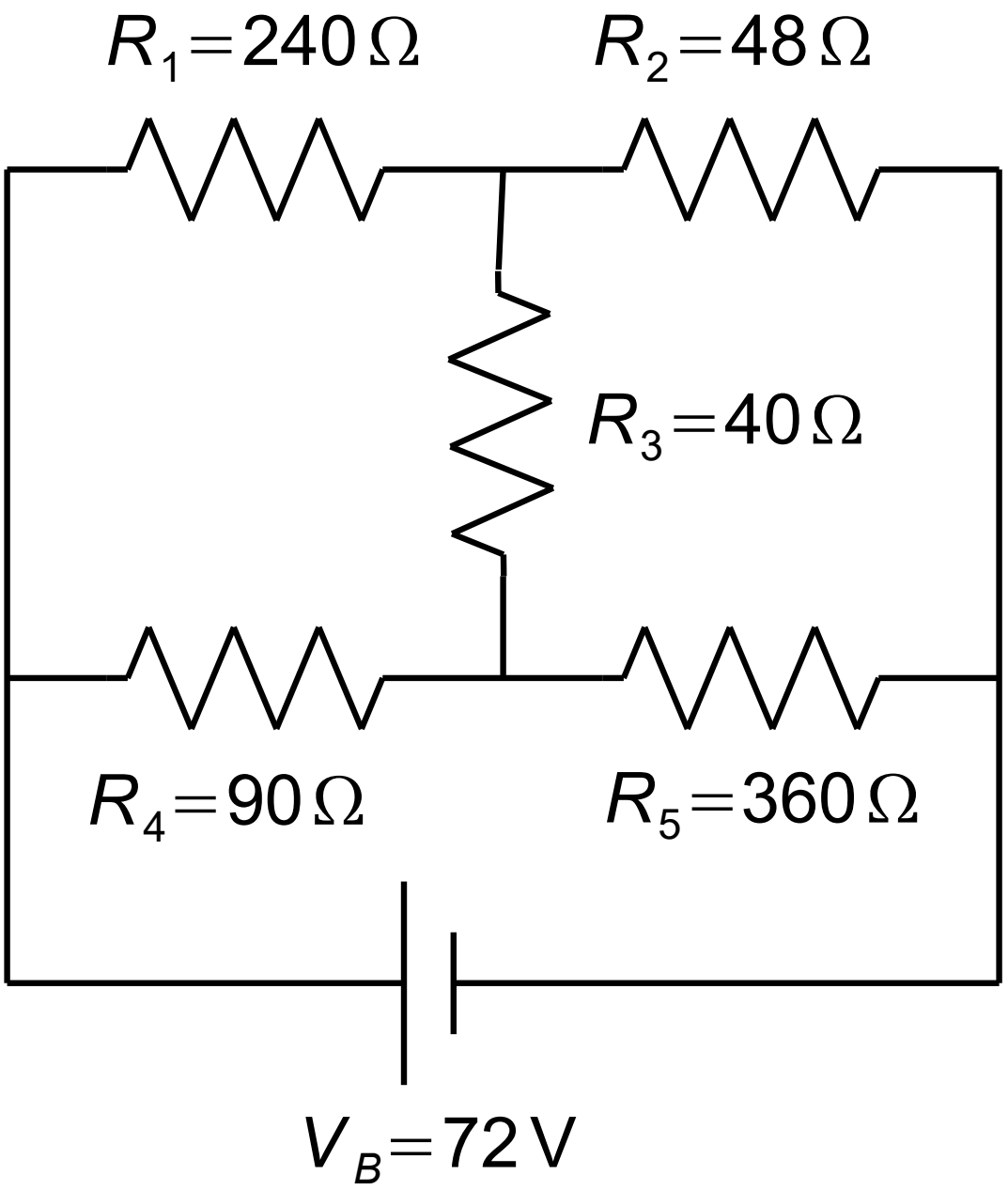
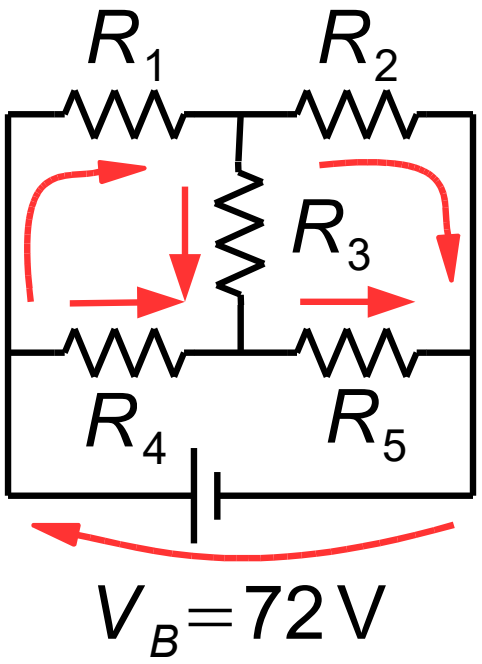
Analyze the given circuit





Determine

- the current through each resistor
- the current through the battery
- the potential across each resistor
- the total resistance

$R_1 = 240\,\Omega$

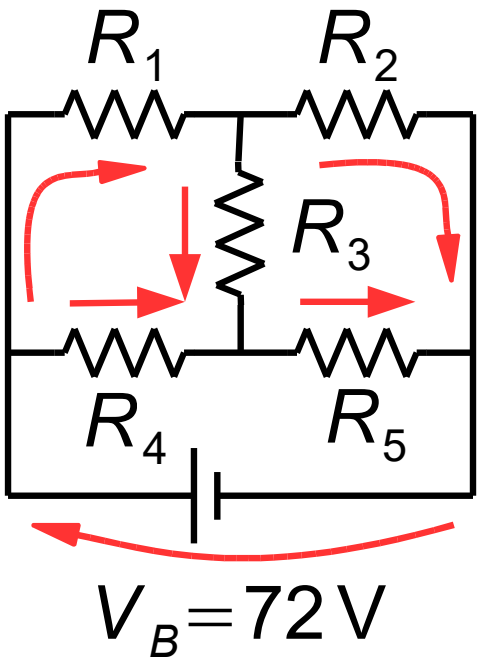
$R_2 = 48\,\Omega$

$R_3 = 40\,\Omega$

$R_4 = 90\,\Omega$

$R_5 = 360\,\Omega$

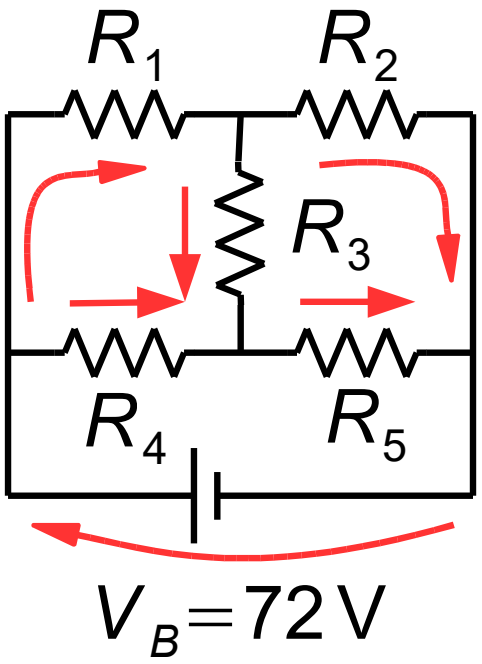
Junction Equations



$$I_B = I_1 + I_4$$
$$I_1 = I_2 + I_3$$
$$I_3 + I_4 = I_5$$
$$I_2 + I_5 = I_B$$

$R_1 = 240\,\Omega$
$R_2 = 48\,\Omega$
$R_3 = 40\,\Omega$
$R_4 = 90\,\Omega$
$R_5 = 360\,\Omega$

Junction Equations



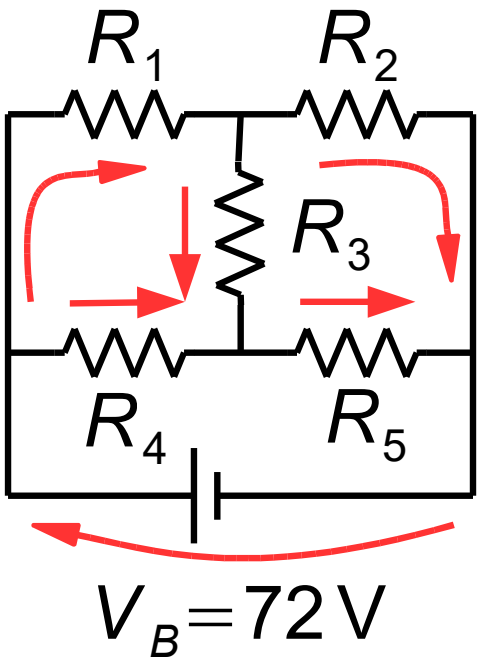
$$I_B = I_1 + I_4$$
$$I_1 = I_2 + I_3$$
$$I_3 + I_4 = I_5$$
$$I_2 + I_5 = I_B$$

Combine

$$I_1 + I_4 = I_2 + I_5$$

$$R_1 = 240\,\Omega$$
$$R_2 = 48\,\Omega$$
$$R_3 = 40\,\Omega$$
$$R_4 = 90\,\Omega$$
$$R_5 = 360\,\Omega$$

Junction Equations



$$I_B = I_1 + I_4$$
$$I_1 = I_2 + I_3$$
$$I_3 + I_4 = I_5$$
$$I_2 + I_5 = I_B$$

Substitute and Simplify

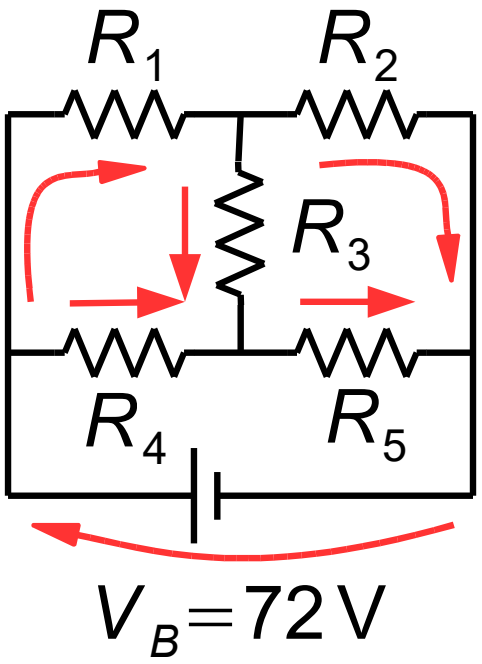
$$I_1 + I_4 = I_2 + I_5$$

$$(I_2 + I_3) + I_4 = I_2 + I_5$$

$$I_3 + I_4 = I_5$$

$$R_1 = 240\,\Omega$$
$$R_2 = 48\,\Omega$$
$$R_3 = 40\,\Omega$$
$$R_4 = 90\,\Omega$$
$$R_5 = 360\,\Omega$$

Junction Equations



$$I_B = I_1 + I_4$$
$$I_1 = I_2 + I_3$$
$$I_3 + I_4 = I_5$$
$$I_2 + I_5 = I_B$$

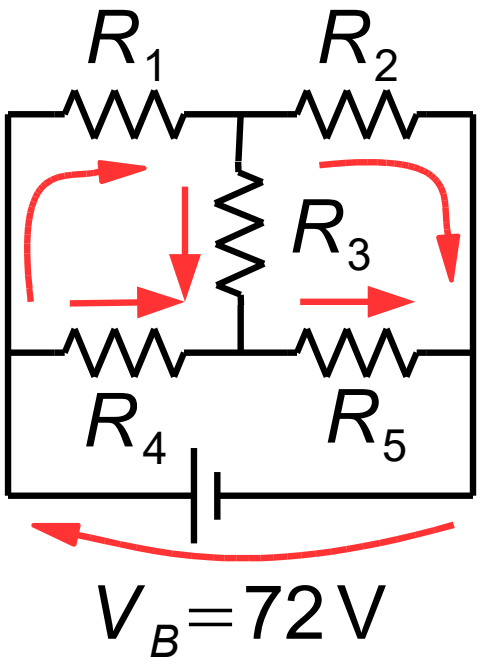
$$I_1 + I_4 = I_2 + I_5$$

$$(I_2 + I_3) + I_4 = I_2 + I_5$$

$$I_3 + I_4 = I_5$$

$R_1 = 240\,\Omega$
$R_2 = 48\,\Omega$
$R_3 = 40\,\Omega$
$R_4 = 90\,\Omega$
$R_5 = 360\,\Omega$

Equations are not independent!



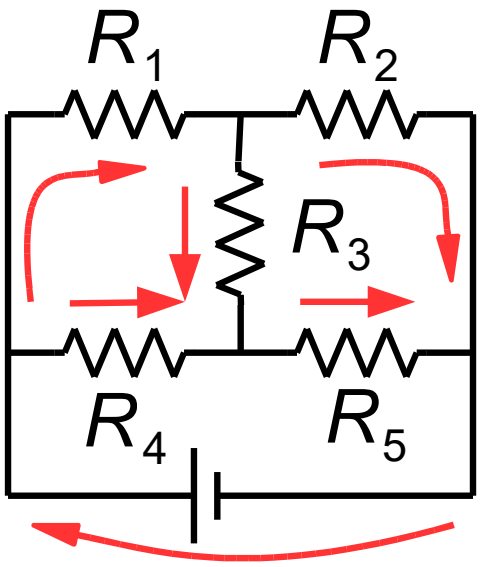
$$I_B = I_1 + I_4$$
$$I_1 = I_2 + I_3$$
$$I_2 + I_5 = I_B$$

There are many possible loop equations, but they are not all independent.

$$R_1 = 240\,\Omega$$
$$R_2 = 48\,\Omega$$
$$R_3 = 40\,\Omega$$
$$R_4 = 90\,\Omega$$
$$R_5 = 360\,\Omega$$

Loop Equations

$$-I_1R_1 - I_3R_3 + I_4R_4 = 0$$
$$-I_2R_2 + I_5R_5 + I_3R_3 = 0$$
$$V_B - I_4R_4 - I_5R_5 = 0$$
$$V_B - I_1R_1 - I_2R_2 = 0$$
$$-I_1R_1 - I_2R_2 + I_5R_5 + I_4R_4 = 0$$
$$V_B - I_4R_4 + I_3R_3 - I_2R_2 = 0$$
$$V_B - I_1R_1 - I_3R_3 - I_5R_5 = 0$$



$V_B = 72\,\mathrm{V}$

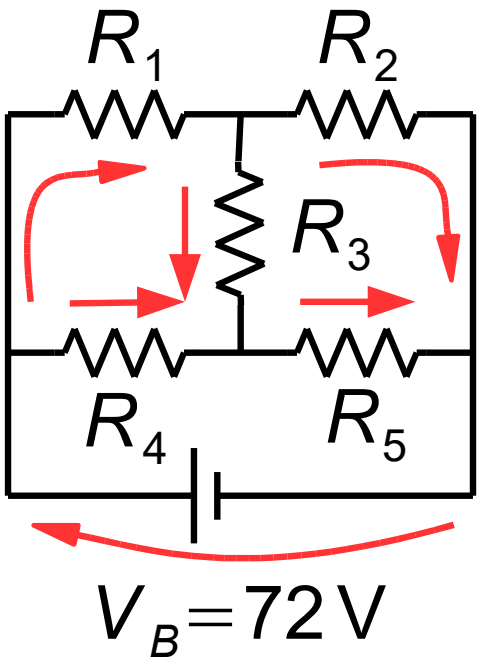
$$R_1 = 240\,\Omega$$
$$R_2 = 48\,\Omega$$
$$R_3 = 40\,\Omega$$
$$R_4 = 90\,\Omega$$
$$R_5 = 360\,\Omega$$

$$I_B = I_1 + I_4$$
$$I_1 = I_2 + I_3$$
$$I_2 + I_5 = I_B$$

Because both I_3 and I_5 only occur in one equation each, I'm guessing these are independent.

Loop Equations

$$-I_1R_1 - I_3R_3 + I_4R_4 = 0$$
$$-I_2R_2 + I_5R_5 + I_3R_3 = 0$$
$$V_B - I_4R_4 - I_5R_5 = 0$$
$$V_B - I_1R_1 - I_2R_2 = 0$$
$$-I_1R_1 - I_2R_2 + I_5R_5 + I_4R_4 = 0$$
$$V_B - I_4R_4 + I_3R_3 - I_2R_2 = 0$$
$$V_B - I_1R_1 - I_3R_3 - I_5R_5 = 0$$

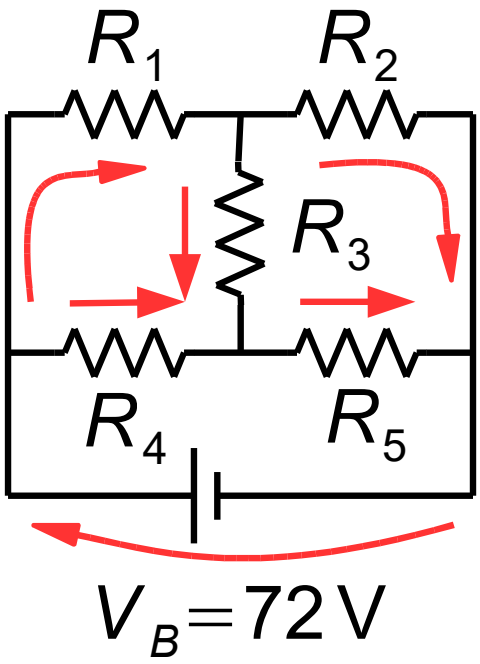


Just a little algebra left to do

$$I_B = I_1 + I_4$$
$$I_1 = I_2 + I_3$$
$$I_2 + I_5 = I_B$$

$$-I_1 R_1 - I_3 R_3 + I_4 R_4 = 0$$
$$V_B - I_1 R_1 - I_2 R_2 = 0$$
$$-I_1 R_1 - I_2 R_2 + I_5 R_5 + I_4 R_4 = 0$$

$$R_1 = 240\,\Omega$$
$$R_2 = 48\,\Omega$$
$$R_3 = 40\,\Omega$$
$$R_4 = 90\,\Omega$$
$$R_5 = 360\,\Omega$$

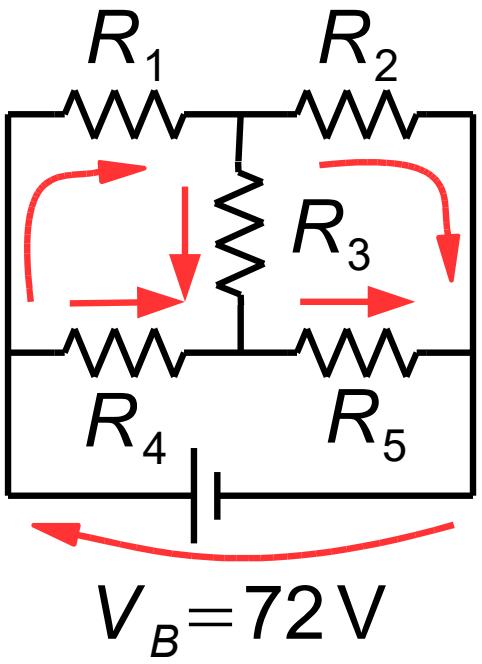


$$I_B = I_1 + I_4$$
$$I_1 = I_2 + I_3$$
$$I_2 + I_5 = I_B$$

$$-I_1 R_1 - I_3 R_3 + I_4 R_4 = 0$$
$$V_B - I_1 R_1 - I_2 R_2 = 0$$
$$-I_1 R_1 - I_2 R_2 + I_5 R_5 + I_4 R_4 = 0$$

$$I_1 + I_4 = I_2 + I_5$$

$$R_1 = 240\,\Omega$$
$$R_2 = 48\,\Omega$$
$$R_3 = 40\,\Omega$$
$$R_4 = 90\,\Omega$$
$$R_5 = 360\,\Omega$$



Substitute for I_1 everywhere.

$$I_1 = I_2 + I_3$$
$$I_1 + I_4 = I_2 + I_5$$

$$-I_1R_1 - I_3R_3 + I_4R_4 = 0$$
$$V_B - I_1R_1 - I_2R_2 = 0$$
$$-I_1R_1 - I_2R_2 + I_5R_5 + I_4R_4 = 0$$

$$(I_2 + I_3) + I_4 = I_2 + I_5$$
$$I_3 + I_4 = I_5$$

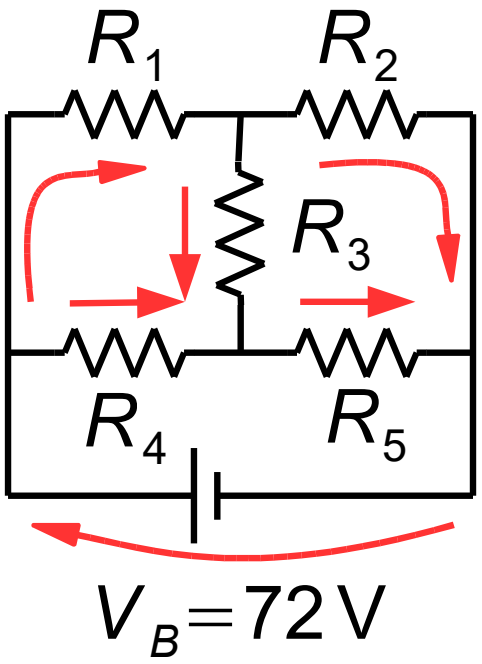
$$-(I_2 + I_3)R_1 - I_3R_3 + I_4R_4 = 0$$
$$-I_2R_1 - I_3(R_1 + R_3) + I_4R_4 = 0$$

$$V_B - (I_2 + I_3)R_1 - I_2R_2 = 0$$
$$V_B - I_2(R_1 + R_2) - I_3R_1 = 0$$

$$-(I_2 + I_3)R_1 - I_2R_2 + I_5R_5 + I_4R_4 = 0$$
$$-I_2(R_1 + R_2) - I_3R_1 + I_5R_5 + I_4R_4 = 0$$

$R_1 = 240\,\Omega$
$R_2 = 48\,\Omega$
$R_3 = 40\,\Omega$
$R_4 = 90\,\Omega$
$R_5 = 360\,\Omega$

Substitute for I_5.

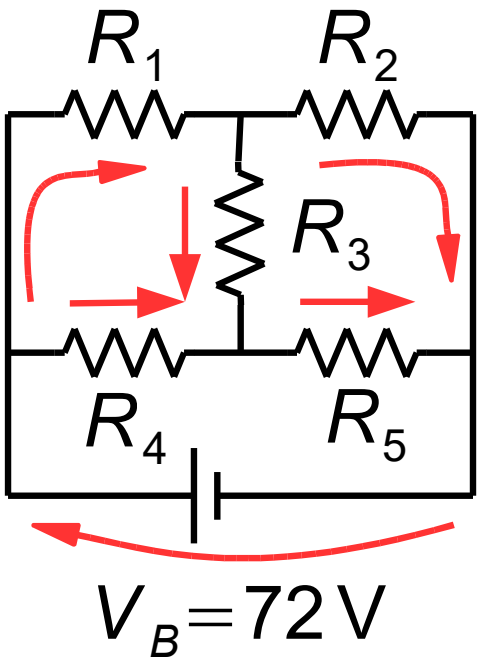


$$I_3 + I_4 = I_5$$

$$-I_2R_1 - I_3(R_1 + R_3) + I_4R_4 = 0$$
$$V_B - I_2(R_1 + R_2) - I_3R_1 = 0$$
$$-I_2(R_1 + R_2) - I_3R_1 + I_5R_5 + I_4R_4 = 0$$

$$R_1 = 240\,\Omega$$
$$R_2 = 48\,\Omega$$
$$R_3 = 40\,\Omega$$
$$R_4 = 90\,\Omega$$
$$R_5 = 360\,\Omega$$

$$-I_2(R_1 + R_2) - I_3R_1 + (I_3 + I_4)R_5 + I_4R_4 = 0$$
$$-I_2(R_1 + R_2) + I_3(R_5 - R_1) + I_4(R_4 + R_5) = 0$$

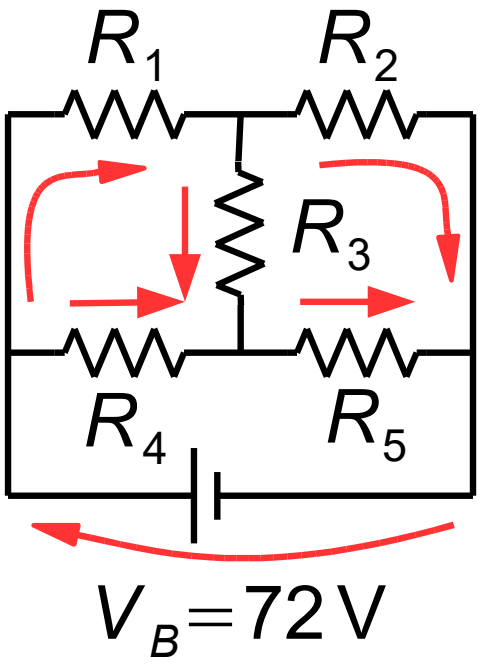


$$R_1 = 240\,\Omega$$
$$R_2 = 48\,\Omega$$
$$R_3 = 40\,\Omega$$
$$R_4 = 90\,\Omega$$
$$R_5 = 360\,\Omega$$

Solve for I_3 and substitute.

$$-I_2R_1 - I_3(R_1+R_3) + I_4R_4 = 0$$
$$V_B - I_2(R_1+R_2) - I_3R_1 = 0$$
$$-I_2(R_1+R_2) + I_3(R_5-R_1) + I_4(R_4+R_5) = 0$$

It is also acceptable to substitute in the values for the resistances before solving the system of equations.



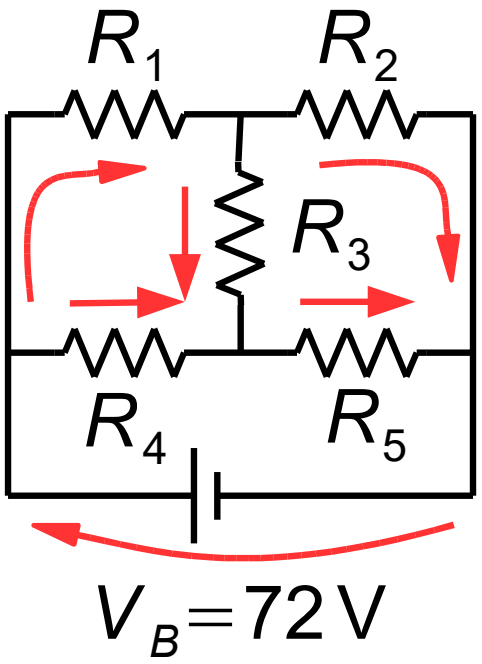
$$R_1 = 240\,\Omega$$
$$R_2 = 48\,\Omega$$
$$R_3 = 40\,\Omega$$
$$R_4 = 90\,\Omega$$
$$R_5 = 360\,\Omega$$

Substituting resistance values.

$$-I_2(240\,\Omega) - I_3(280\,\Omega) + I_4(90\,\Omega) = 0$$
$$72\,\mathrm{V} - I_2(288\,\Omega) - I_3(240\,\Omega)$$
$$-I_2(288\,\Omega) + I_3(120\,\Omega) + I_4(450\,\Omega) = 0$$

Solve for I_3.

$$\frac{72\,\mathrm{V} - I_2(288\,\Omega)}{240\,\Omega} = I_3$$
$$0.3\,\mathrm{A} - (1.2)I_2 = I_3$$



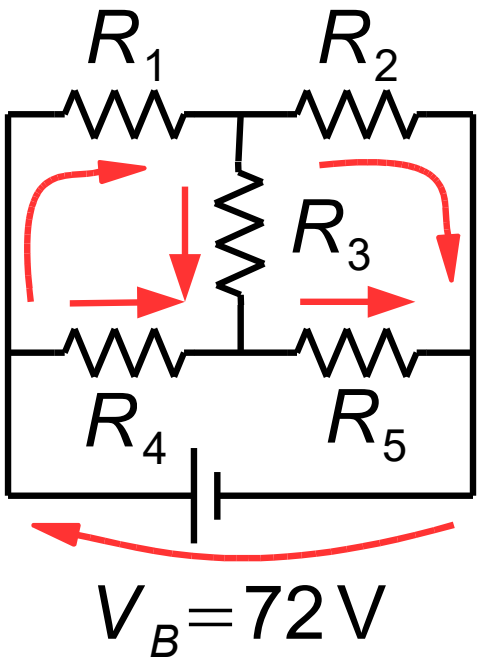
$$R_1 = 240\,\Omega$$
$$R_2 = 48\,\Omega$$
$$R_3 = 40\,\Omega$$
$$R_4 = 90\,\Omega$$
$$R_5 = 360\,\Omega$$

Substituting for I_3.

$$-I_2(240\,\Omega) - I_3(280\,\Omega) + I_4(90\,\Omega) = 0$$
$$0.3\,\mathrm{A} - (1.2)I_2 = I_3$$
$$-I_2(288\,\Omega) + I_3(120\,\Omega) + I_4(450\,\Omega) = 0$$

$$-I_2(240\,\Omega) - [0.3\,\mathrm{A} - (1.2)I_2](280\,\Omega) + I_4(90\,\Omega) = 0$$
$$-84\,\mathrm{V} + I_2(96\,\Omega) + I_4(90\,\Omega) = 0$$

$$-I_2(288\,\Omega) + [0.3\,\mathrm{A} - (1.2)I_2](120\,\Omega) + I_4(450\,\Omega) = 0$$
$$36\,\mathrm{V} - I_2(432\,\Omega) + I_4(450\,\Omega) = 0$$



$$R_1 = 240\,\Omega$$
$$R_2 = 48\,\Omega$$
$$R_3 = 40\,\Omega$$
$$R_4 = 90\,\Omega$$
$$R_5 = 360\,\Omega$$

$$-84\,\text{V} + I_2(96\,\Omega) + I_4(90\,\Omega) = 0$$
$$36\,\text{V} - I_2(432\,\Omega) + I_4(450\,\Omega) = 0$$

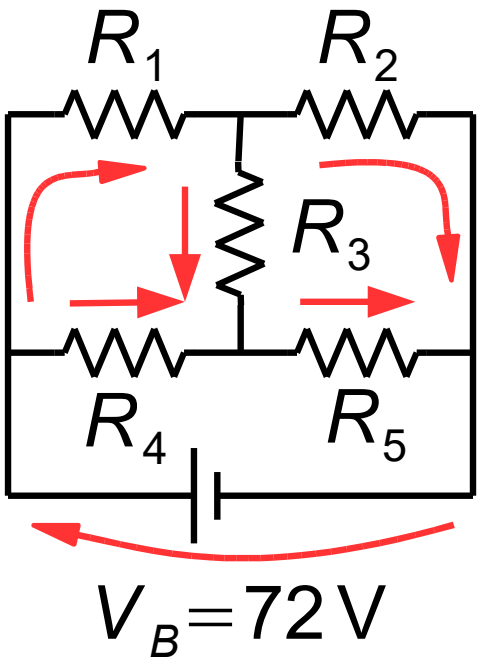
Solve for I_2.

$$\frac{36\,\text{V} + I_4(450\,\Omega)}{432\,\Omega} = I_2$$

$$0.0833\,\text{A} + (1.042)I_4 = I_2$$

Substitute.

$$-84\,\text{V} + [0.0833\,\text{A} + (1.042)I_4](96\,\Omega) + I_4(90\,\Omega) = 0$$
$$-76\,\text{V} + I_4(190\,\Omega) = 0$$
$$0.4\,\text{A} = I_4$$



$$0.4\,\mathrm{A} = I_4$$

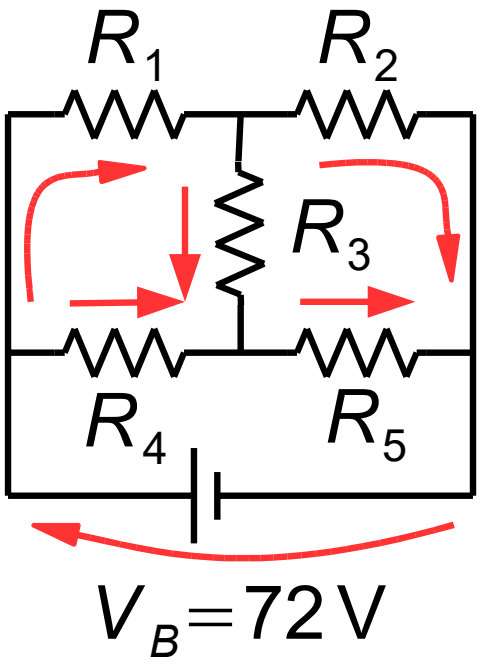
$$0.0833\,\mathrm{A} + (1.042) I_4 = I_2$$

$$0.08333\,\mathrm{A} + (1.042)(0.4\,\mathrm{A}) = I_2$$

$$\boxed{0.5\,\mathrm{A} = I_2}$$

$R_1 = 240\,\Omega$
$R_2 = 48\,\Omega$
$R_3 = 40\,\Omega$
$R_4 = 90\,\Omega$
$R_5 = 360\,\Omega$

$I_1 = ?$
$I_2 = 0.5\,\mathrm{A}$
$I_3 = ?$
$I_4 = 0.4\,\mathrm{A}$
$I_5 = ?$



$$0.3\,\text{A} - (1.2) I_2 = I_3$$

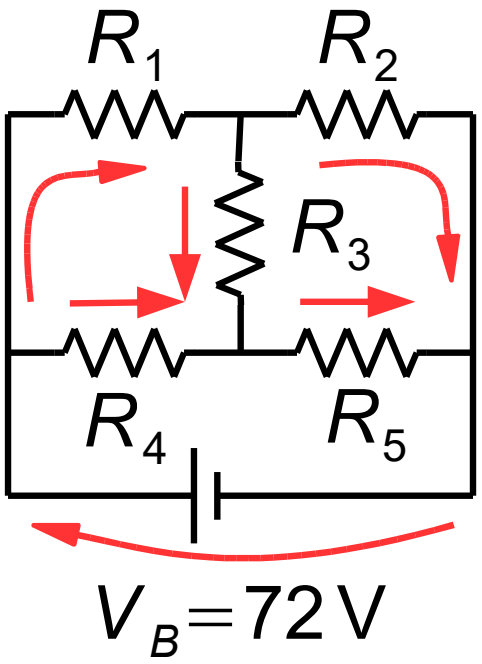
$$0.3\,\text{A} - (1.2)(0.5\,\text{A}) = I_3$$

$$\boxed{-\,0.3\,\text{A} = I_3}$$

The negative sign indicates that the direction is opposite to the direction we chose for I_3.

$R_1 = 240\,\Omega$
$R_2 = 48\,\Omega$
$R_3 = 40\,\Omega$
$R_4 = 90\,\Omega$
$R_5 = 360\,\Omega$

$I_1 = ?$
$I_2 = 0.5\,\text{A}$
$I_3 = -\,0.3\,\text{A}$
$I_4 = 0.4\,\text{A}$
$I_5 = ?$



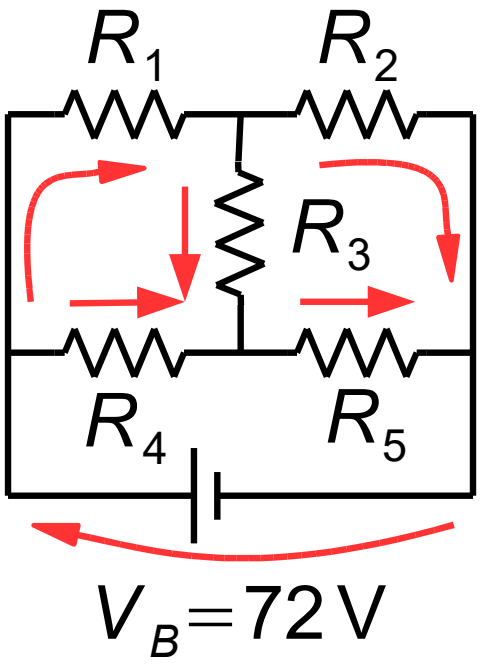
$$I_3 + I_4 = I_5$$

$$-0.3\,\mathrm{A} + 0.4\,\mathrm{A} = I_5$$

$$0.1\,\mathrm{A} = I_5$$

$R_1 = 240\,\Omega$
$R_2 = 48\,\Omega$
$R_3 = 40\,\Omega$
$R_4 = 90\,\Omega$
$R_5 = 360\,\Omega$

$I_1 = ?$
$I_2 = 0.5\,\mathrm{A}$
$I_3 = -0.3\,\mathrm{A}$
$I_4 = 0.4\,\mathrm{A}$
$I_5 = 0.1\,\mathrm{A}$



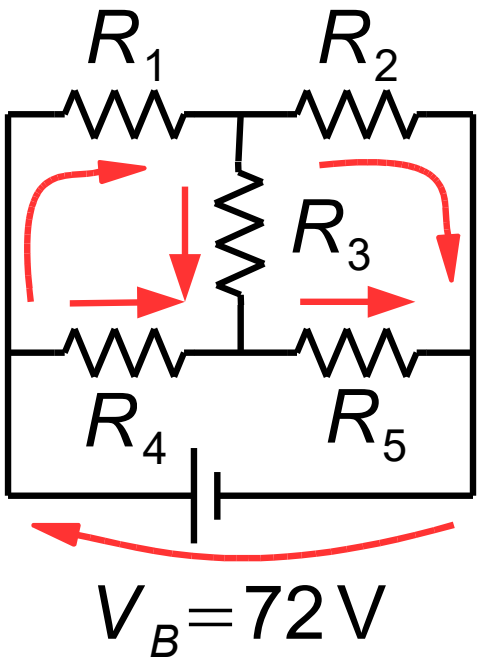
$$I_1 = I_2 + I_3$$

$$I_1 = 0.5\,\mathrm{A} - 0.3\,\mathrm{A}$$

$$\boxed{I_1 = 0.2\,\mathrm{A}}$$

$$R_1 = 240\,\Omega$$
$$R_2 = 48\,\Omega$$
$$R_3 = 40\,\Omega$$
$$R_4 = 90\,\Omega$$
$$R_5 = 360\,\Omega$$

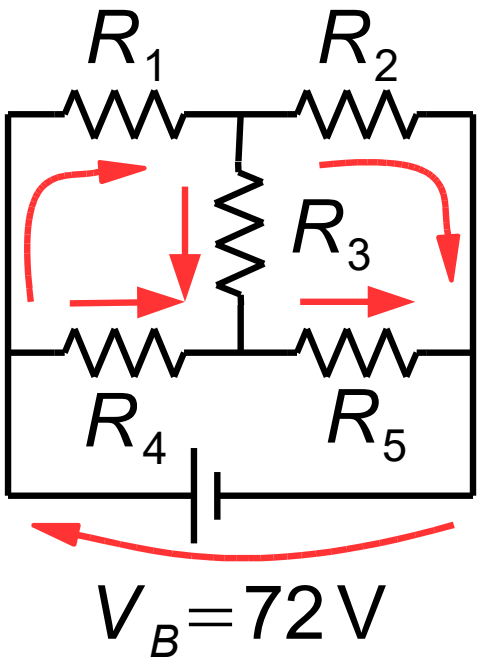
$$I_1 = 0.2\,\mathrm{A}$$
$$I_2 = 0.5\,\mathrm{A}$$
$$I_3 = -\,0.3\,\mathrm{A}$$
$$I_4 = 0.4\,\mathrm{A}$$
$$I_5 = 0.1\,\mathrm{A}$$



$$I_B = I_1 + I_4$$
$$I_B = 0.2\,\mathrm{A} + 0.4\,\mathrm{A}$$
$$\boxed{I_B = 0.6\,\mathrm{A}}$$

$$R_1 = 240\,\Omega$$
$$R_2 = 48\,\Omega$$
$$R_3 = 40\,\Omega$$
$$R_4 = 90\,\Omega$$
$$R_5 = 360\,\Omega$$

$$I_1 = 0.2\,\mathrm{A}$$
$$I_2 = 0.5\,\mathrm{A}$$
$$I_3 = -\,0.3\,\mathrm{A}$$
$$I_4 = 0.4\,\mathrm{A}$$
$$I_5 = 0.1\,\mathrm{A}$$
$$I_B = 0.6\,\mathrm{A}$$



Apply Ohm's Law … again … and again … and ...

$R_1 = 240\,\Omega$	$I_1 = 0.2\,\mathrm{A}$	$V_1 = 48\,\mathrm{V}$
$R_2 = 48\,\Omega$	$I_2 = 0.5\,\mathrm{A}$	$V_2 = 24\,\mathrm{V}$
$R_3 = 40\,\Omega$	$I_3 = -\,0.3\,\mathrm{A}$	$\lvert V_3 \rvert = 12\,\mathrm{V}$
$R_4 = 90\,\Omega$	$I_4 = 0.4\,\mathrm{A}$	$V_4 = 36\,\mathrm{V}$
$R_5 = 360\,\Omega$	$I_5 = 0.1\,\mathrm{A}$	$V_5 = 36\,\mathrm{V}$
$R_T = 120\,\Omega$	$I_B = 0.6\,\mathrm{A}$	$V_B = 72\,\mathrm{V}$