

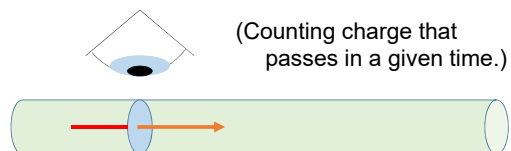
From Course Handbook:

**Regrade policy. Requests for regrades must be submitted no later than the end of the second recitation meeting after the general return of the graded material, ...**

### Electric Current (rate of charge flow)

Average Current:

$$I_{ave} = \frac{\Delta Q}{\Delta t}$$

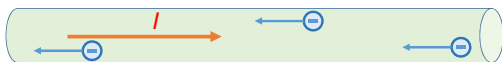


### Electric Current (rate of charge flow)

Average Current:

$$I_{ave} = \frac{\Delta Q}{\Delta t}$$

Positive current refers to flow of positive charges. If moving charges are negative (electrons), the current is in the opposite "direction".

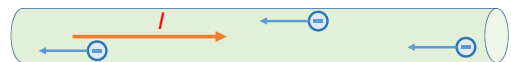


### Electric Current (rate of charge flow)

Average Current:

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It is odd to speak of direction associated with current. Current is a scalar, but it does have a sign.



### Electric Current (rate of charge flow)

Average Current:

$$I_{ave} = \frac{\Delta Q}{\Delta t}$$

Instantaneous Current:

$$I = \frac{dQ}{dt}$$

Unit of current is ampere (A) or amp.  $1A = \frac{1C}{1s}$

### Electric Current (rate of charge flow)

Typical Currents:

- 100W light bulb                    1A
- Automobile starter motor       200A
- Electronics                        nA - mA

### Current and Current Density

The current in a wire is the sum of all the charge per time passing through a cross-section of the wire.

$$\text{Current} = \frac{\text{charge}}{(\text{area})(\text{time})} (\text{area})$$

$$I = \int \vec{j} \cdot d\vec{A}$$

$\vec{j}$  is current density.

### Current and Current Density

$$I = \int \vec{j} \cdot d\vec{A}$$

In many applications,

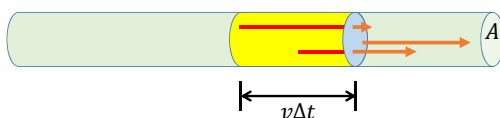
$\vec{j}$  is uniform and parallel to  $d\vec{A}$ .

$$I = \int \vec{j} \cdot d\vec{A} = J \int dA = JA$$

or

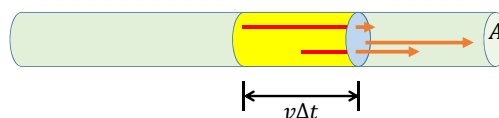
$$J = \frac{I}{A}$$

### Current A microscopic view



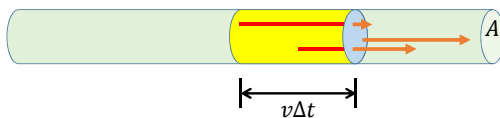
- Charges passing a given point in time,  $\Delta t$ , are those initially in a volume,  $(v\Delta t)A$ .

### Current A microscopic view



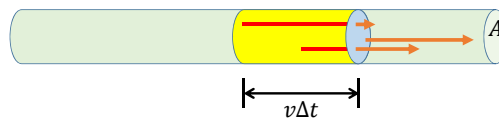
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### Current A microscopic view



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- The number of passing charges depends on the charge density,  $n$ .  $N = n(v\Delta t)A$
- The amount of passing charge depends on the charge per carrier,  $q$ .  $\Delta Q = q(nv\Delta t)A$

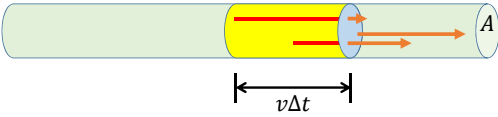
### Current A microscopic view



$$I = \frac{\Delta Q}{\Delta t} = nqvA$$

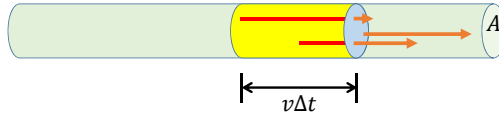
- The amount of passing charge depends on the charge per carrier,  $q$ .  $\Delta Q = q(nv\Delta t)A$

**Current**  
A microscopic view



$I = \frac{\Delta Q}{\Delta t} = nqvA$        $J = \frac{I}{A} = nqv$   
(not quite correct)


**Current Density Corrected**



$I = \frac{\Delta Q}{\Delta t} = nqvA$        $\vec{J} = nq\vec{v}_d$

$\vec{v}_d$  is the average velocity, called **drift velocity**.

**Current Density Corrected**



$\vec{J} = nq\vec{v}_d$

Some free electron densities:

- Silver       $n = 5.86 \times 10^{28} /\text{m}^3$
- Gold       $n = 5.90 \times 10^{28} /\text{m}^3$
- Copper     $n = 8.47 \times 10^{28} /\text{m}^3$
- Aluminum  $n = 18.1 \times 10^{28} /\text{m}^3$

Example: 12-gauge copper wire (common in home wiring) has a cross-sectional area of  $3.31 \times 10^{-6} \text{m}^2$  and carries a current of 10A. Determine the drift speed of the electrons. [Free electron density in copper is  $8.47 \times 10^{28} \text{m}^{-3}$ .]

**Ohm's Law**

Current caused by electric field in conductor

Amount of current depends on

- Strength of field
- How conductive the conductor is

$\vec{j} = \sigma \vec{E}$

$\sigma$  is electrical conductivity

**Ohm's Law**

$\vec{j} = \sigma \vec{E}$

$\sigma$  is electrical conductivity

Alternatively written as

$\vec{j} = \frac{1}{\rho} \vec{E}$

$\rho$  is electrical resistivity


Either version may be referred to as Ohm's Law

(In this context,  $\sigma$  and  $\rho$  are NOT charge densities.)


**Ohm's Law**

$$\vec{j} = \frac{1}{\rho} \vec{E}$$

Some materials follow Ohm's Law.  
Ohmic materials



Other materials do not follow Ohm's Law.  
Non-Ohmic materials



(Ohm's Law is not a Law of Nature.)

Example: An 8m long 12-gauge copper wire ( $A = 3.31 \times 10^{-6} \text{m}^2$ ) carries a current of 10A when connected across a potential difference of 0.406V.

- a) Determine the current density in the wire.
- b) Determine the electric field in the wire.
- c) Determine the resistivity of the wire.

**Ohm's Law**

$$\vec{j} = \frac{1}{\rho} \vec{E}$$

Resistivity depends on temperature.

$$\rho = \rho_0 [1 + \alpha(T - T_0)]$$

$\alpha$  is the temperature coefficient and  $\rho_0$  is the resistivity at  $T_0$ .

Example: A wire is connected across a constant potential difference. It is noted that as the wire heats up from 20°C to 50°C, the current drops from 0.500A to 0.443A. Determine the temperature coefficient of the material.

Material	Conductivity ( $\times 10^7 / \Omega\text{m}$ )	Resistivity ( $\times 10^{-8} \Omega\text{m}$ )	Temperature Coefficient ( $^{\circ}\text{C}$ )
Aluminum	3.77	2.65	0.00429
Gold	4.1	2.44	0.0034
Copper	5.95	1.68	0.00386
Silver	6.29	1.59	0.0038
Silicon*	$1.56 \times 10^{-3} / \Omega\text{m}$	$6.4 \times 10^2 \Omega\text{m}$	$-0.075 / ^{\circ}\text{C}$

\*Silicon values depend strongly on impurities.

**Microscopic vs. Macroscopic View**

**Microscopic**

- Material
- Resistivity
- Current Density

$$\vec{E} = \rho \vec{j}$$

**Macroscopic**

- Device
- Resistance
- Current

$$V = IR$$

**Connections**

$$J = \frac{I}{A}$$

$$R = \rho \frac{L}{A}$$

**Resistance**

$$R = \rho \frac{L}{A}$$

Resistance

- Proportional to length
- Inversely proportional to area
- Unit is Ohm ( $\Omega$ )

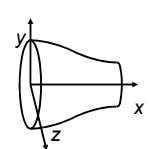
Applications: Jumper cables and holiday lights.

**Resistance**

Resistance depends on the material (resistivity) and the geometry of the device.

$$R = \rho \frac{L}{A}$$

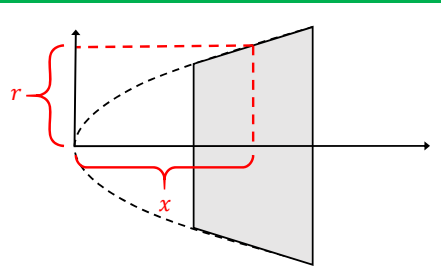
Integrate if the cross-section is not uniform.



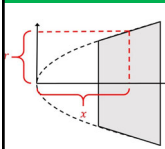
$$R = \int_0^L \rho \frac{dx}{A}$$

(A is a function of x.)

Example: The radius of a conductor is given by  $r = \frac{1}{4}x^2$  between  $x = 2\text{mm}$  and  $x = 4\text{mm}$ . Determine the resistance of the conductor for a current flowing in the  $x$ -direction.



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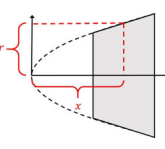
$$R = \rho \frac{L}{A} \rightarrow R = \int \rho \frac{dx}{A}$$

$$R = \int_{r_0}^{r_f} \rho \frac{dx}{\pi r^2}$$

$$R = \int_{r_0}^{r_f} \rho \frac{dx}{\pi \left(\frac{1}{4}x^2\right)^2} = \int_{r_0}^{r_f} \rho \frac{16dx}{\pi x^4}$$

$$R = -\frac{16\rho}{3\pi} \left( \frac{1}{r_f^3} - \frac{1}{r_0^3} \right)$$

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Be careful to enter limits in meters.

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