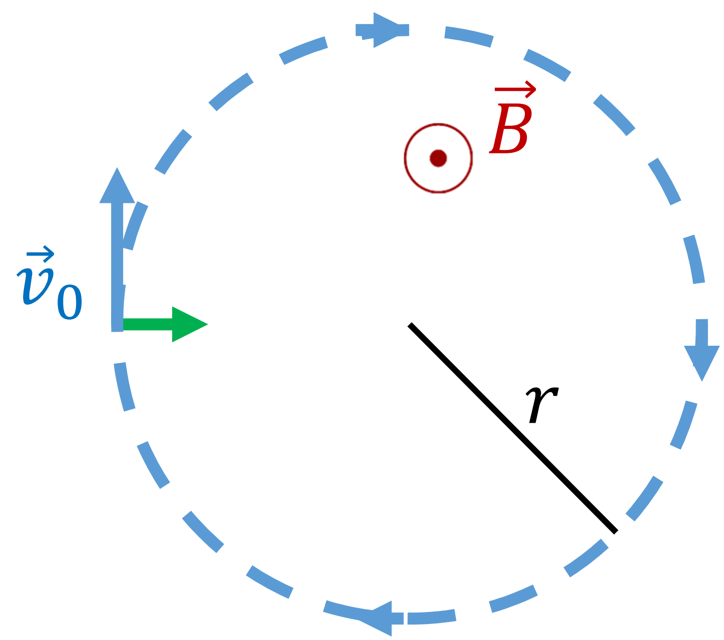
Force, $F_r = -|q|vB$

Radius, $r = \frac{mv_0}{|q|B}$

Period:

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{\left(\frac{|q|Br}{m}\right)}$$

$$T = \frac{2\pi m}{|q|B}$$



Motion of Charged Object in a Uniform Magnetic Field

Initially moving perpendicular to field:

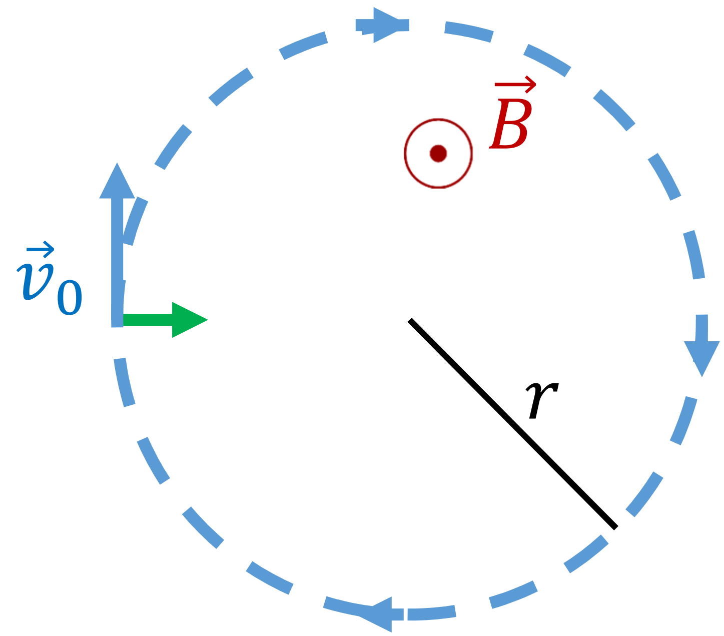
Force, $F_r = -|q|vB$

Radius, $r = \frac{mv_0}{|q|B}$

Period, $T = \frac{2\pi m}{|q|B}$

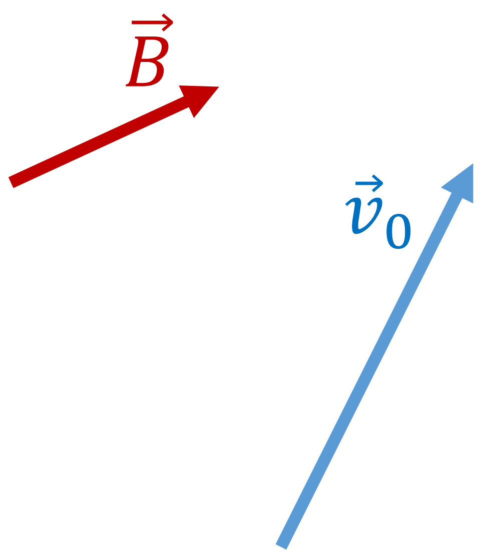
Frequency:

$$f = \frac{1}{T} = \frac{|q|B}{2\pi m}$$



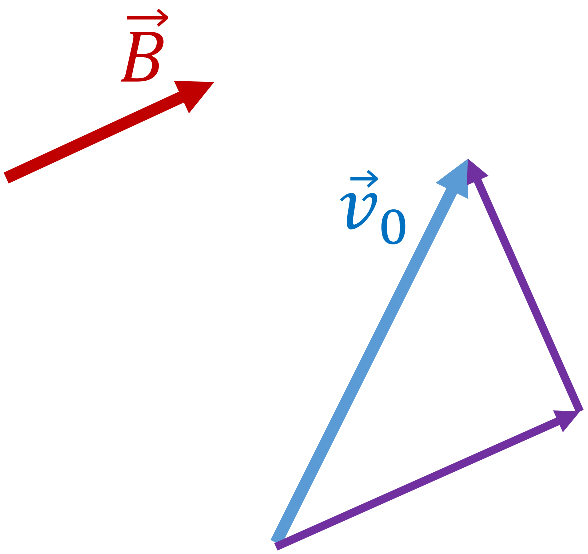
Motion of Charged Object in a Uniform Magnetic Field

Motion neither parallel nor perpendicular to field:



Motion of Charged Object in a Uniform Magnetic Field

Motion neither parallel nor perpendicular to field:



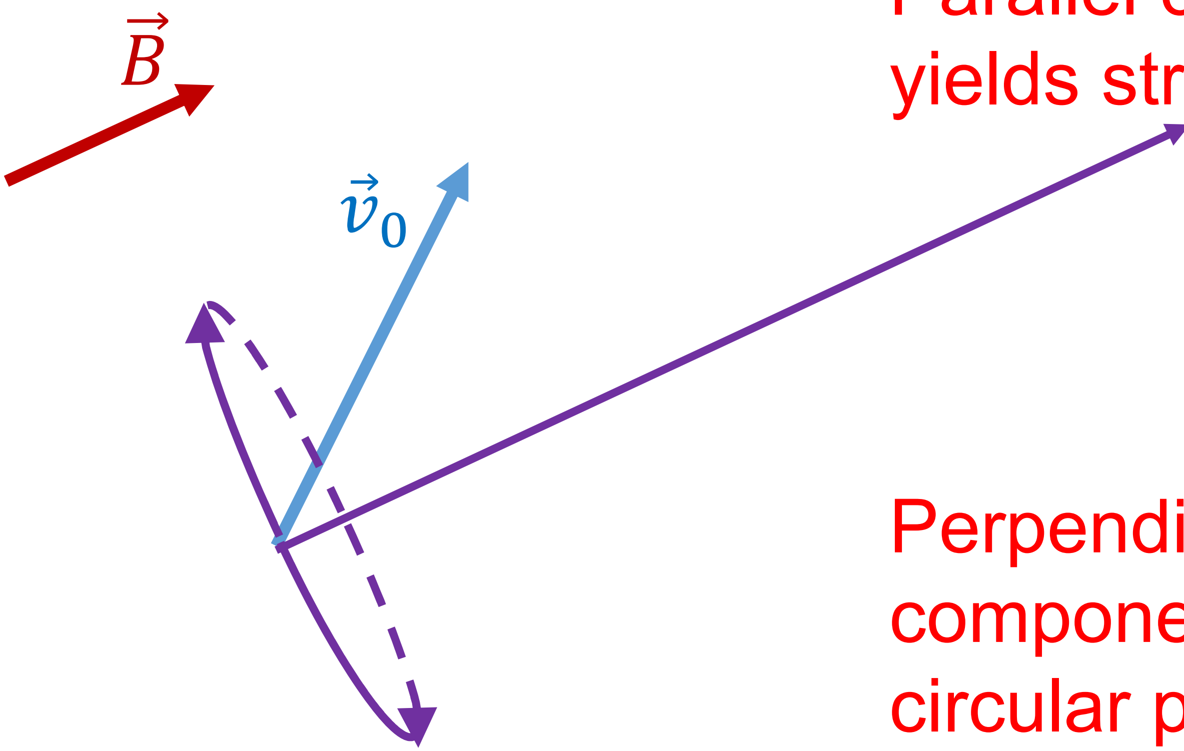
Velocity broken into two components

- Parallel to $\vec{B}$
- Perpendicular to $\vec{B}$

Motion of Charged Object in a Uniform Magnetic Field

Motion neither parallel nor perpendicular to field:

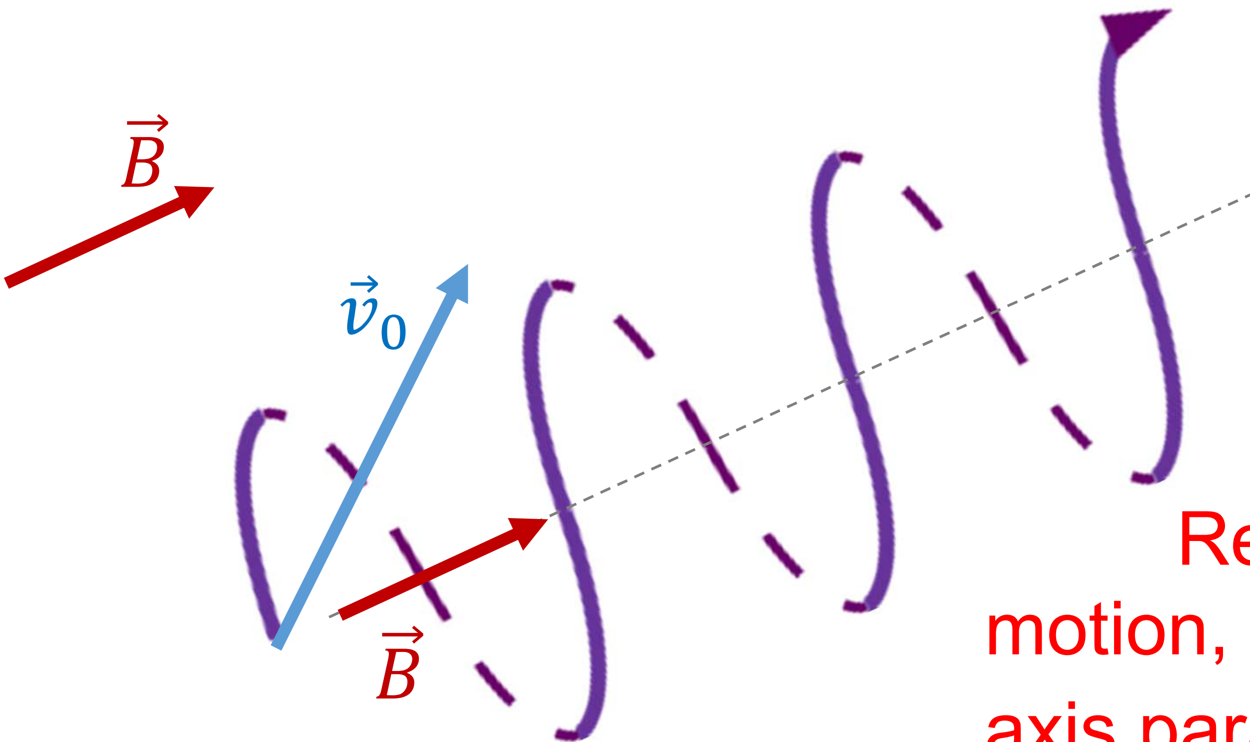
Parallel component yields straight line path



Perpendicular component yields circular path.

Motion of Charged Object in a Uniform Magnetic Field

Motion neither parallel nor perpendicular to field:



Result is helical motion, centered on axis parallel to $\vec{B}$.

Motion in Electric and Magnetic Fields
Lorentz Force

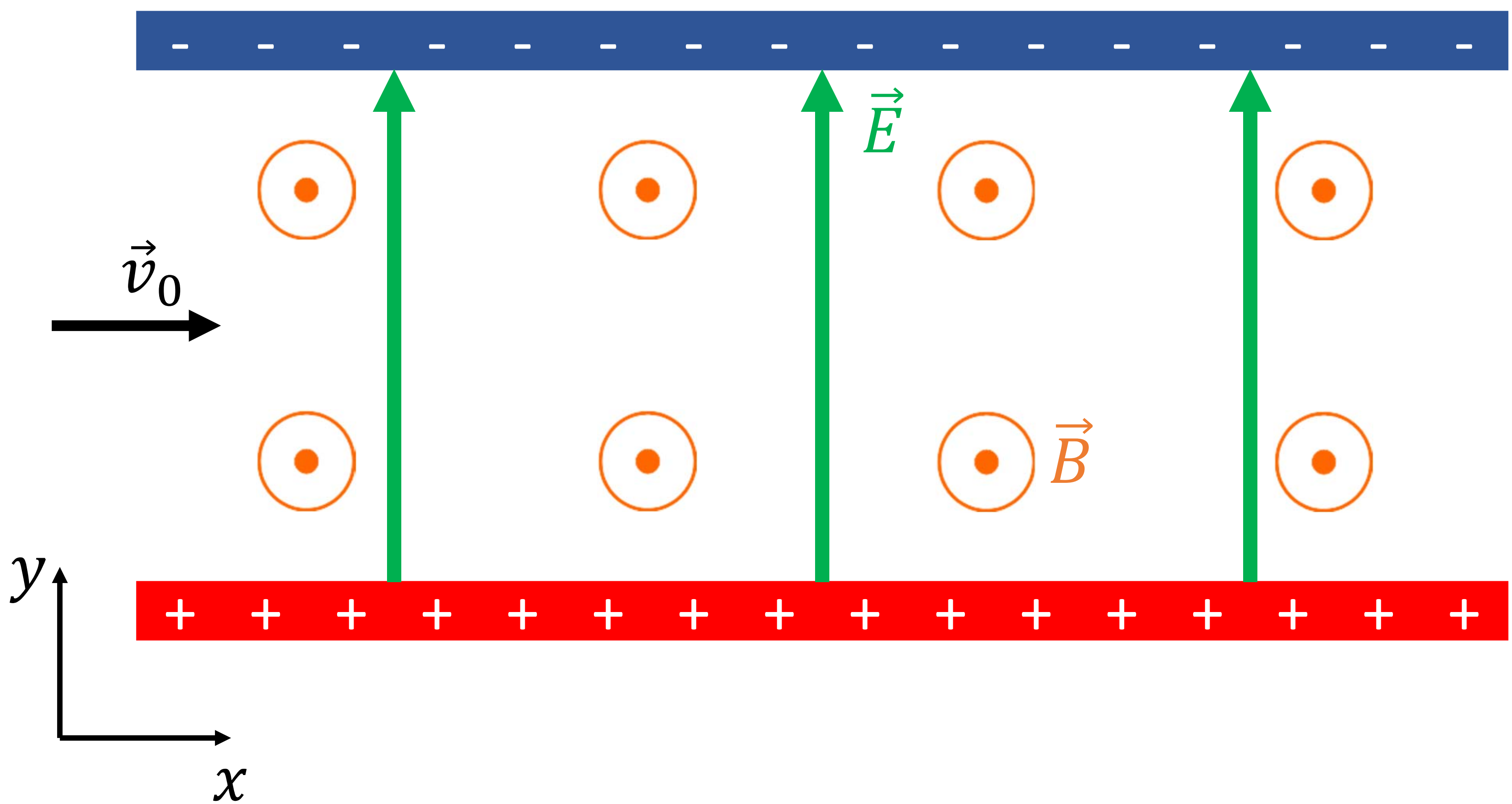
Combining $\vec{F} = q\vec{E}$ and $\vec{F} = q\vec{v} \times \vec{B}$

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

Some conditions result in simple motion:

- Uniform $\vec{E}$ with $\vec{B} = 0$
- Uniform $\vec{B}$ with $\vec{E} = 0$
- $\vec{E} \perp \vec{v}_0$, $\vec{B} \perp \vec{v}_0$ and $\vec{B} \perp \vec{E}$ (Crossed fields)

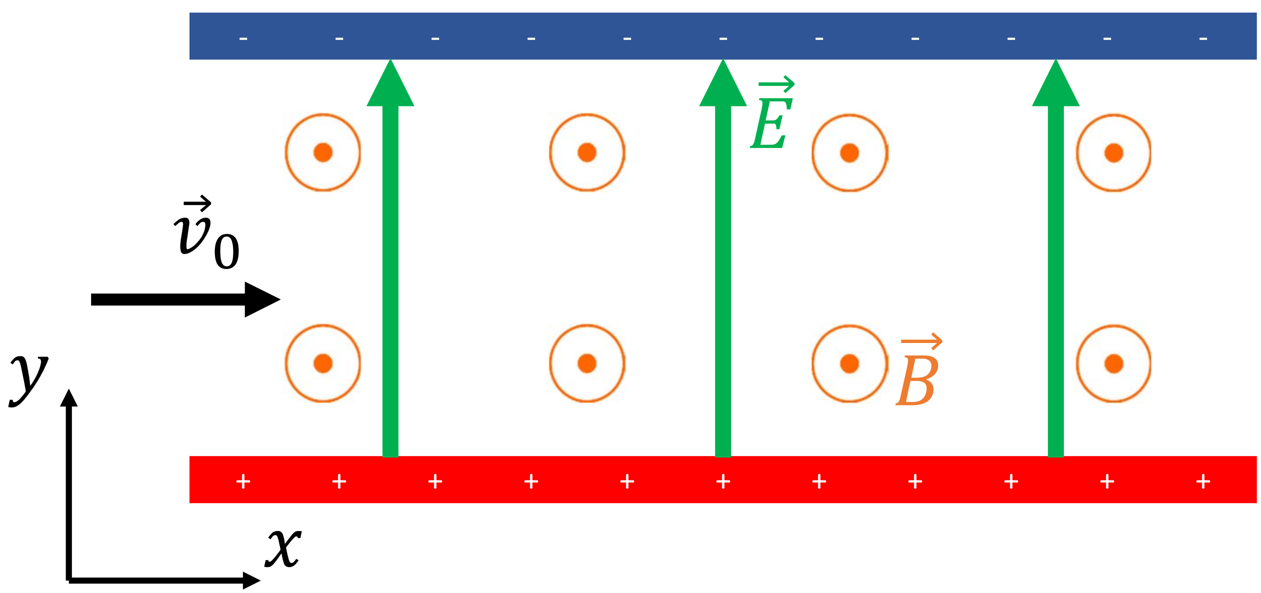
Crossed Fields
Velocity Selector



Crossed Fields
Velocity Selector

$$\vec{v} = v_0\hat{\imath}$$
$$\vec{E} = E\hat{\jmath}$$
$$\vec{B} = B\hat{k}$$

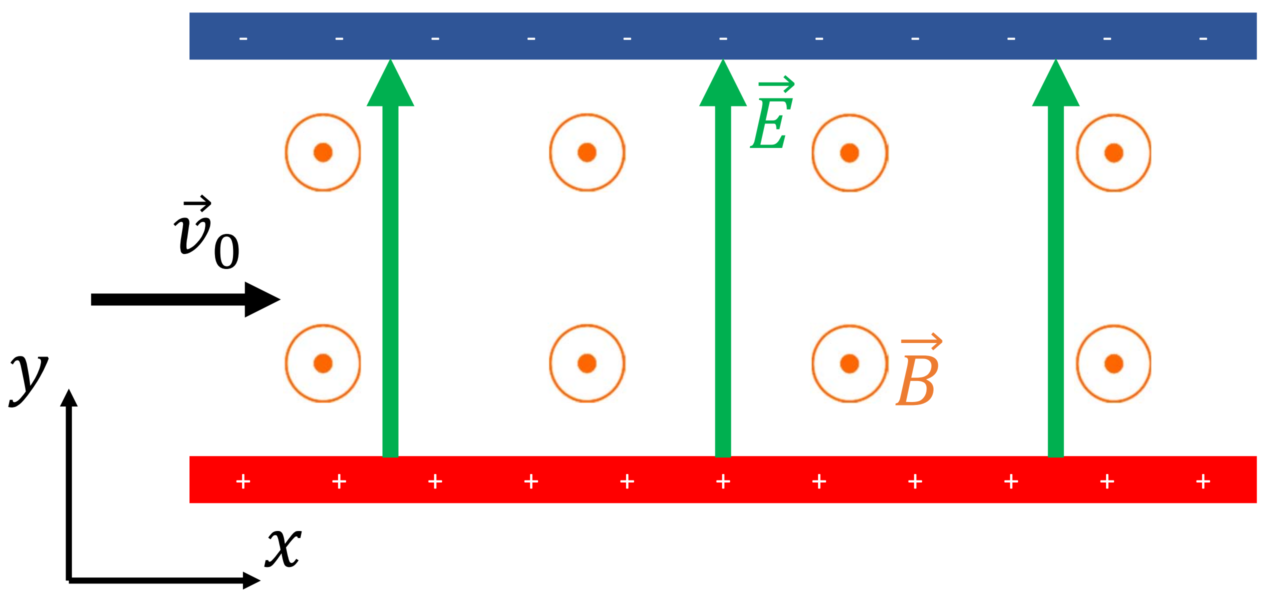
$$\vec{F} = q\left(E\hat{\jmath} + v_0\hat{\imath} \times B\hat{k}\right)$$
$$\vec{F} = q(E - v_0 B)\hat{\jmath}$$



Crossed Fields
Velocity Selector

$$\left.\begin{aligned}\vec{v} &= v_0\hat{\imath}\\ \vec{E} &= E\hat{\jmath}\\ \vec{B} &= B\hat{k}\end{aligned}\right\} \quad \begin{aligned}\vec{F} &= q\left(E\hat{\jmath} + v_0\hat{\imath} \times B\hat{k}\right)\\ \vec{F} &= q(E - v_0 B)\hat{\jmath}\end{aligned}$$

If $v_0 = \frac{E}{B}$, then $\vec{F} = 0$ and the trajectory is a straight line.



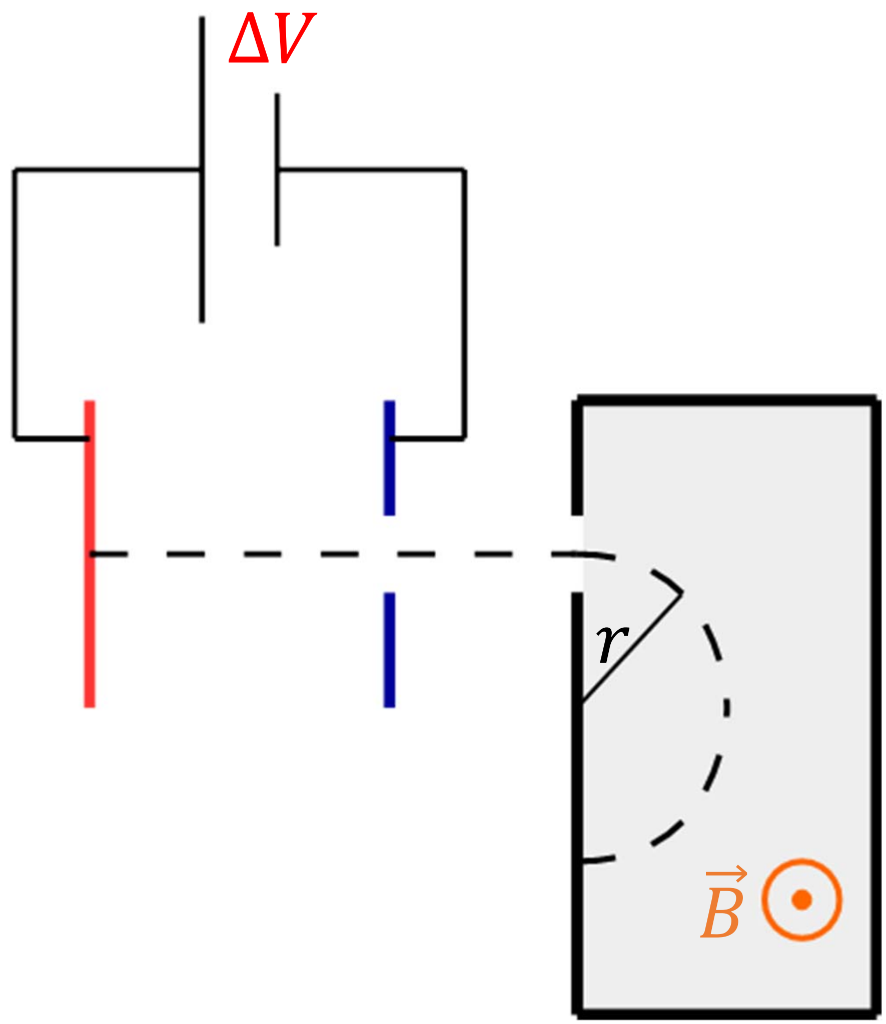
Mass Spectrometer

Charge accelerates between plates, gaining energy, $\Delta \mathrm{U} = \mathrm{q}\Delta V$.

Velocity is function of ΔV.

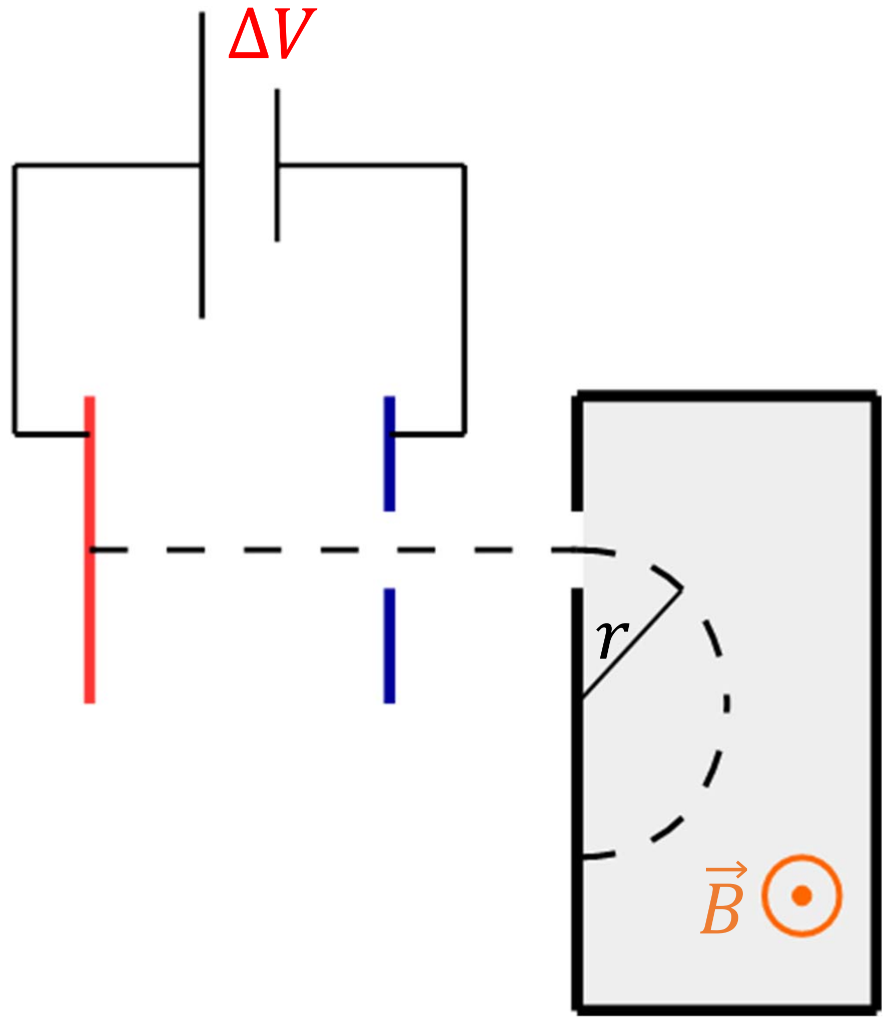
$$\frac{1}{2}mv^2 = q\Delta V$$

$$v = \sqrt{\frac{2q\Delta V}{m}}$$



Mass Spectrometer

Charge moves in circular path in spectrometer.



Radius of path is function of mass.

$$qvB = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB}$$

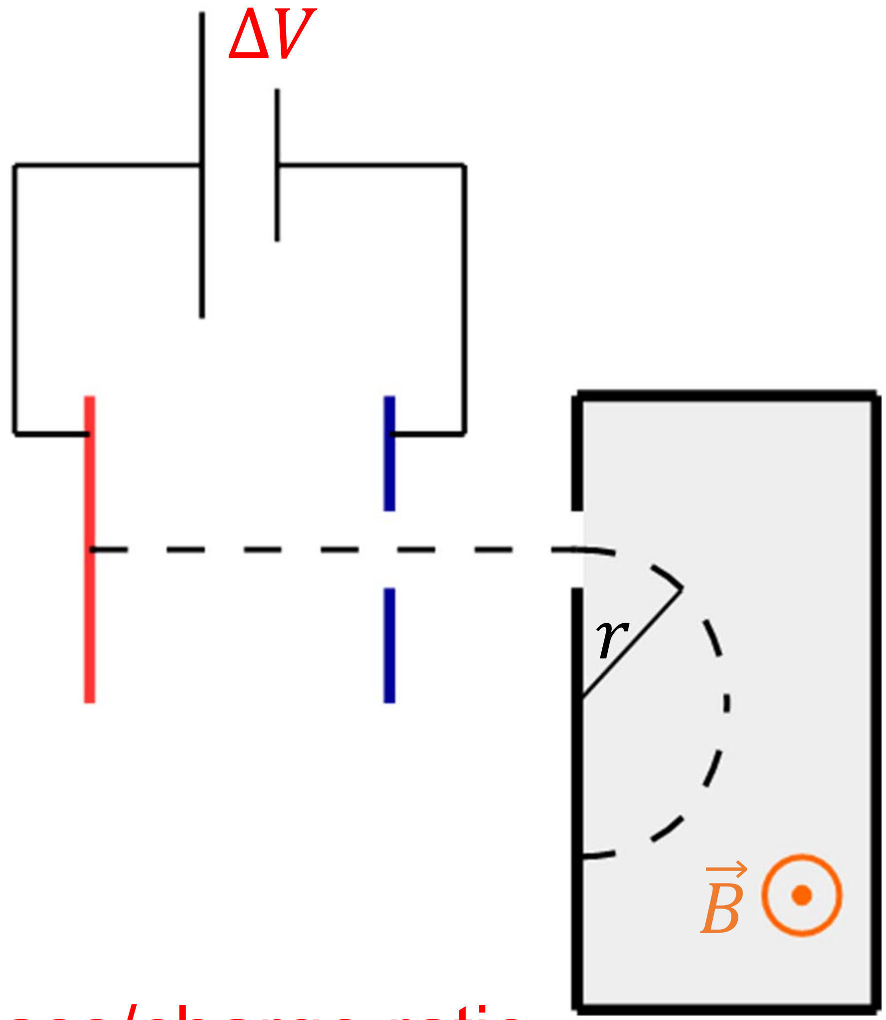
Mass Spectrometer

Combining

$$v = \sqrt{\frac{2q\Delta V}{m}} \quad \& \quad r = \frac{mv}{qB}$$

yields

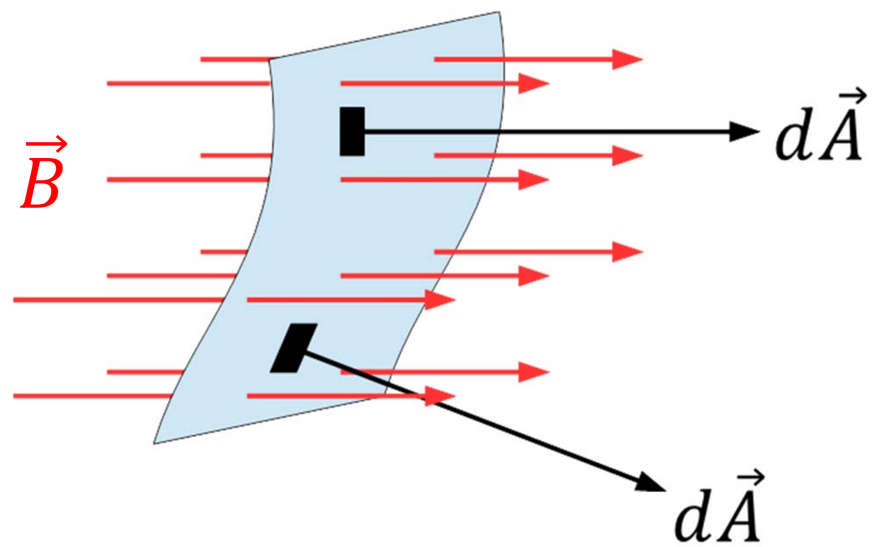
$$r = \frac{1}{B}\sqrt{\frac{2m\Delta V}{q}}$$



Radius is a function of mass/charge ratio.

Magnetic Flux
“Counting field lines” through a surface

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$



Area vector
- Magnitude of area
- Direction normal (perpendicular) to surface

Gauss's Law for Magnetism

$$\Phi_B = \oint \vec{B} \cdot d\vec{A} \sim q_{B,\text{enclosed}}$$

- q_B refers to magnetic charge.
- Magnetic charges are called poles (North/South) and come in pairs with net magnetic charge of 0.
- A single magnetic charge is called a monopole. No monopole has ever been observed.

$$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0$$

Gauss's Law for Magnetism

$$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0$$

- There are no magnetic monopoles.
- All magnetic field lines entering a closed surface also exit the closed surface.

Magnetic flux through an open surface will be of more importance in this course than total flux through a closed surface.

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$