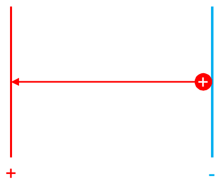
Energy Storage

Consider the work required to move a small positive charge between plates of a capacitor.

Single Charge

$$W_{ext} = q \Delta V$$



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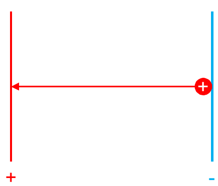


Total Q

$$W_{ext} = \int_0^Q V \, dq$$

$$W_{ext} = \int_0^Q \frac{q}{C} \, dq$$

$$W_{ext} = \frac{1}{2} \frac{Q^2}{C}$$

Energy Storage

The energy stored in a capacitor is

$$U = W_{ext} = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} C V^2 = \frac{1}{2} Q V$$

Energy Storage

Where is the energy stored?

$U = \frac{1}{2}\frac{Q^2}{C}$ Energy is stored in the charge arrangement.

Energy Storage

Where is the energy stored?

$$U = \frac{1}{2} C V^2$$

$$U = \frac{1}{2}\left(\frac{A\epsilon_0}{d}\right)(Ed)^2$$

$$U = \frac{1}{2}\epsilon_0 (Ad) E^2$$

$Ad = \text{Volume}$

$$u = \frac{U}{\text{Volume}} = \frac{1}{2}\epsilon_0 E^2$$

Energy Storage

Where is the energy stored?

$U = \frac{1}{2}\frac{Q^2}{C}$ Energy is stored in the charge arrangement.

OR

$u = \frac{1}{2}\epsilon_0 E^2$ Energy is stored in the electric field.

(Note: I use either interpretation as convenient, but do not use a combination or sum of the two interpretations.)

Material in Capacitors

Conductors:

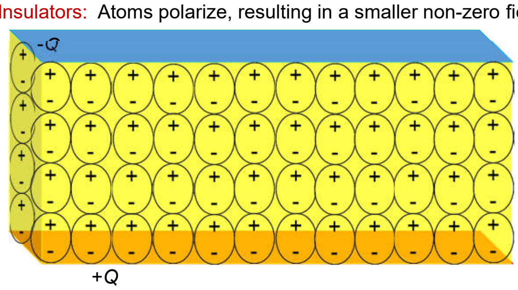
When calculating ΔV, the integral of $\int \vec{E} \cdot d\vec{s} = 0$ across the conductor, resulting in an effective gap in the capacitor of $(d - w)$.

$$C = \frac{A\epsilon_0}{d - w}$$

(See previous lecture.)

Material in Capacitors

Insulators: Atoms polarize, resulting in a smaller non-zero field.



$$C = \frac{\kappa A\epsilon_0}{d}$$

Material in Capacitors

Conductors:

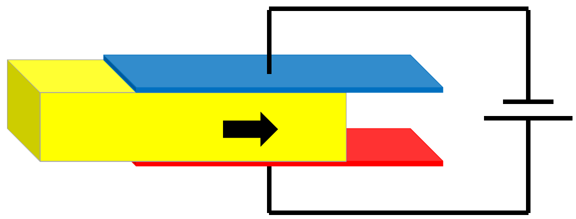
$$C = \frac{A\epsilon_0}{d - w}$$

Insulators:

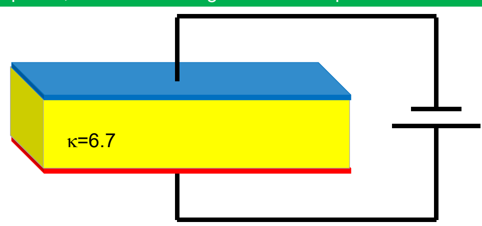
$$C = \frac{\kappa A\epsilon_0}{d} \qquad (\kappa \geq 1)$$

- Dielectric constant depends on the properties of the insulator.
- Insulator formula assumes gap is completely filled with the insulator.
- Conductor formula assumes gap is partially filled with the conductor.

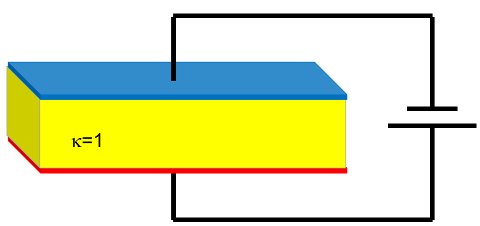
Example: While a capacitor is connected to a battery, a dieletric material is inserted. What happens to the amount of charge stored on the capacitor?



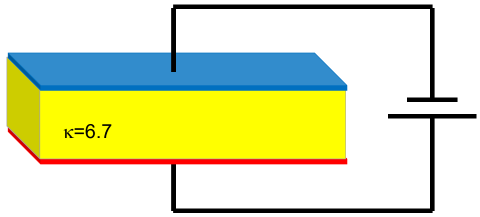
Example: a parallel plate capacitor has an area of 10 cm^2 and plate separation 5 mm. A 300 V battery is connected to its plates. If neoprene is inserted between its plates, how much charge does the capacitor hold.



Example: how much charge would the capacitor on the previous slide hold if the dielectric were air?

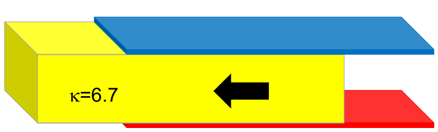


Example: a parallel plate capacitor has an area of 10 cm^2 and plate separation 5 mm. A 300 V battery is connected to its plates. How much energy is stored in the capacitor?

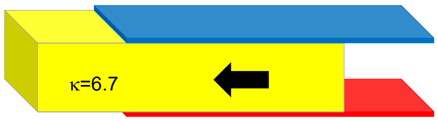


Example: The battery is disconnected. (a) How much energy is stored in the capacitor? (b) What is the capacitance of the capacitor? (c) What is the potential difference across the plates? How much charge is stored on the plates?

Example: The dielectric is removed while the battery is disconnected. (a) After complete removal, how much energy is stored in the capacitor? (b) What is the capacitance of the capacitor? (c) What is the potential difference across the plates? (d) How much charge is on the plates?



Example: The dielectric is removed while the battery is disconnected. (a) After complete removal, how much energy is stored in the capacitor? (b) What is the capacitance of the capacitor? (c) What is the potential difference across the plates? (d) How much charge is on the plates?



What was the source of the additional energy?