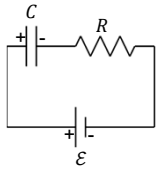
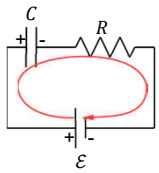
RC Circuits

What happens when capacitor is connected to a resistor in a circuit?



RC Circuits

Apply Kirchhoff's loop rule.



$$\mathcal{E} - \frac{Q}{C} - IR = 0$$

RC Circuits

$$\mathcal{E} - \frac{Q}{C} - IR = 0$$

$$I = +\frac{dQ}{dt}$$

Positive because current through the resistor is associated with an increase of charge on the capacitor.

$$\mathcal{E} - \frac{Q}{C} - \frac{dQ}{dt}R = 0$$

RC Circuits

$$\mathcal{E} - \frac{Q}{C} - \frac{dQ}{dt}R = 0$$

$$\mathcal{E} - \frac{Q}{C} = \frac{dQ}{dt}R$$

$$\frac{\mathcal{E}C - Q}{RC} = \frac{dQ}{dt}$$

$$\frac{dt}{RC} = \frac{dQ}{\mathcal{E}C - Q}$$

RC Circuits

$$\frac{dt}{RC} = \frac{dQ}{\mathcal{E}C - Q}$$

$$\int_0^t \frac{dt}{RC} = \int_0^Q \frac{dQ}{\mathcal{E}C - Q}$$

$$\frac{t}{RC} = -\ln[\mathcal{E}C - Q]_0^Q$$

$$-\frac{t}{RC} = \ln\left(\frac{\mathcal{E}C - Q}{\mathcal{E}C}\right)$$

RC Circuits

$$-\frac{t}{RC} = \ln\left(\frac{\mathcal{E}C - Q}{\mathcal{E}C}\right)$$

$$e^{-\frac{t}{RC}} = \frac{\mathcal{E}C - Q}{\mathcal{E}C}$$

$$\mathcal{E}Ce^{-\frac{t}{RC}} = \mathcal{E}C - Q$$

$$Q = \mathcal{E}C\left(1 - e^{-\frac{t}{RC}}\right)$$

RC Circuits

$$-\frac{t}{RC} = \ln\left(\frac{\mathcal{E}C - Q}{\mathcal{E}C}\right)$$

$$e^{-\frac{t}{RC}} = \frac{\mathcal{E}C - Q}{\mathcal{E}C}$$

$$\mathcal{E}Ce^{-\frac{t}{RC}} = \mathcal{E}C - Q$$

$$Q = \mathcal{E}C\left(1 - e^{-\frac{t}{RC}}\right)$$

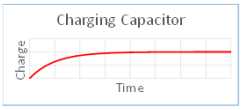
$$Q = Q_f\left(1 - e^{-\frac{t}{RC}}\right)$$

RC Circuits

Charging a capacitor

- Charge on capacitor

$$Q = Q_f\left(1 - e^{-\frac{t}{\tau}}\right)$$



$(\tau \equiv RC)$

RC Circuits

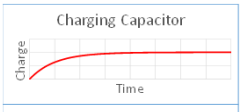
Charging a capacitor

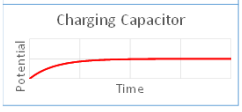
- Charge on capacitor

$$Q = Q_f\left(1 - e^{-\frac{t}{\tau}}\right)$$

- Voltage across capacitor

$$V = \frac{Q}{C} = \mathcal{E}\left(1 - e^{-\frac{t}{\tau}}\right)$$





$(\tau \equiv RC)$

RC Circuits

Charging a capacitor

- Charge on capacitor

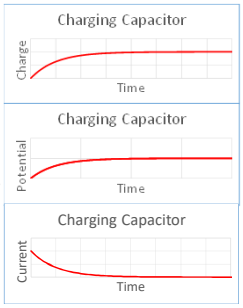
$$Q = Q_f\left(1 - e^{-\frac{t}{\tau}}\right)$$

- Voltage across capacitor

$$V = \frac{Q}{C} = \mathcal{E}\left(1 - e^{-\frac{t}{\tau}}\right)$$

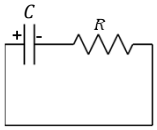
- Current through resistor

$$I = \frac{dQ}{dt} = \frac{\mathcal{E}}{R}e^{-\frac{t}{\tau}}$$



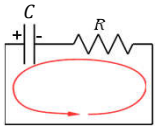
RC Circuits

What happens when charged capacitor is connected to a resistor in a circuit without a battery?



RC Circuits

Apply Kirchhoff's loop rule.



$$\frac{Q}{C} - IR = 0$$

RC Circuits

$$\frac{Q}{C} - IR = 0$$

$$I = -\frac{dQ}{dt}$$

Negative because current through the resistor is associated with a decrease of charge on the capacitor.

$$\frac{Q}{C} + \frac{dQ}{dt}R = 0$$

RC Circuits

$$\frac{Q}{C} + \frac{dQ}{dt}R = 0$$

$$\frac{dQ}{Q} = -\frac{dt}{RC}$$

$$\ln\left(\frac{Q}{Q_0}\right) = -\frac{t}{RC}$$

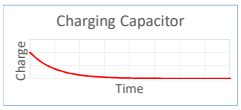
$$Q = Q_0 e^{-\frac{t}{RC}}$$

RC Circuits

Discharging a capacitor

- Charge on capacitor

$$Q = Q_0 e^{-\frac{t}{\tau}}$$



$(\tau \equiv RC)$

RC Circuits

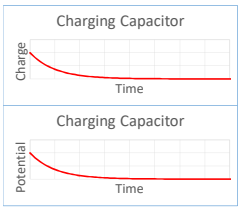
Discharging a capacitor

- Charge on capacitor

$$Q = Q_0 e^{-\frac{t}{\tau}}$$

- Voltage across capacitor

$$V = \frac{Q}{C} = \frac{Q_0}{C} e^{-\frac{t}{\tau}}$$



$(\tau \equiv RC)$

RC Circuits

Discharging a capacitor

- Charge on capacitor

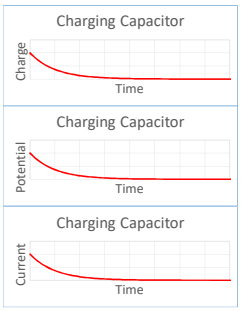
$$Q = Q_0 e^{-\frac{t}{\tau}}$$

- Voltage across capacitor

$$V = \frac{Q}{C} = \frac{Q_0}{C} e^{-\frac{t}{\tau}}$$

- Current through resistor

$$I = -\frac{dQ}{dt} = \frac{Q_0}{RC} e^{-\frac{t}{\tau}}$$



Example: An 8μF capacitor is connected in series with a 50Ω resistor and a 60V battery. (a) When does the capacitor become fully charged? (b) What is the charge on the capacitor when it is fully charged?

Example: An 8μF capacitor is connected in series with a 50Ω resistor and a 60V battery. (a) When does the capacitor become fully charged? (b) What is the charge on the capacitor when it is fully charged? (c) When does the capacitor hold a third of its ultimate full charge? (d) What is the current in the resistor when the capacitor holds a third of its ultimate full charge?