Today's agenda:

Measuring Instruments: ammeter, voltmeter, ohmmeter.

You must be able to calculate currents and voltages in circuits that contain "real" measuring instruments.

RC Circuits.

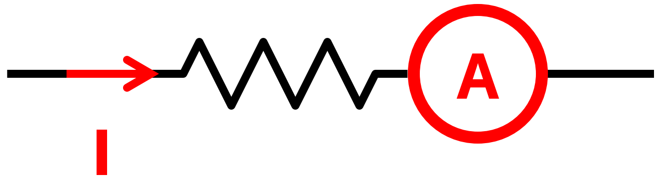
You must be able to calculate currents and voltages in circuits containing both a resistor and a capacitor. You must be able to calculate the time constant of an RC circuit, or use the time constant in other calculations.

Measuring current, voltage, and resistance

Ammeter:

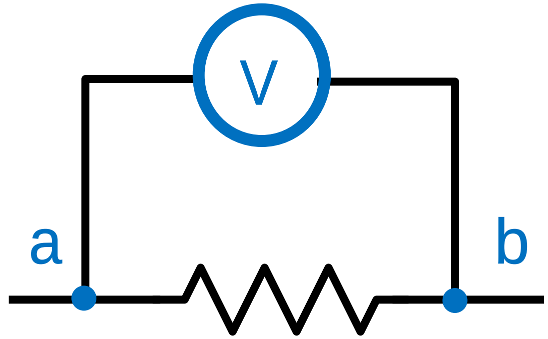
- measures current (A)
- connected **in series**

(current must go through instrument)



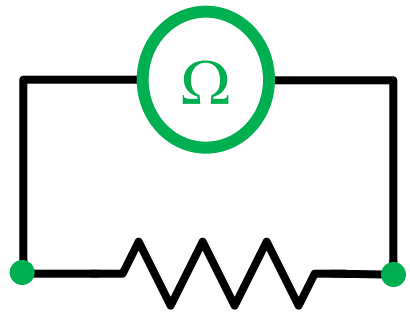
Voltmeter:

- measures potential difference (V)
- connected **in parallel**



Ohmmeter:

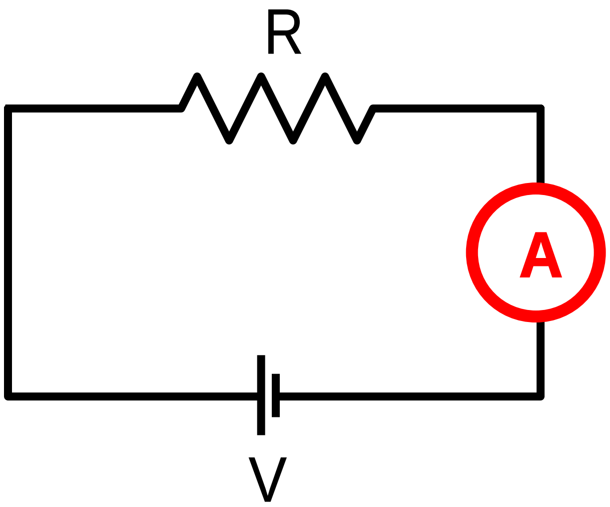
- measures resistance of an isolated resistor (not in a working circuit)



Effect of ammeter on circuit

Measuring current in a simple circuit:

- connect ammeter in series

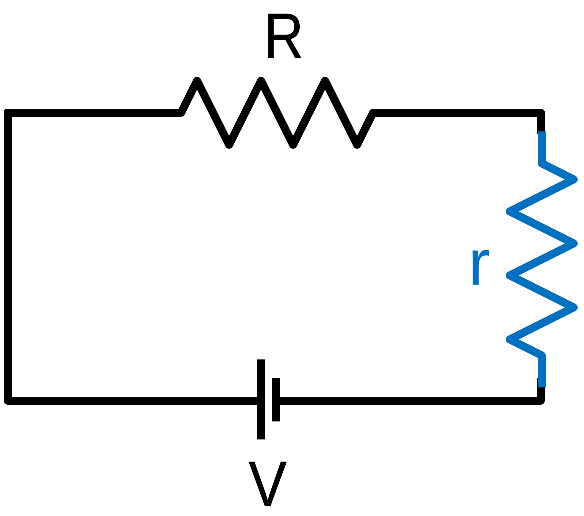


Are we measuring the correct current?

(the current in the circuit without ammeter)

Effect of ammeter on circuit

Measuring current in a simple circuit:



- connect ammeter in series

Are we measuring the correct current?
(the current in the circuit without ammeter)

- any ammeter has **some resistance r.**
- current in presence of ammeter is $I = \frac{V}{R + r}$.
- current without the ammeter would be $I = \frac{V}{R}$.

To minimize error, ammeter resistance r must be very small.
(ideal ammeter would have zero resistance)

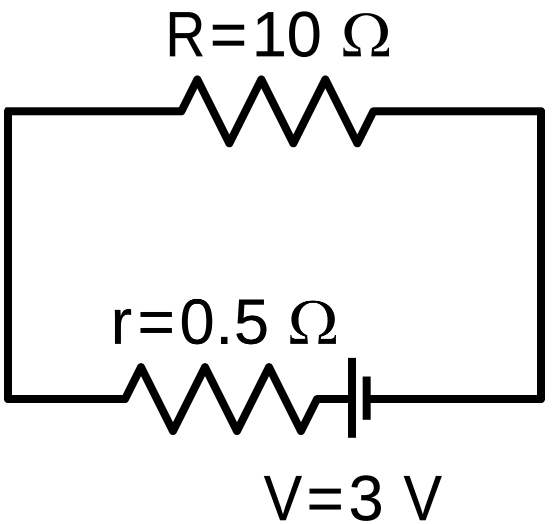
Example: an ammeter of resistance 10 mΩ is used to measure the current through a 10 Ω resistor in series with a 3 V battery that has an internal resistance of 0.5 Ω. What is the relative (percent) error caused by the ammeter?

Actual current **without** ammeter:

$$I = \frac{V}{R + r}$$

$$I = \frac{3}{10 + 0.5} A$$

$$I = 0.2857 \text{ A} = 285.7 \text{ mA}$$



You might see the symbol ε used instead of V.

Current with ammeter:

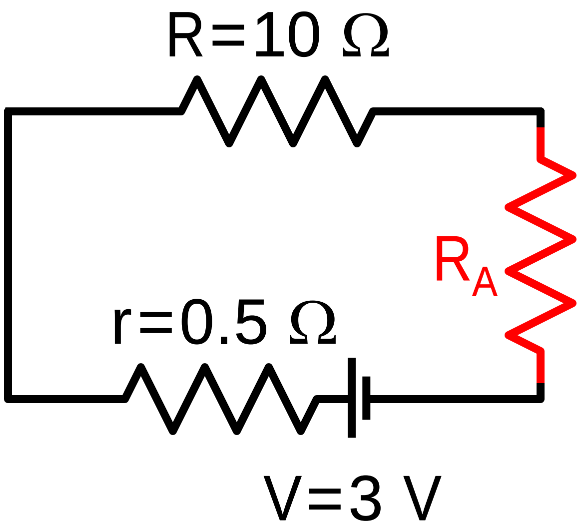
$$I = \frac{V}{R + r + R_A}$$

$$I = \frac{3}{10 + 0.5 + 0.01} A$$

$$I = 0.2854 \text{ A} = 285.4 \text{ mA}$$

$$\% \text{ Error} = \frac{0.2857 - 0.2854}{0.2857} \times 100$$

$$\% \text{ Error} = 0.1 \%$$

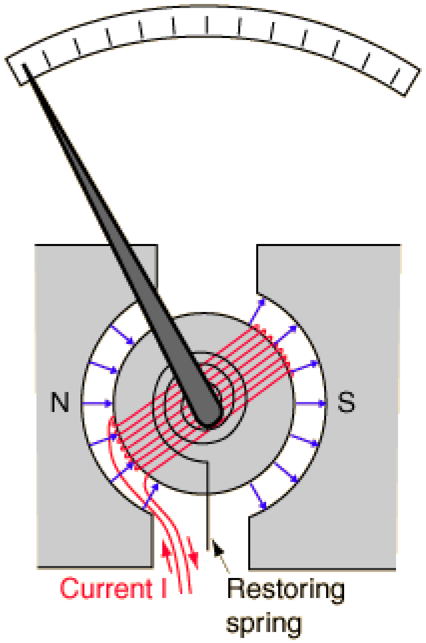


Designing an ammeter

Galvanometer:

- current flows through a coil in a magnetic field
- coil experiences a torque, connected needle deflects

  (see later chapters of this class)

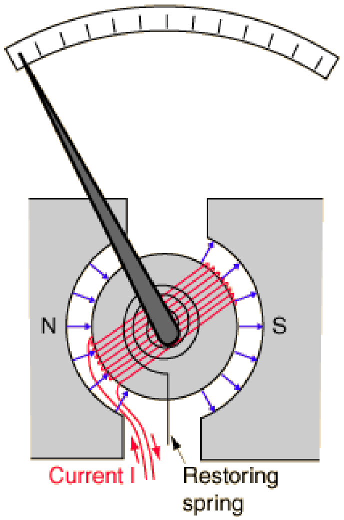


http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/galvan.html#c1

Designing an ammeter

- ammeter can be based on galvanometer
  (for electronic instrument, use electronic sensor instead, analysis still applies)

- simplest case: send current directly through galvanometer, observe deflection of needle

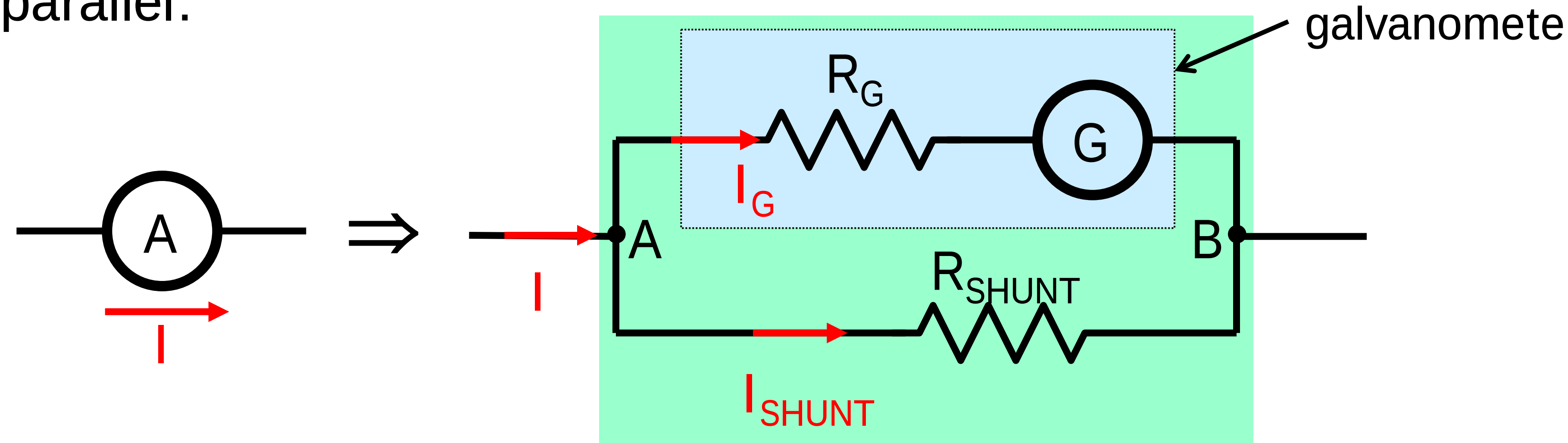


Needle deflection is proportional to current. Each galvanometer has a certain maximum current corresponding to full needle deflection.

What if you need to measure a larger current?

- use shunt resistor

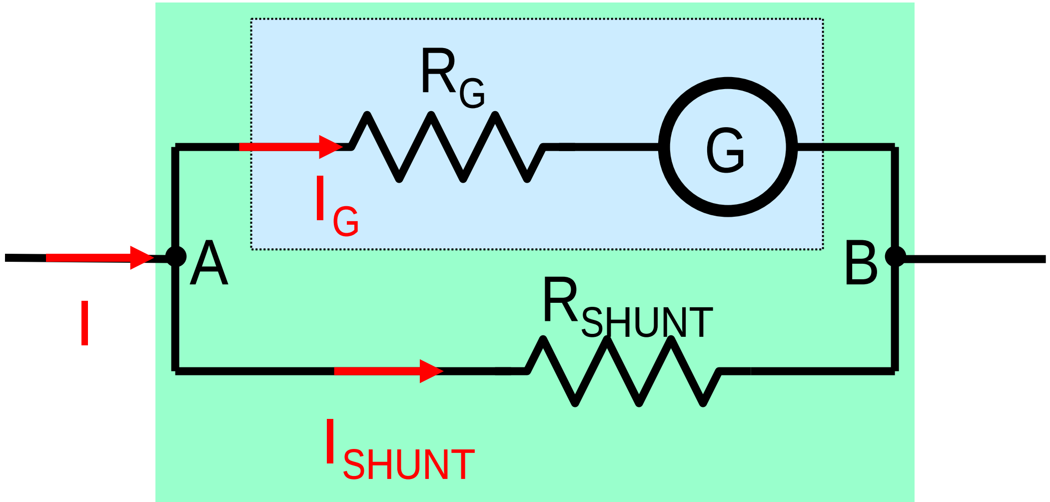
Ammeter uses a galvanometer and a shunt, connected in parallel:



Everything inside the green box is the ammeter.

- Current I gets split into I_{shunt} and I_G

Homework hint:
If your galvanometer reads 1A full scale but you want the ammeter to read 5A full scale, then R_{SHUNT} must result in I_G=1A when I=5A. What are I_{SHUNT} and V_{SHUNT}?



Shunt also reduces resistance of the ammeter:

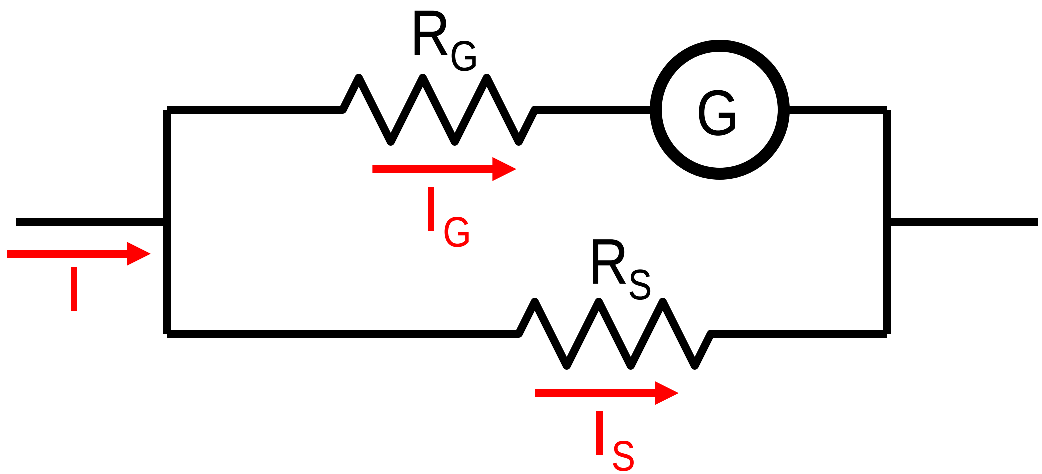
$$\frac{1}{R_A} = \frac{1}{R_G} + \frac{1}{R_{SHUNT}}$$

$$R_A = \frac{R_G \, R_{SHUNT}}{R_G + R_{SHUNT}}$$

Example: what shunt resistance is required for an ammeter to have a resistance of 10 mΩ, if the galvanometer resistance is 60 Ω?

$$\frac{1}{R_A} = \frac{1}{R_G} + \frac{1}{R_S}$$

$$\frac{1}{R_S} = \frac{1}{R_A} - \frac{1}{R_G}$$

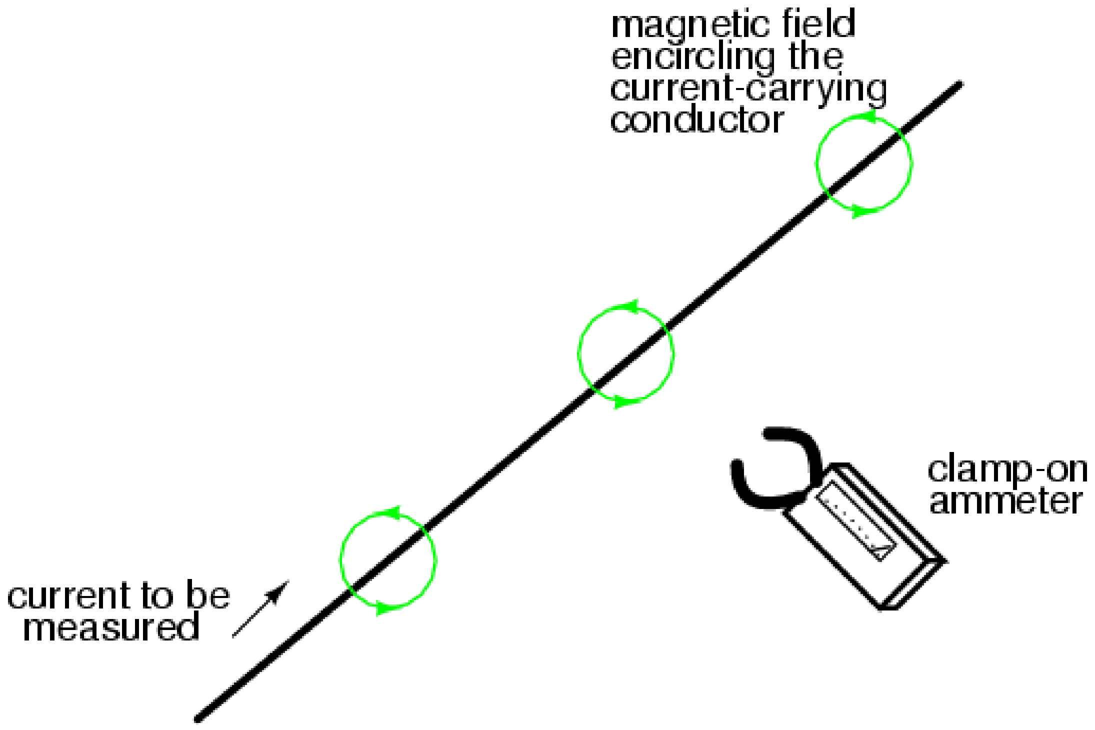


$$R_S = \frac{R_G R_A}{R_G - R_A} = \frac{(60)(.01)}{60 - .01} = 0.010\ \Omega$$

(actually 0.010002 Ω)

To achieve such a small resistance, the shunt is probably a large-diameter wire or solid piece of metal.

Web links: ammeter design, ammeter impact on circuit, clamp-on ammeter (based on principles we will soon be studying).



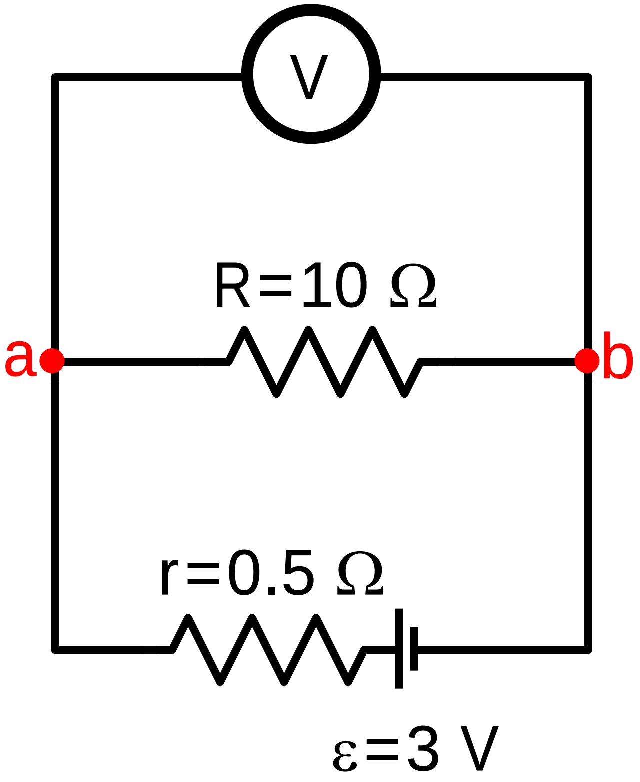
Effect of voltmeter on circuit

Measuring voltage (potential difference) V_{ab} in a simple circuit:

- connect voltmeter in parallel

Are we measuring the correct voltage?

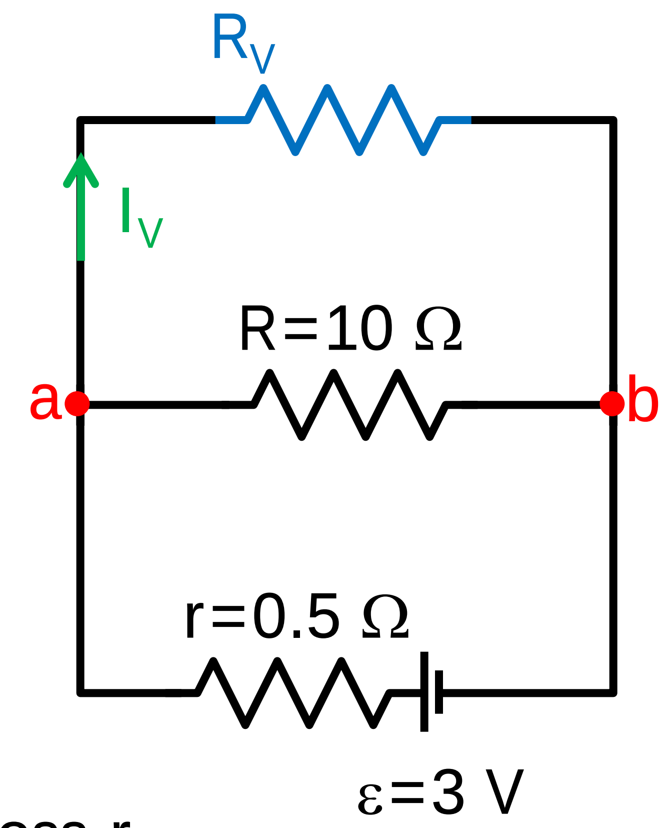
(the voltage in the circuit without voltmeter)



Effect of voltmeter on circuit

Measuring voltage (potential difference) V_{ab} in a simple circuit:

- connect voltmeter in parallel



Are we measuring the correct voltage?
(the voltage in the circuit without voltmeter)

- voltmeter has **some resistance R_V**
- **current** I_V flows through voltmeter
- extra current changes voltage drop across r and thus V_{ab}

To minimize error, voltmeter resistance r must be very large.
(ideal voltmeter would have infinite resistance)

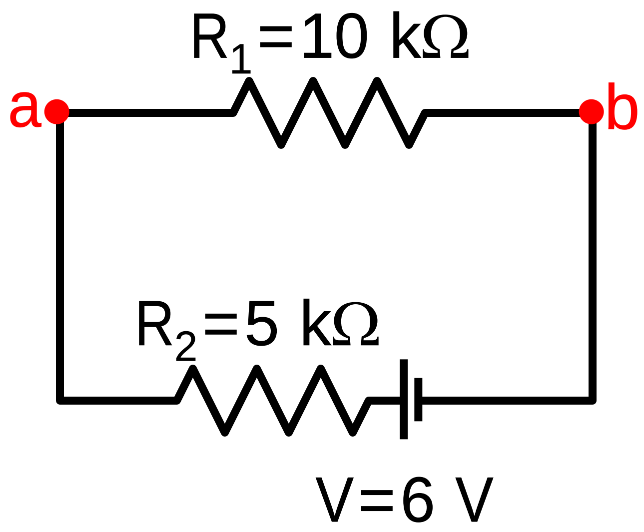
Example: a galvanometer of resistance 60 Ω is used to measure the voltage drop across a 10 kΩ resistor in series with an ideal 6 V battery and a 5 kΩ resistor. What is the relative error caused by the nonzero resistance of the galvanometer?

Actual voltage drop without instrument:

$$R_{eq} = R_1 + R_2 = 15 \times 10^3\ \Omega$$

$$I = \frac{V}{R_{eq}} = \frac{6\ \text{V}}{15 \times 10^3\ \Omega} = 0.4 \times 10^{-3}\ \text{A}$$

$$V_{ab} = IR = \left(0.4 \times 10^{-3}\right)\left(10 \times 10^3\ \Omega\right) = 4\ \text{V}$$



The measurement is made with the galvanometer.

60 Ω and 10 kΩ resistors in parallel are equivalent to 59.6 Ω resistor.

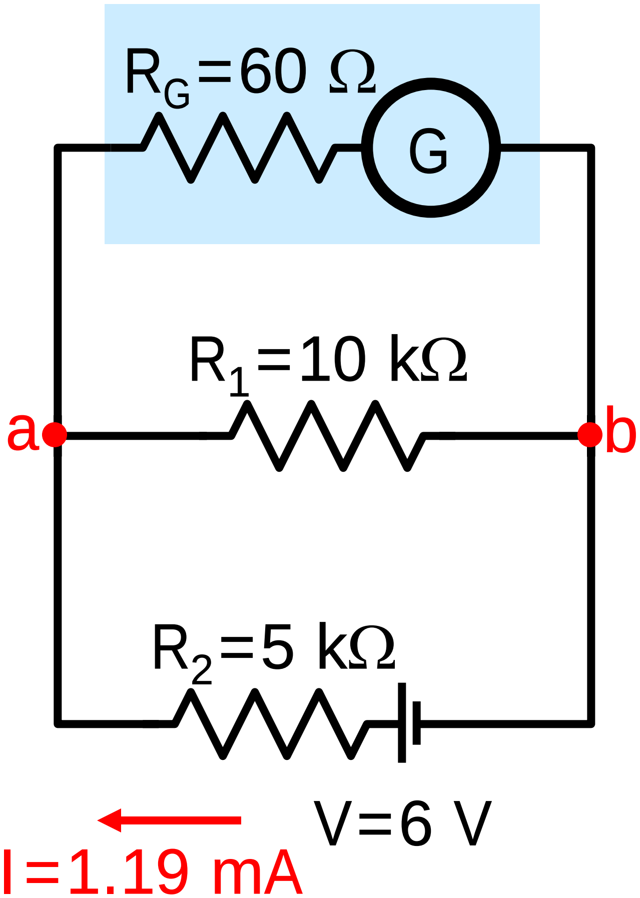
Total equivalent resistance: 5059.6 Ω

Total current: $I = 1.186 \times 10^{-3}$ A

$V_{ab} = 6V - IR_2 = 0.07$ V.

The relative error is:

$$\% \text{ Error} = \frac{4 - .07}{4} \times 100 = 98\%$$

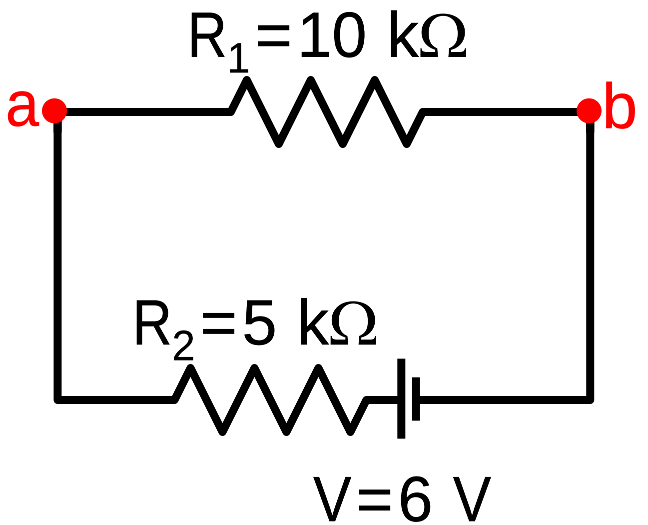


Would you pay for this voltmeter?
We need a better instrument!

Example: a voltmeter of resistance 100 kΩ is used to measure the voltage drop across a 10 kΩ resistor in series with an ideal 6 V battery and a 5 kΩ resistor. What is the percent error caused by the nonzero resistance of the voltmeter?

We already calculated the actual voltage drop (2 slides back).

$$V_{ab} = IR = \left(0.4\times10^{-3}\right)\left(10\times10^{3}\ \Omega\right) = 4\ \text{V}$$



The measurement is now made with the "better" voltmeter.

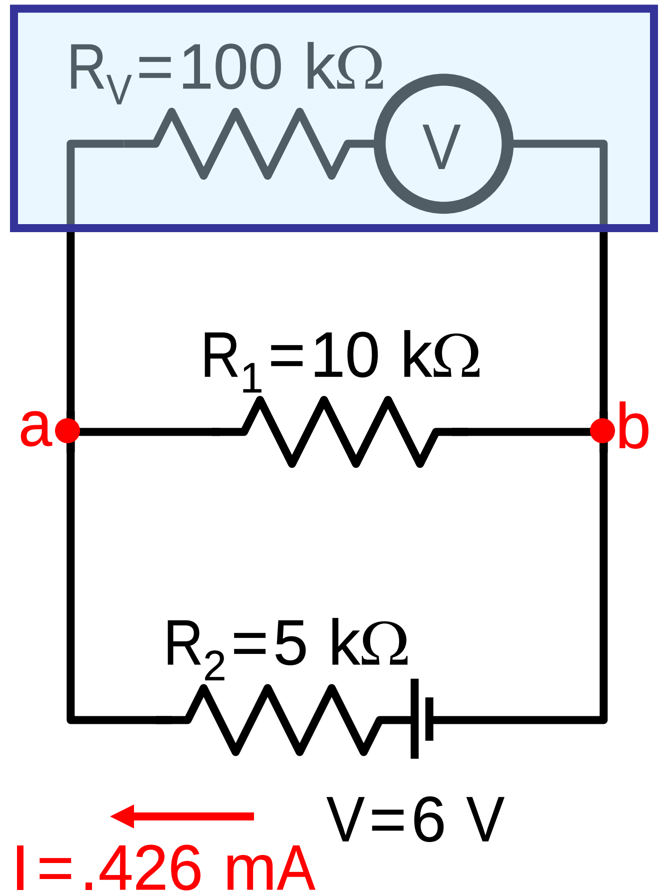
100 kΩ and 10 kΩ resistors in parallel are equivalent to an 9090 Ω resistor.
Total equivalent resistance: 14090 Ω

Total current: $I=4.26\times10^{-4}$ A

The voltage drop from a to b:
$6-(4.26\times10^{-4})(5000)=3.87$ V.

The percent error is.

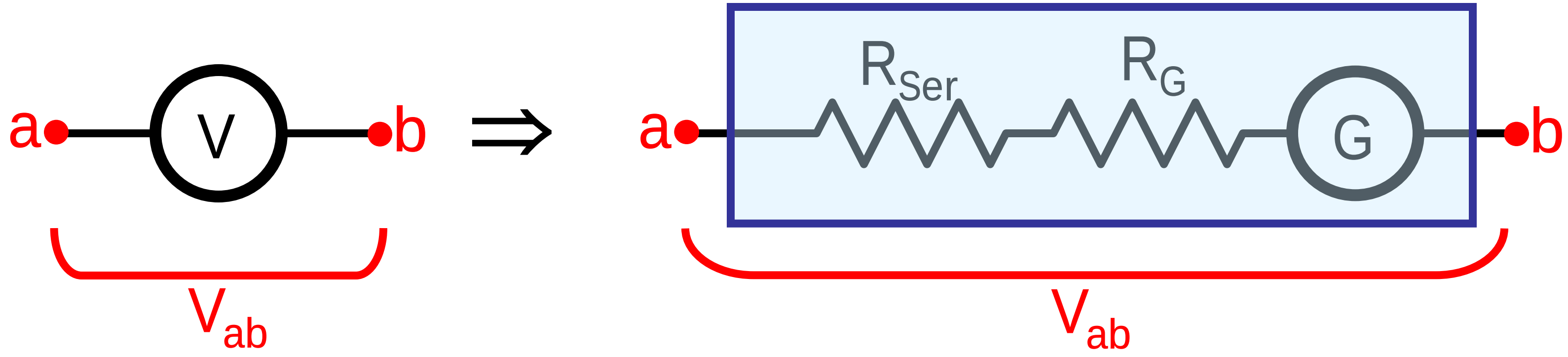
$$\%\ \text{Error} = \frac{4-3.87}{4}\times 100 = 3.25\%$$



Not great, but much better.

Designing a voltmeter

- voltmeter must have a very large resistance
- voltmeter can be made from galvanometer in series with a large resistance



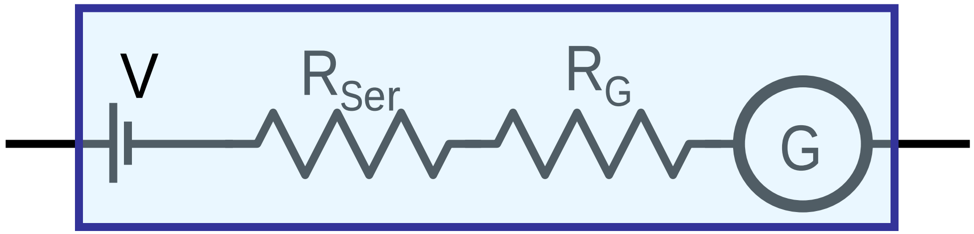
Everything inside the blue box is the voltmeter.

Homework hints: "the galvanometer reads 1A full scale" would mean a current of I_G=1A would produce a full-scale deflection of the galvanometer needle.

If you want the voltmeter shown to read 10V full scale, then the selected R_{Ser} must result in I_G=1A when V_{ab}=10V.

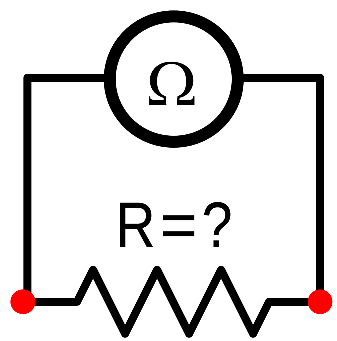
Measuring Instruments: Ohmmeter

- Ohmmeter measures resistance of isolated resistor
- Ohmmeter can be made from a galvanometer, a series resistance, and a battery (active device).



Everything inside the blue box is the ohmmeter.

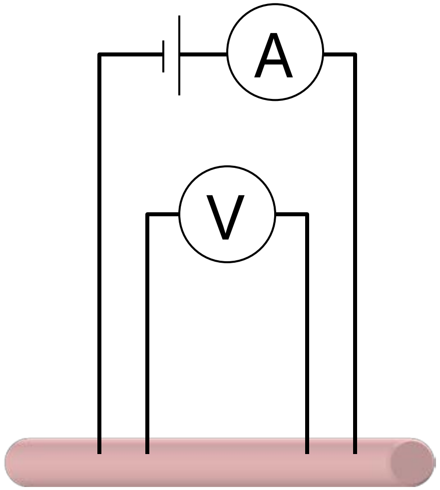
- Terminals of ohmmeter are connected to unknown resistor
- battery causes current to flow and galvanometer to deflect
- $V = I\ (R_{ser} + R_G + R)$ solve for unknown R



Alternatively:

- separately measure current and voltage for resistor
- Apply Ohm's law

Four-point probe:



reference: http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/movcoil.html#c4

Today’s agenda:

Measuring Instruments: ammeter, voltmeter, ohmmeter.

You must be able to calculate currents and voltages in circuits that contain “real” measuring instruments.

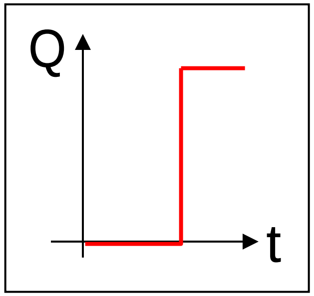
RC Circuits.

You must be able to calculate currents and voltages in circuits containing both a resistor and a capacitor. You must be able to calculate the time constant of an RC circuit, or use the time constant in other calculations.

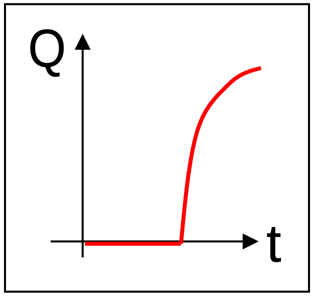
Charging and discharging a capacitor

What happens if we connect a capacitor to a voltage source?

- so far, we have assumed that charge instantly appears on capacitor



- in reality, capacitor does **not** change instantaneously
- charging speed depends on capacitance C and on resistance R between the battery and the capacitor



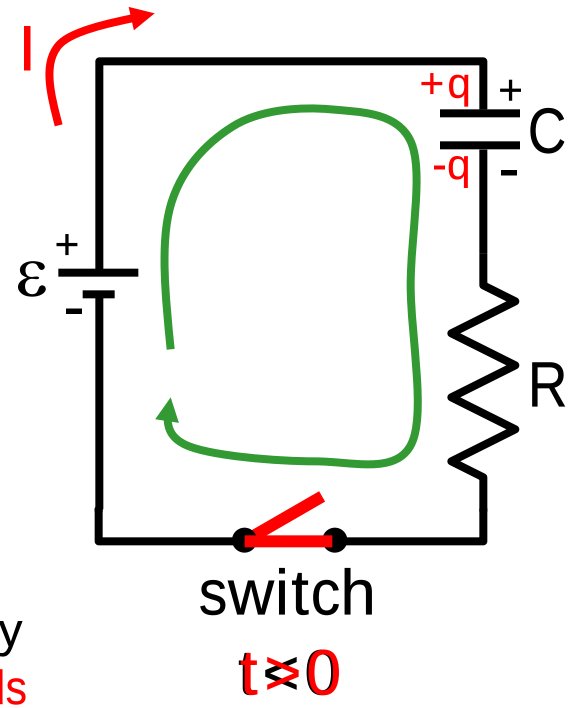
Charging and discharging are time-dependent phenomena!

RC circuit: Charging a Capacitor

Switch open, no current flows.

Close switch, current flows.

Kirchoff's loop rule* (green loop) at the time when charge on C is q.



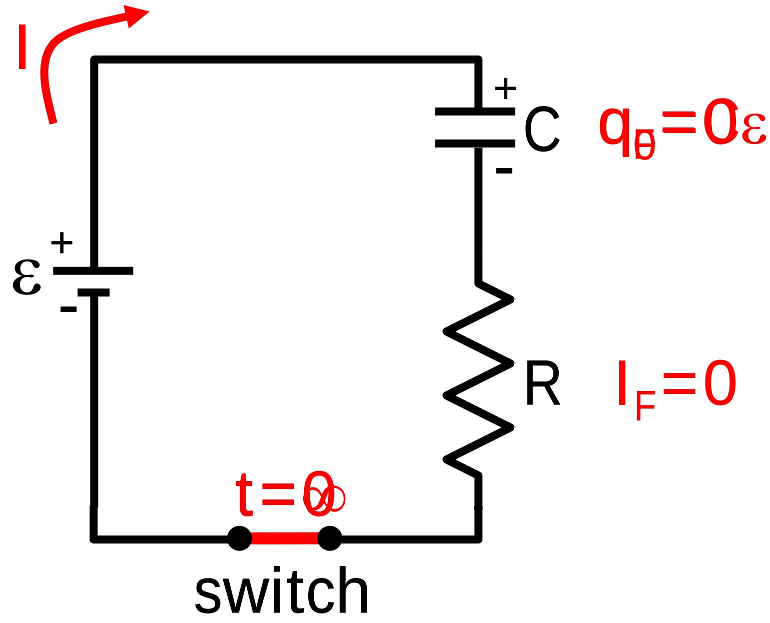
$$\varepsilon - \frac{q}{C} - IR = 0$$

This equation is deceptively complex because I depends on q and both depend on time.

*Sign convention for capacitors is the same as for batteries: Voltage counts positive if going across from - to +.

Limiting cases:

$$\varepsilon - \frac{q}{C} - IR = 0$$



Empty capacitor:

(right after closing the switch)

$q = 0$

$V_C = 0,$ $\qquad V_R = \varepsilon$

$I = I_0 = \varepsilon / R$

Full capacitor:

(after a very long time)

$V_C = \varepsilon,$ $\qquad V_R = 0$

$Q = C\varepsilon$

$I = 0$

Distinguish capacitor and resistor voltages V_C and V_R. They are not equal but $V_C + V_R = \varepsilon$.

Arbitrary time:

- loop rule: $\varepsilon - \frac{q}{C} - IR = 0$
- using $I = \frac{dq}{dt}$ gives $\varepsilon - \frac{q}{C} - R\frac{dq}{dt} = 0$

Differential equation for q(t)

Solution:

$$\frac{dq}{dt} = \frac{\varepsilon}{R} - \frac{q}{RC} = \frac{C\varepsilon - q}{RC}$$

Separation of variables

$$\frac{dq}{C\varepsilon - q} = \frac{dt}{RC}$$

$$\frac{dq}{q - C\varepsilon} = -\frac{dt}{RC}$$

More math:

$$\int_0^q \frac{dq'}{q - C\varepsilon} = -\int_0^t \frac{dt'}{RC}$$

$$\ln\left(q' - C\varepsilon\right)\Big|_0^q = -\frac{1}{RC}t'\Big|_0^t$$

$$\ln\left(\frac{q - C\varepsilon}{-C\varepsilon}\right) = -\frac{t}{RC}$$

$$q - C\varepsilon = -C\varepsilon\, e^{-\frac{t}{RC}}$$

$$q(t) = Q_{final}\left(1 - e^{-\frac{t}{RC}}\right)$$

$$Q_{final} = C\varepsilon$$

τ= RC: **time constant** of the circuit; it tells us “how fast” the capacitor charges and discharges.

Current as a function of time:

- take derivative:

$$I(t) = \frac{dq}{dt} = C\varepsilon\left(\frac{1}{RC}e^{-\frac{t}{RC}}\right) = \frac{C\varepsilon}{RC}e^{-\frac{t}{RC}} = \frac{\varepsilon}{R}e^{-\frac{t}{RC}} = \frac{\varepsilon}{R}e^{-\frac{t}{\tau}}$$

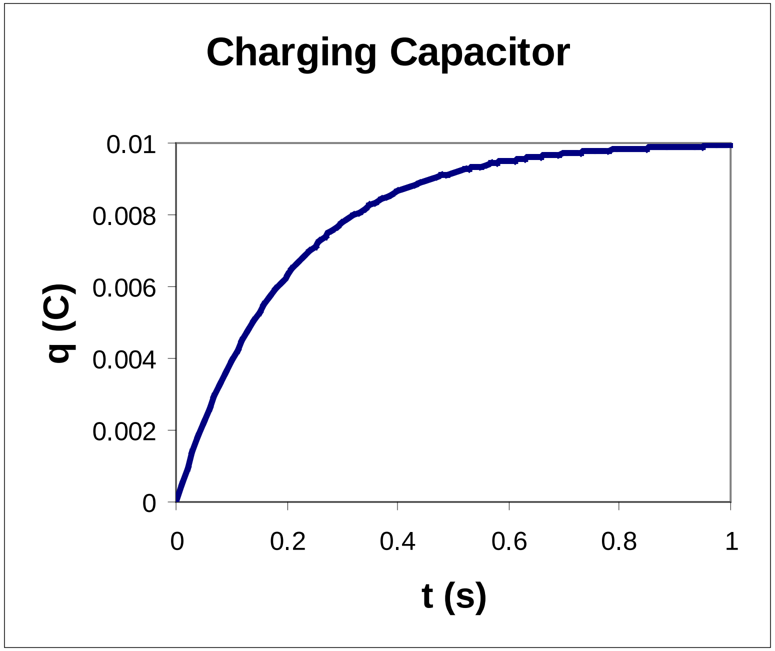
$$I(t) = \frac{\varepsilon}{R}e^{-\frac{t}{\tau}} = I_0 e^{-\frac{t}{\tau}}$$

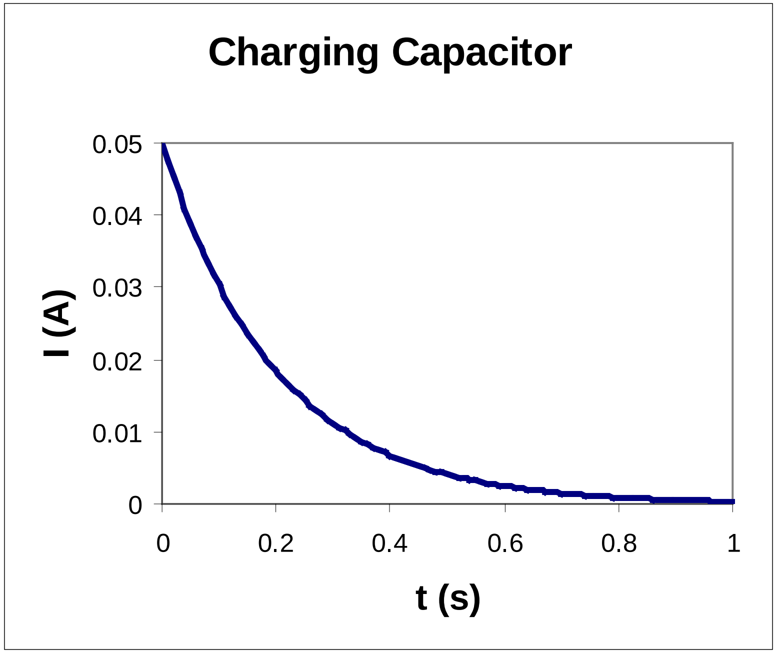
Charging a capacitor; summary:

$$q(t) = Q_{final}\left(1 - e^{-\frac{t}{RC}}\right)$$

recall that this is I_0, also called I_{max}

$$I(t) = \frac{\varepsilon}{R} e^{-\frac{t}{RC}}$$

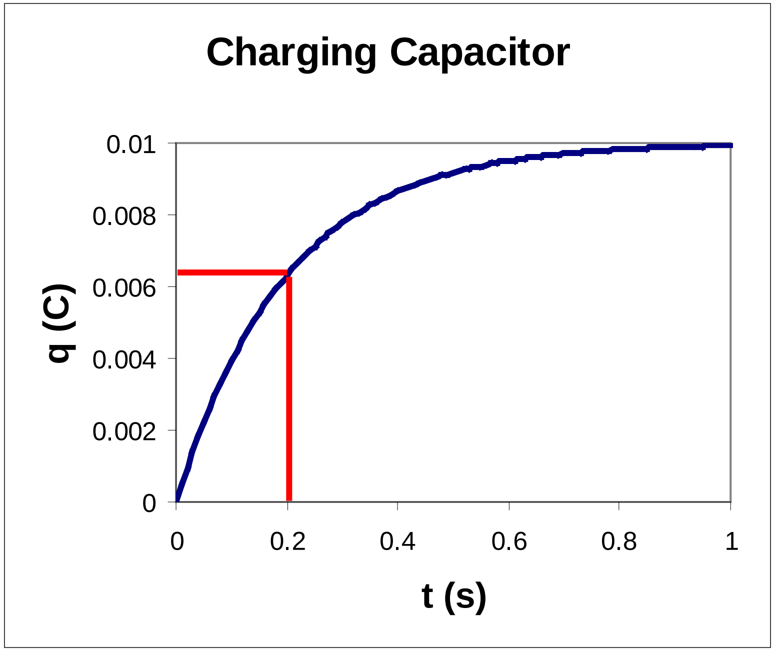


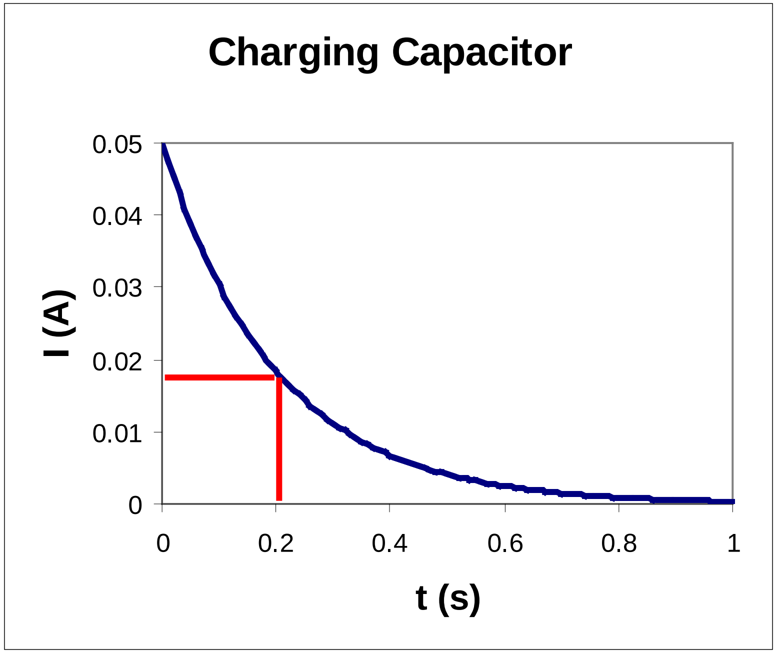


Sample plots with ε=10 V, R=200 Ω, and C=1000 μF.
RC=0.2 s

In a time t=RC, the capacitor charges to $Q_{final}(1-e^{-1})$ or 63% of its capacity...

...and the current drops to $I_{max}(e^{-1})$ or 37% of its maximum.





RC=0.2 s

τ=RC is called the **time constant** of the RC circuit

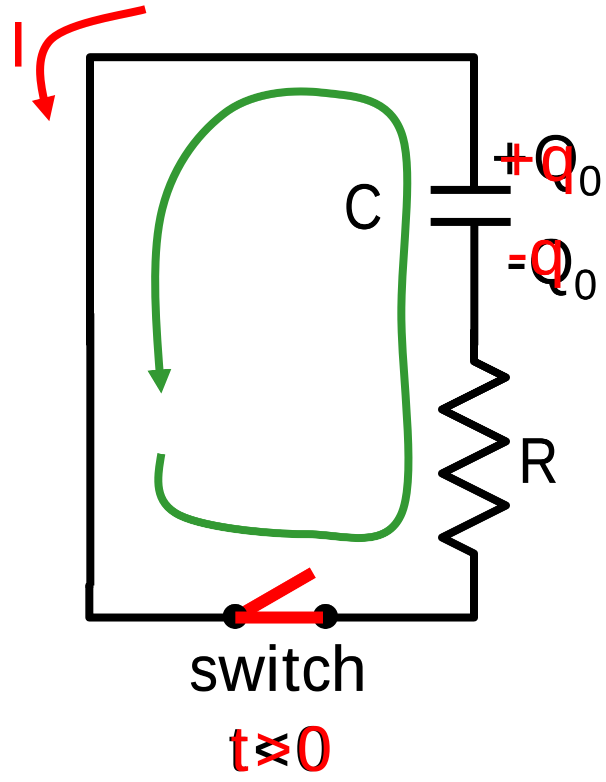
Discharging a Capacitor

Capacitor charged, switch open, no current flows.

Close switch, current flows.

Kirchoff's loop rule* (green loop) at the time when charge on C is q.

$$\frac{q}{C} - IR = 0$$



*Sign convention for capacitors is the same as for batteries: Voltage counts positive if going across from - to +.

Arbitrary time:

- loop rule: $\frac{q}{C} - IR = 0$
- using $I = -\frac{dq}{dt}$ gives $\boxed{\frac{q}{C} + R\frac{dq}{dt} = 0}$

Differential equation for q(t)

negative because current decreases charge on C

Solve:

$$-R\frac{dq}{dt} = \frac{q}{C}$$

$$\frac{dq}{q} = -\frac{dt}{RC}$$

More math:

$$\int_{Q_0}^{q} \frac{dq'}{q'} = -\int_0^t \frac{dt'}{RC} = -\frac{1}{RC}\int_0^t dt'$$

$$\ln(q')\Big|_{Q_0}^{q} = -\frac{1}{RC}\int_0^t dt$$

$$\ln\left(\frac{q}{Q_0}\right) = -\frac{t}{RC}$$

$$\boxed{q(t) = Q_0\, e^{-\frac{t}{RC}}}$$

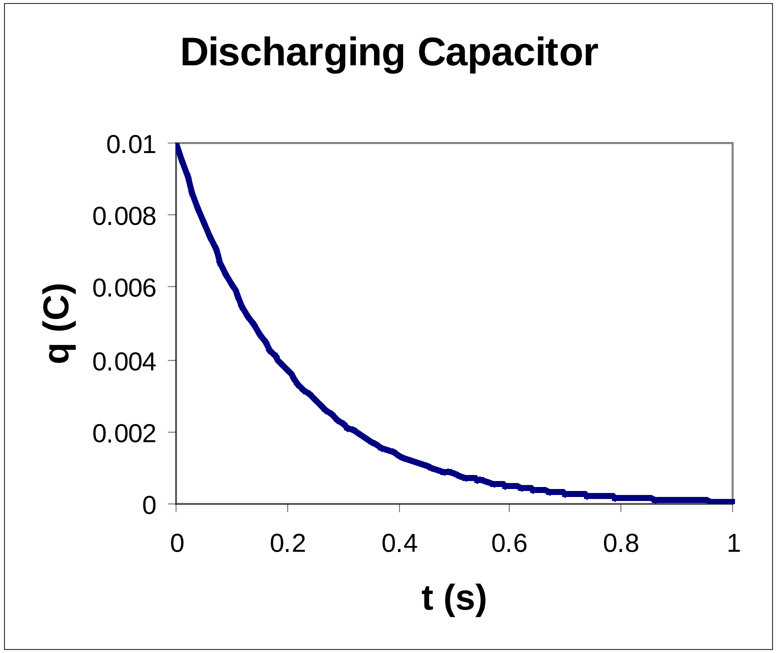
$$I(t) = -\frac{dq}{dt} = \frac{Q_0}{RC}\, e^{-\frac{t}{RC}} = \boxed{I_0\, e^{-\frac{t}{RC}}}$$

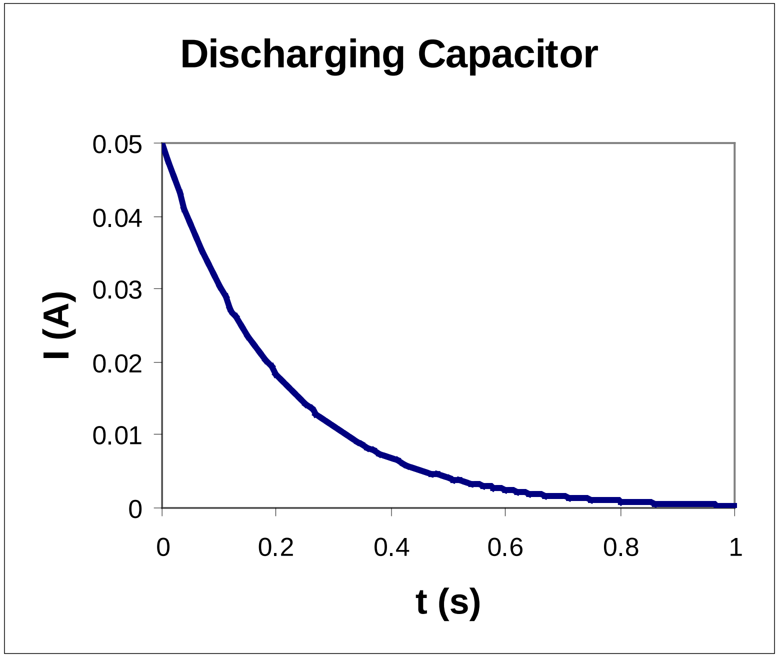
same equation as for charging

Discharging a capacitor; summary:

$$q(t) = Q_0 \, e^{-\frac{t}{RC}}$$

$$I(t) = I_0 e^{-\frac{t}{RC}}$$

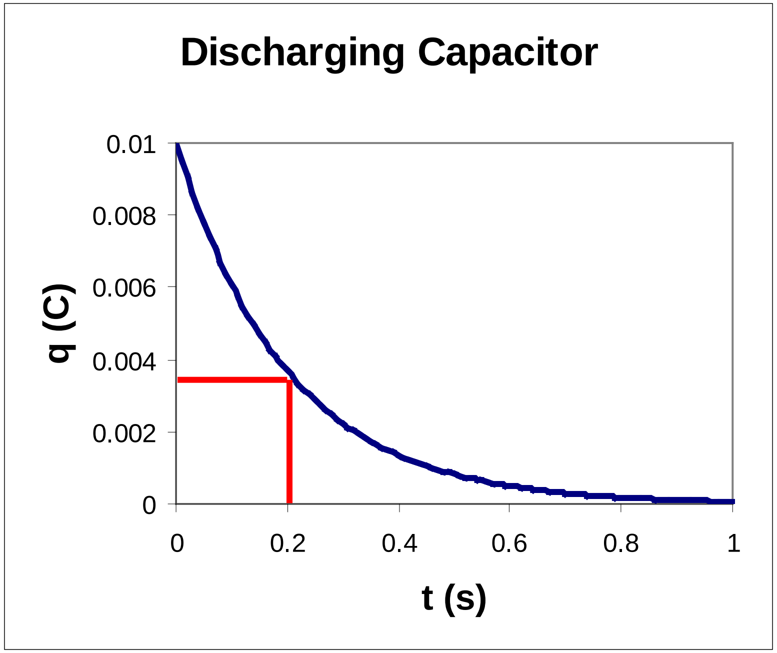


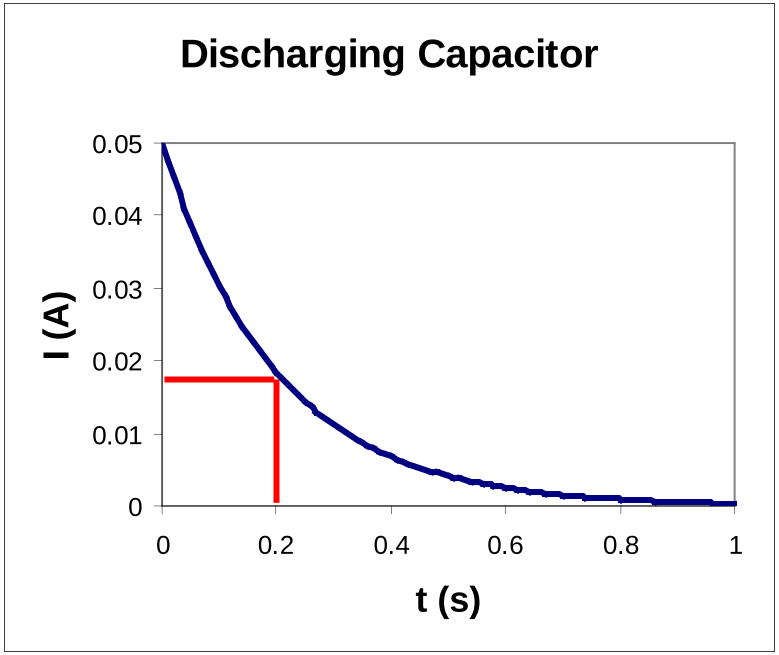


Sample plots with ε=10 V, R=200 Ω, and C=1000 μF.
RC=0.2 s

In a time t=RC, the capacitor discharges to Q_0e^{-1} or 37% of its initial value...

...and the current drops to $I_{max}(e^{-1})$ or 37% of its maximum.





RC=0.2 s

	Charging	Discharging
Charge $Q(t)$	$Q(t) = Q_{final}(1-e^{-t/\tau})$	$Q(t) = Q_0\, e^{-t/\tau}$
Capacitor voltage $V_C(t)$	$V_C(t) = \varepsilon(1-e^{-t/\tau})$	$V_C(t) = V_0\, e^{-t/\tau}$ $= (Q_0/C)\, e^{-t/\tau}$
Resistor voltage $V_R(t)$	$V_R(t) = \varepsilon - V_C(t)$ $= \varepsilon\, e^{-t/\tau}$	$V_R(t) = V_C(t) = V_0\, e^{-t/\tau}$ $= (Q_0/C)\, e^{-t/\tau}$
Current $I(t)$	$I(t) = I_0\, e^{-t/\tau}$ $= (\varepsilon/R)\, e^{-t/\tau}$	$I(t) = I_0\, e^{-t/\tau}$ $= [Q_0/(RC)]\, e^{-t/\tau}$

$\tau = RC$

Only the equations for the charge Q(t) are starting equations. You must be able to derive the other quantities.

Homework Hints

$Q(t) = CV(t)$

This is always true for a capacitor.

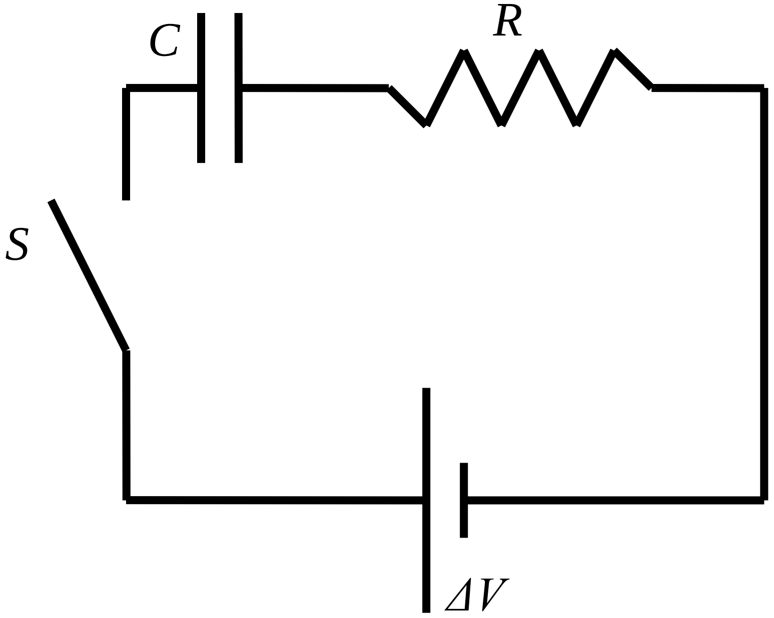
$V = IR$

Ohm's law applies to resistors, not capacitors. Can give you the current only if you know V across the resistor.

In a series RC circuit, the same current I flows through both the capacitor and the resistor.

Example: For the circuit shown $C = 8\ \mu F$ and $\Delta V = 30$ V. Initially the capacitor is uncharged. The switch S is then closed and the capacitor begins to charge. Determine the charge on the capacitor at time $t = 0.693RC$, after the switch is closed. (From a prior test.) Also determine the current through the capacitor and voltage across the capacitor terminals at that time.

To be worked at the blackboard in lecture.



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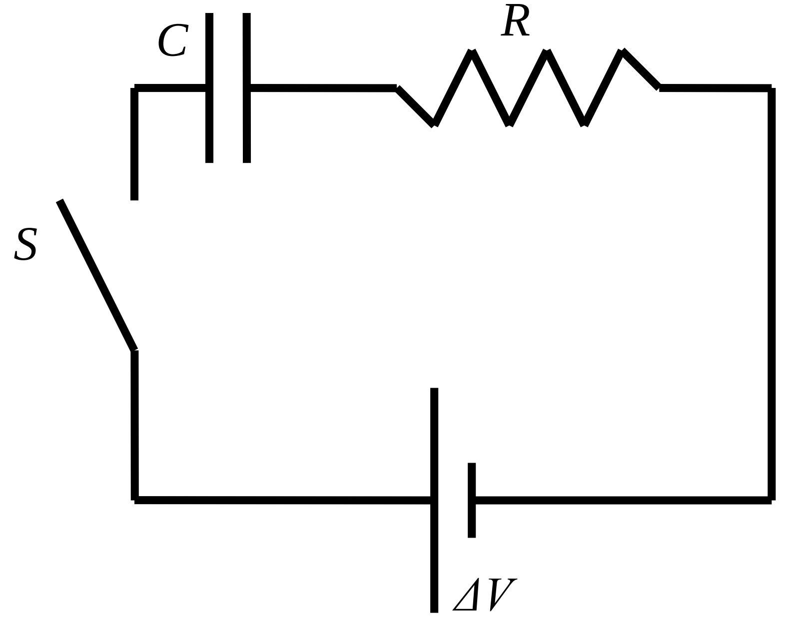
Highlighted text tells us this is a charging capacitor problem.



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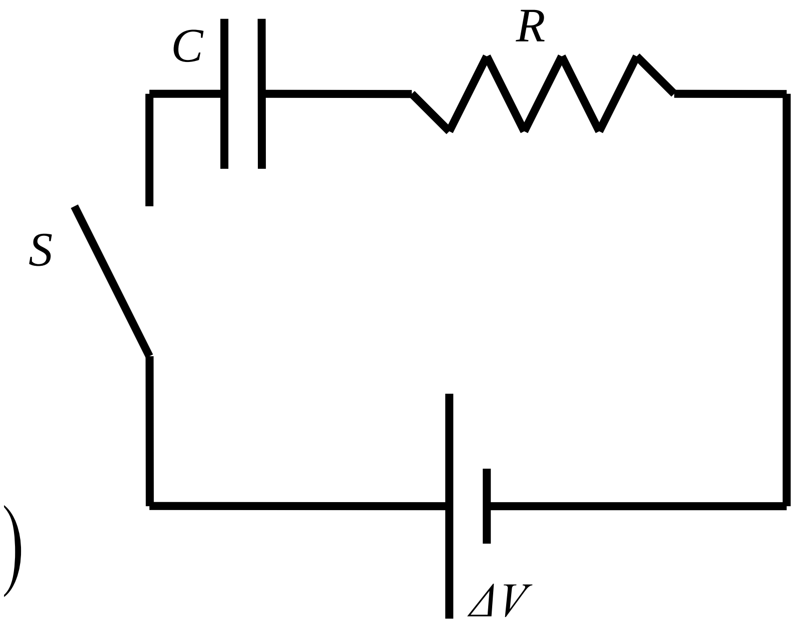


$$q(t) = Q_{final}\left(1 - e^{-\frac{t}{RC}}\right)$$

$$q(0.693\,RC) = C\,\Delta V\left(1 - e^{-\frac{0.693\,RC}{RC}}\right)$$

$$q(0.693\,RC) = (8x10^{-6})(30)(1 - e^{-0.693})$$

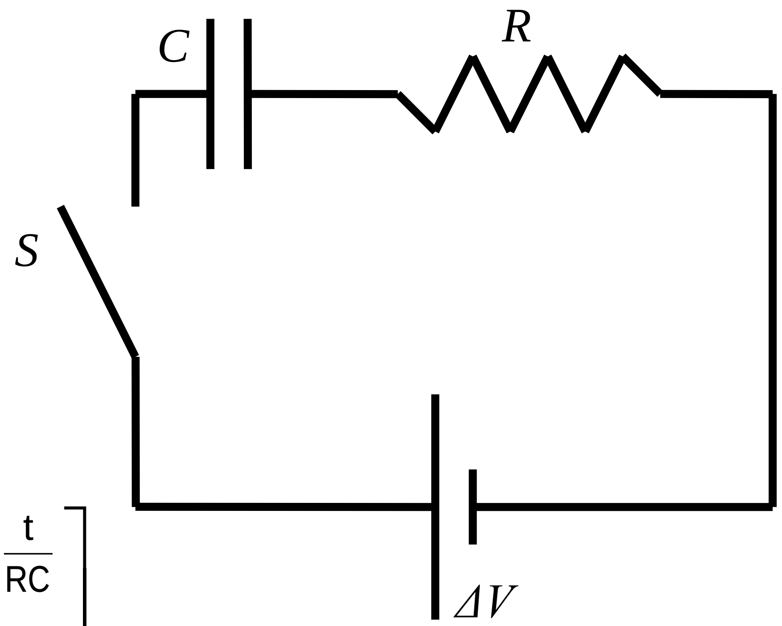
$$q(0.693\,RC) = 240x10^{-6}(1 - 0.5)$$

Nuc E's should recognize that $e^{-0.693} = ½$.

$$q(0.693\,RC) = 120\ \mu C$$

Example: For the circuit shown C = 8 μF and ΔV = 30 V.
Determine the current through the capacitor at $t = 0.693RC$.

You can't use ΔV = IR! (Why?)

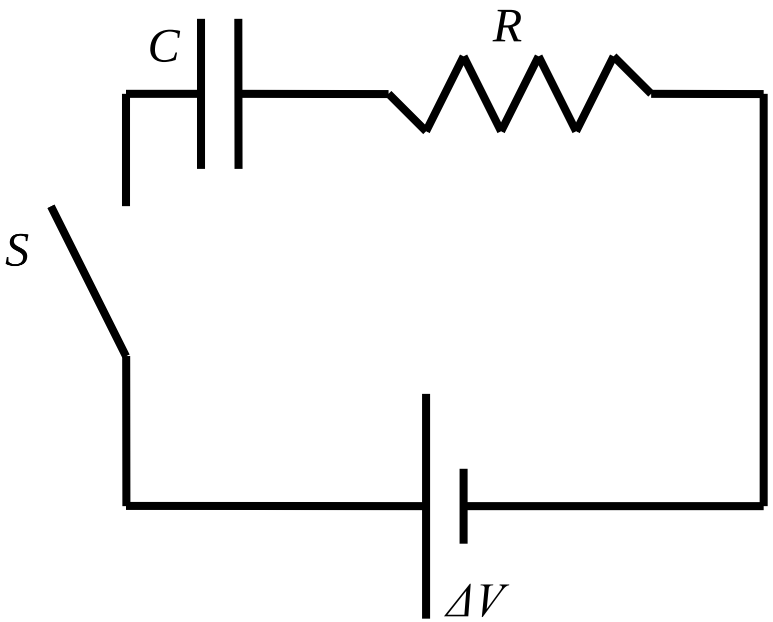


$$I(t) = \frac{dq(t)}{dt}$$

$$I(t) = \frac{d}{dt}\left[Q_{final}\left(1 - e^{-\frac{t}{RC}}\right)\right] = \frac{d}{dt}\left[-Q_{final}e^{-\frac{t}{RC}}\right]$$

$$I(t) = \frac{d}{dt}e^{-\frac{t}{RC}} = -Q_{final}\ e^{-\frac{t}{RC}}\ \frac{d}{dt}\left(-\frac{t}{RC}\right) = -Q_{final}\ e^{-\frac{t}{RC}}\ \left(-\frac{1}{RC}\right)$$

Example: For the circuit shown $C = 8\ \mu F$ and $\Delta V = 30$ V.
Determine the current through the capacitor at $t = 0.693RC$.



$$I(t) = \frac{Q_{final}}{RC} e^{-\frac{t}{RC}} = \frac{C\,\Delta V}{RC} e^{-\frac{t}{RC}}$$

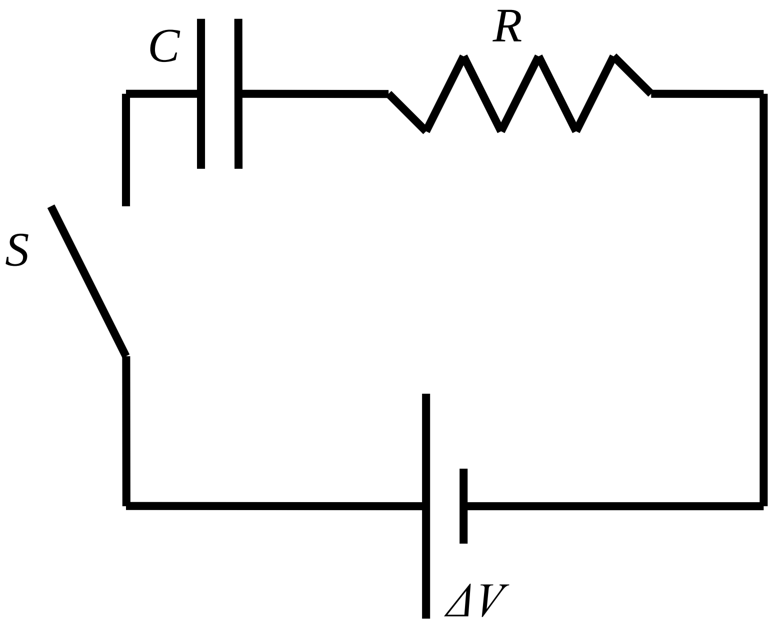
$$I(t) = \frac{\Delta V}{R} e^{-\frac{t}{RC}}$$

$$I(0.693\,RC) = \frac{\Delta V}{R} e^{-\frac{0.693\,RC}{RC}} = \frac{\Delta V}{R}\left(\frac{1}{2}\right)$$

$$I(0.693\,RC) = \frac{1}{2}\frac{\Delta V}{R} = \frac{1}{2} I_0$$

We can't provide a numerical answer because R (and therefore I_0) is not given.

Example: For the circuit shown C = 8 μF and ΔV = 30 V. Determine the voltage across the capacitor terminals at time t = 0.693RC, after the switch is closed.



$$q(t) = Q_{final}\left(1 - e^{-\frac{t}{RC}}\right)$$

$$C\,V(t) = C\,\Delta V\left(1 - e^{-\frac{t}{RC}}\right)$$

$$V(t) = \Delta V\left(1 - e^{-\frac{t}{RC}}\right)$$

ΔV, ε, and V_0 usually mean the same thing, but check the context!

$$V(t) = \varepsilon\left(1 - e^{-\frac{t}{RC}}\right) = V_0\left(1 - e^{-\frac{t}{RC}}\right)$$

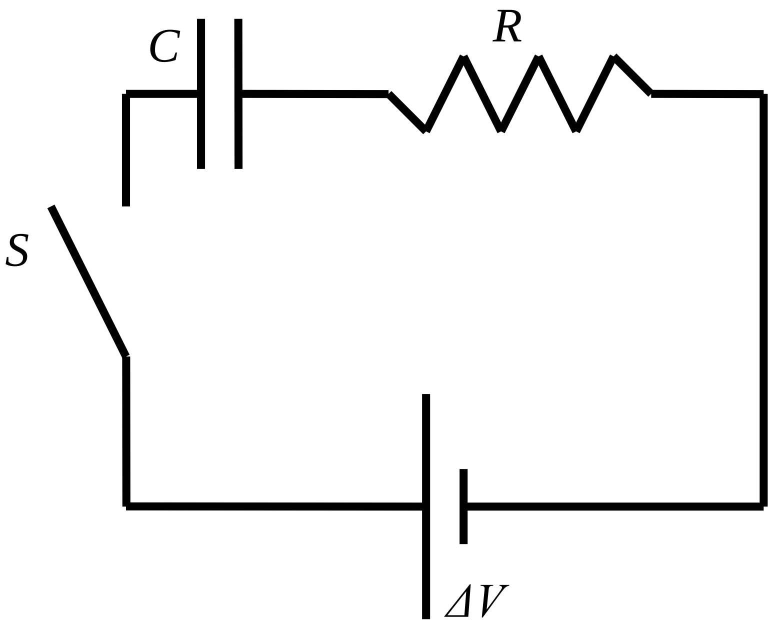
We just derived an equation for V across the capacitor terminals as a function of time! Handy!

Example: For the circuit shown C = 8 μF and ΔV = 30 V. Determine the voltage across the capacitor terminals at time t = 0.693RC, after the switch is closed.

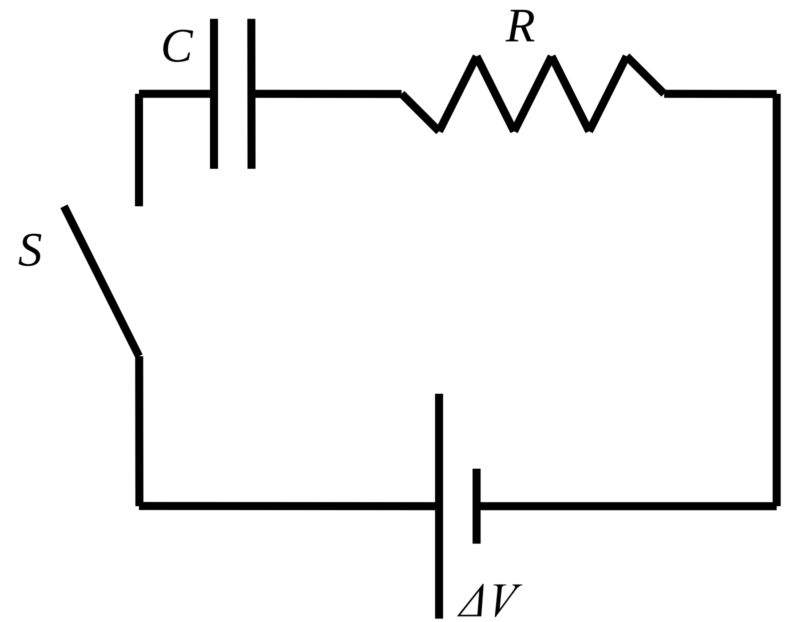
$$V(t) = \Delta V\left(1 - e^{-\frac{t}{RC}}\right)$$

$$V(0.693\,RC) = 30\left(1 - e^{-\frac{0.693\,RC}{RC}}\right)$$

$$V(0.693\,RC) = 30\left(1 - \frac{1}{2}\right) = 15\ \text{V}$$



Example: For the circuit shown $C = 8\ \mu F$ and $\Delta V = 30$ V. Determine the voltage across the capacitor terminals at time $t = 0.693RC$, after the switch is closed.



Digression...

$$V(0.693\,RC) = 15\ V$$

Note that $V_R + V_C = \Delta V$, so

$$V_R(0.693\,RC) = \Delta V - V_C(0.693\,RC)$$

$$V_R(0.693\,RC) = 30 - 15 = 15\ V$$

$$I(0.693\,RC) = \frac{V(0.693\,RC)}{R} = \frac{15}{R}$$

An alternative way to calculate I(0.693 RC), except we still don't know R.

Example: For the circuit shown $C = 8\ \mu F$ and $\Delta V = 30$ V. Determine the voltage across the capacitor terminals at time $t = 0.693RC$, after the switch is closed.

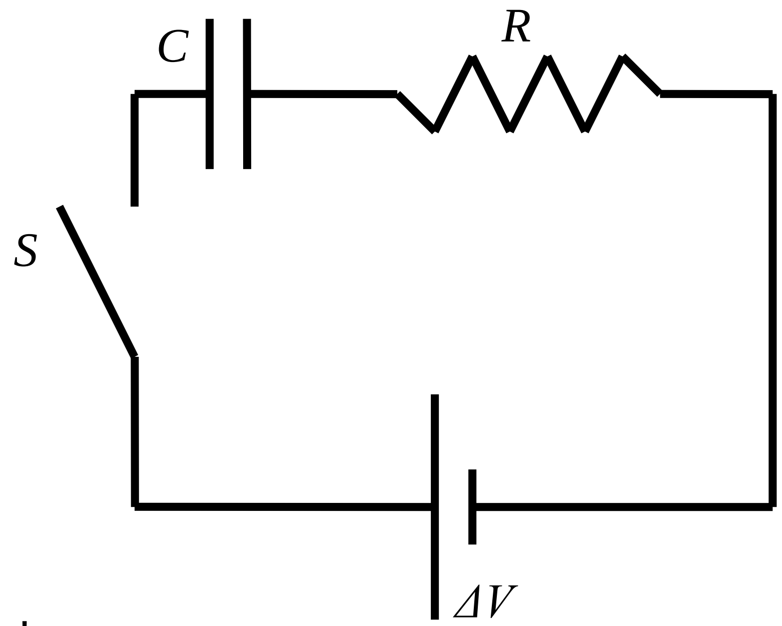
A different way to calculate V(t)...

$$q(0.693\,RC) = 120\ \mu C$$

$$C = \frac{q(t)}{V(t)} \Rightarrow V(t) = \frac{q(t)}{C}$$

$$V(0.693\,RC) = \frac{120 \times 10^{-6}}{8 \times 10^{-6}} = 15\ V$$

Easier!



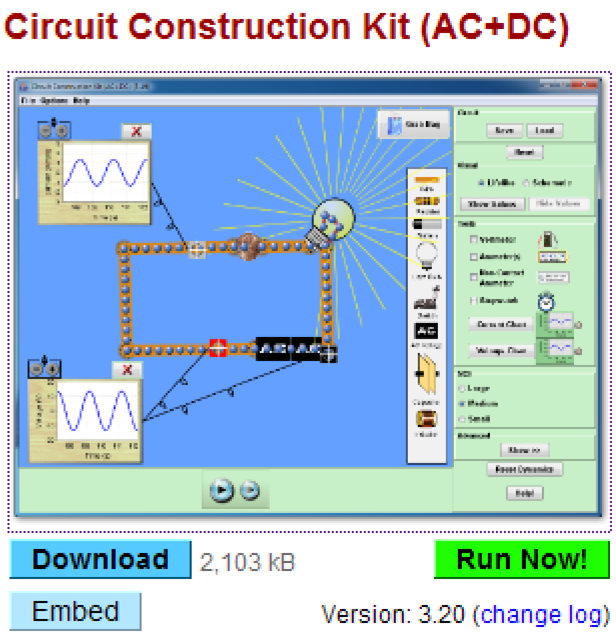
Demo

Charging and discharging a capacitor.

Instead of doing a physical demo, if I have time I will do a virtual demo using the applet linked on the next slide. The applet illustrates the same principles as the physical demo.

make your own capacitor circuits

http://phet.colorado.edu/en/simulation/circuit-construction-kit-ac



For a “pre-built” RC circuit that lets you both charge and discharge (through separate switches), download this file, put it in your “my documents” folder, run the circuit construction applet (link above), maximize it, then select “load” in the upper right. Click on the “capacitor_circuit” file and give the program permission to run it. You can put voltmeters and ammeters in your circuit. You can change values or R, C, and V. Also, click on the “current chart” button for a plot of current (you can have more than one in your applet) or the “voltage chart” button for a plot of voltage.

more applets

http://webphysics.davidson.edu/physlet_resources/bu_semester2/c11_RC.html

http://subaru.univ-lemans.fr/AccesLibre/UM/Pedago/physique/02/electri/condo2.html

http://www.phy.ntnu.edu.tw/ntnujava/index.php?topic=31.0