Announcements

- grade spreadsheets with exam 1 scores will be posted today on the Physics 2135 web site
- you need your PIN to find your grade

- preliminary exam average is about 80% Nicely Done!

- exam 1 will be returned in recitation Thursday
- check that points were added correctly
- regrade requests are **due by next Thursday's** recitation

On a separate sheet of paper, explain the reason for your request. This should be based on the work shown on paper, not what was in your head. Attach to the exam and hand it to your recitation instructor by next Thursday.

Today's agenda:

Energy Storage in Capacitors.

You must be able to calculate the energy stored in a capacitor, and apply the energy storage equations to situations where capacitor configurations are altered.

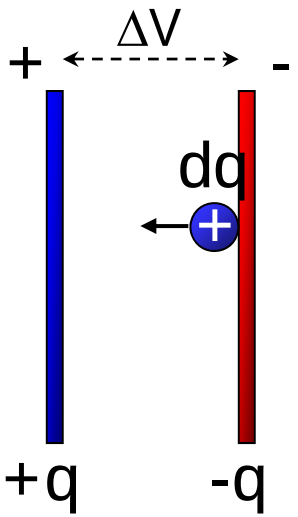
Dielectrics.

You must understand why dielectrics are used, and be able include dielectric constants in capacitor calculations.

Energy Storage in Capacitors

work to charge a capacitor:

- capacitor already has charge q, voltage (difference) ΔV
- move extra charge element dq from one plate to the other
- external work required: $dW = dq\,\Delta V$.



$$dW = \Delta V\ dq = \frac{q}{C}dq$$

from $q=C\Delta V$

- start with zero charge, end up with Q:

$$W = \int_0^Q dW = \int_0^Q \frac{q}{C}dq = \frac{q^2}{2C}\bigg|_0^Q = \frac{Q^2}{2C}\ .$$

- work required to charge the capacitor = change in potential energy

$$U_f - U_i = W_{ext}$$

- when starting from empty capacitor: $U_i = 0$

potential energy stored in capacitor: $U = \frac{Q^2}{2C}$.

Using Q=CV, three equivalent expressions:

$$U = \frac{Q^2}{2C} = \frac{CV^2}{2} = \frac{QV}{2} .$$

All three equations are valid; use the one most convenient for the problem at hand.

- four quantities for a capacitor: C, Q, V, and U
- if you know any two of them, you can find the other two

Example: a camera flash unit stores energy in a 150 μF capacitor at 200 V. How much electric energy can be stored?

$$U = \frac{CV^2}{2}$$

$$U = \frac{\left(150 \times 10^{-6}\right)\left(200^2\right)}{2} \text{ J}$$

$$\boxed{U = 3 \text{ J}}$$

If you keep everything in SI (mks) units, the result is “automatically” in SI units.

Energy stored in capacitor vs. energy stored in battery

12 V, 100 Ah car battery

- charge: 3.6×10^5 C, energy: 4.3×10^6 J

we'll learn how to calculate that later in the course

100 μF capacitor at 12 V

- charge: $Q=CV= 1.2 \times 10^{-3}$ C, energy: $U=CV^2/2=7.2 \times 10^{-3}$ J

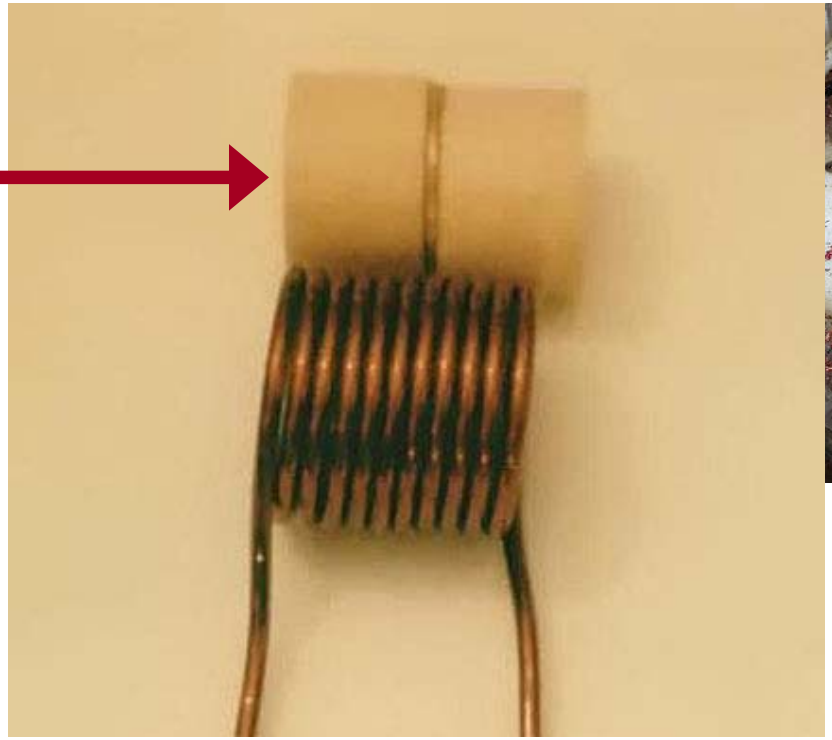
If batteries store so much more energy, why use capacitors?

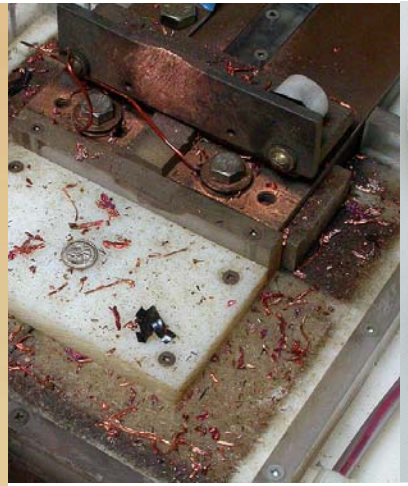
- capacitor stores charge physically, battery stores charge chemically
- capacitor can **release** stored charge and energy **much faster**

Application #1:
Short pulse magnets at the National Magnet Laboratory,

10^6 joules of energy are stored at **high voltage** in capacitor banks, and released during a period of a few milliseconds. The enormous current produces incredibly high magnetic fields.

Application #2: quarter shrinker.





Application #3: can crusher.

Some links: shrinking, shrinking (can you spot the physics mistake), can crusher,. Don't do this at home. Or this.

Where does the stored energy reside?

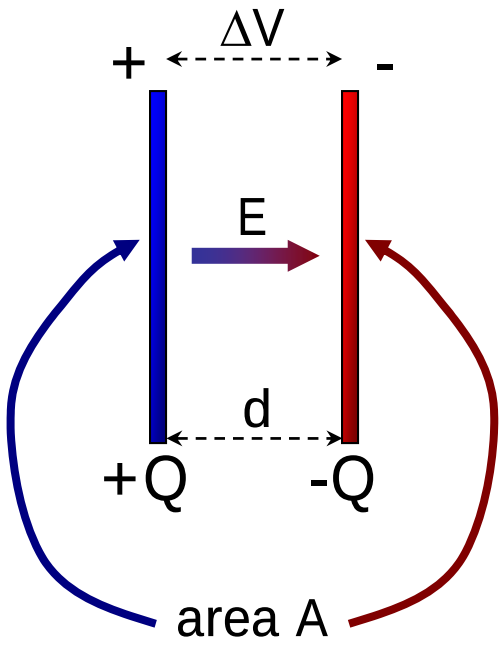
Energy is stored in the capacitor:

$$U = \frac{1}{2} C (\Delta V)^2$$

$$C = \frac{\varepsilon_0 A}{d} \quad \text{and} \quad \Delta V = Ed$$

$$U = \frac{1}{2} \left(\frac{\varepsilon_0 A}{d} \right) (Ed)^2$$

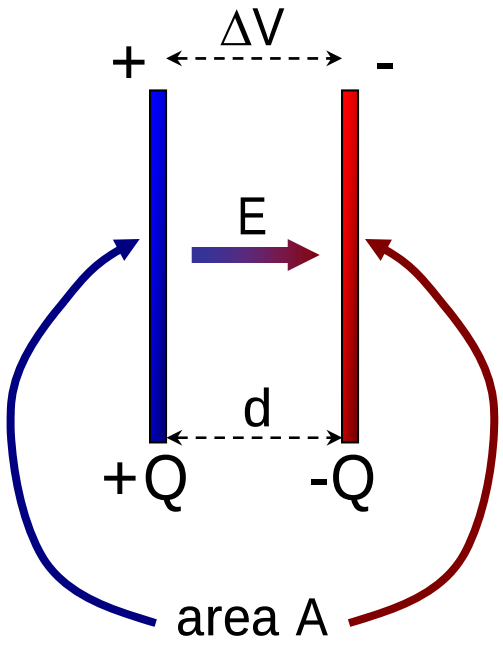
$$U = \frac{1}{2} (\varepsilon_0 Ad) E^2$$



The “volume of the capacitor” is Volume=Ad

energy density u (energy per unit volume):

$$u = \frac{U}{Ad} = \frac{\frac{1}{2}\left(\varepsilon_0 Ad\right)E^2}{Ad} = \frac{1}{2}\varepsilon_0 E^2$$



energy resides in the electric field between the plates

$$u = \frac{1}{2}\varepsilon_0 E^2$$

This is on page 2 of your OSE sheet. For now, use it only if you **really** know what you are doing.)

Today's agenda:

Energy Storage in Capacitors.

You must be able to calculate the energy stored in a capacitor, and apply the energy storage equations to situations where capacitor configurations are altered.

Dielectrics.

You must understand why dielectrics are used, and be able include dielectric constants in capacitor calculations.

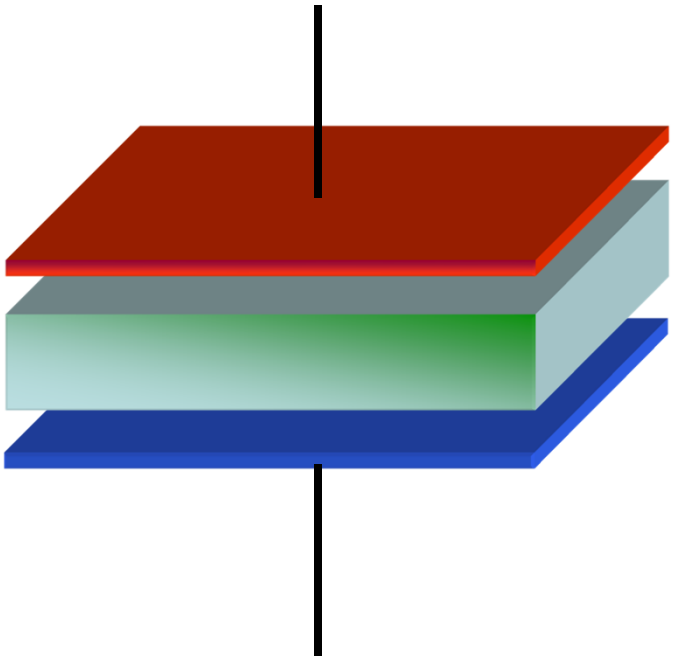
Dielectrics

- if insulating material ("dielectric") is placed between capacitor plates capacitance increases by factor κ
- κ (greek letter kappa) is the **dielectric constant**

$$C = \frac{\kappa \varepsilon_0 A}{d}.$$

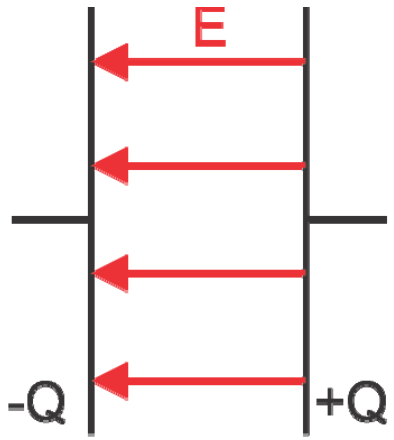
- κ depends on the material

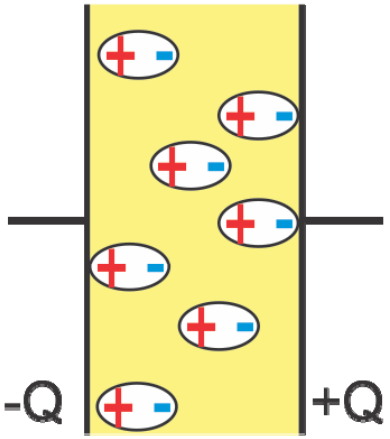
vacuum	$\kappa =$	1
air	$\kappa =$	1.0006
PVC	$\kappa =$	3.18
$SrTiO_3$	$\kappa =$	310

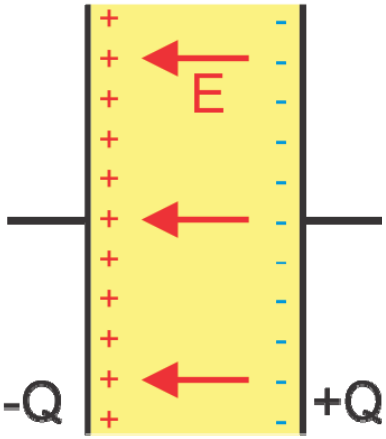


How does a dielectric work?

- dielectric material contains **dipoles**
- dipoles align in electric field
- **induced charges** partially compensate charges on plates
- electric field and voltage in capacitor reduced $(\Delta V = Ed)$
- $C = \frac{Q}{\Delta V}$ capacitance increases







Homework hints:

- if you charge a capacitor and then **disconnect the battery, Q stays the same** (charge cannot leave the plates!)
  C, V, and U may change

- if you charge a capacitor and **keep the battery connected**, **V stays the same** (voltage is fixed by the battery)
  C, Q, and U may change

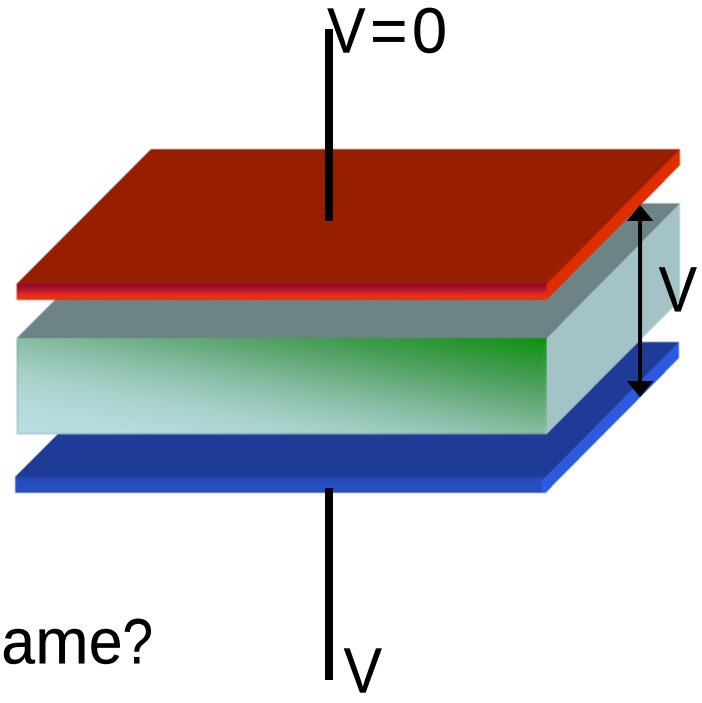
Homework hints:

If you charge two capacitors, then remove the battery and reconnect the capacitors with oppositely-charged plates connected together...

draw a circuit diagram before and after, and use conservation of charge to determine the total charge on each plate before and after.

Conceptual Example

A capacitor connected to a voltage source as shown acquires a charge Q.



While the capacitor is still connected to the battery, a dielectric material is inserted.

Will Q increase, decrease, or stay the same?

Why?

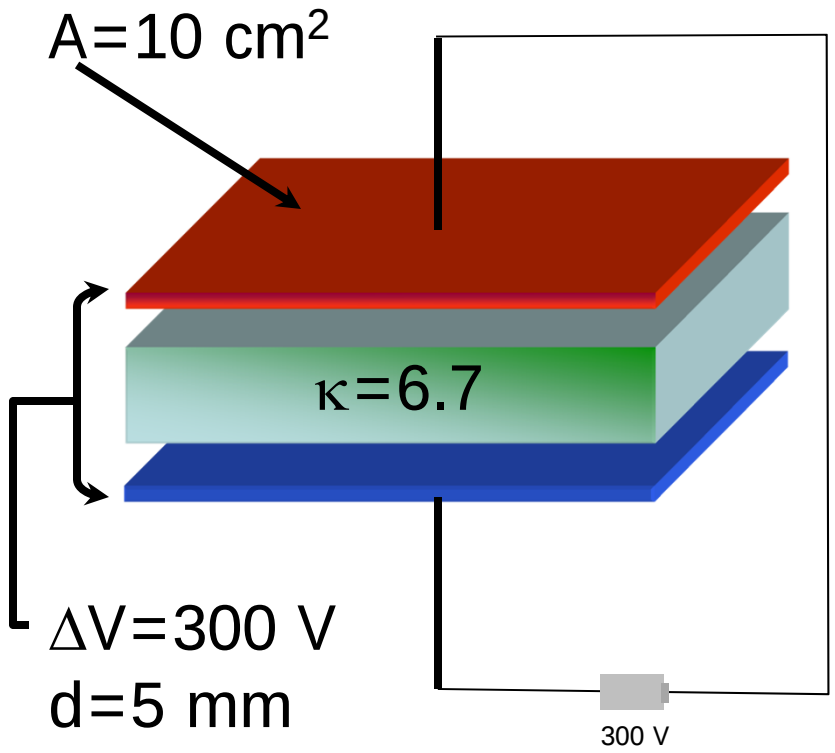
Example: a parallel plate capacitor has an area of 10 cm^2 and plate separation 5 mm. A 300 V battery is connected to its plates. If neoprene is inserted between its plates, how much charge does the capacitor hold.

$$C = \frac{\kappa \varepsilon_0 A}{d}$$

$$C = \frac{(6.7)(8.85\times10^{-12})(10\times10^{-4})}{5\times10^{-3}}\,F$$

$$C = 1.19\times10^{-11}F$$

$$Q = CV$$



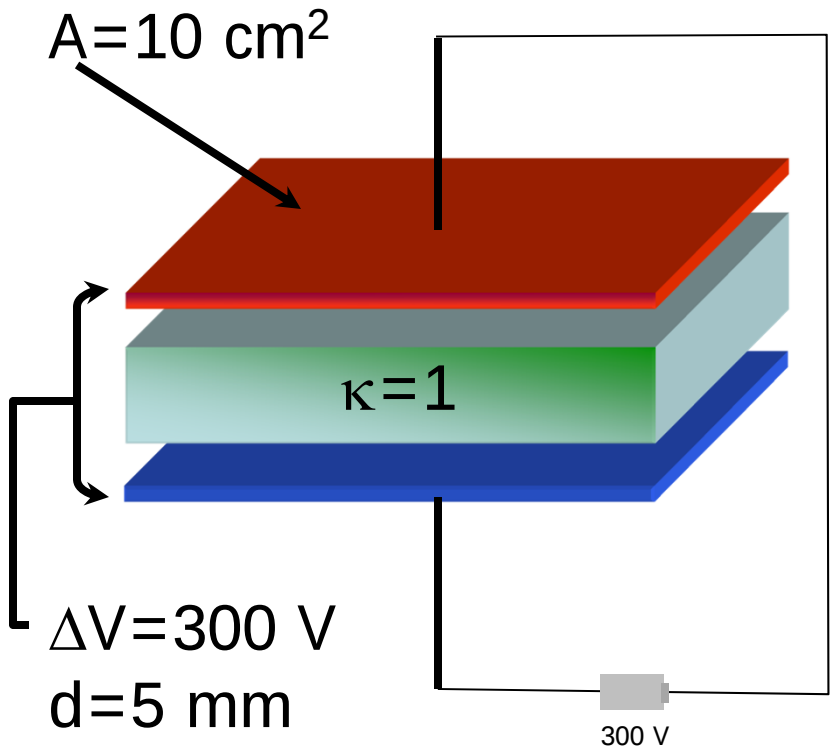
$$Q = (1.19\times10^{-11})(300)C = (3.56\times10^{-9}\ C) = 3.56\,nC$$

Example: how much charge would the capacitor on the previous slide hold if the dielectric were air?

The calculation is the same, except replace 6.7 by 1.

Or just divide the charge on the previous page by 6.7 to get...

$Q = 0.53\ \text{nC}$



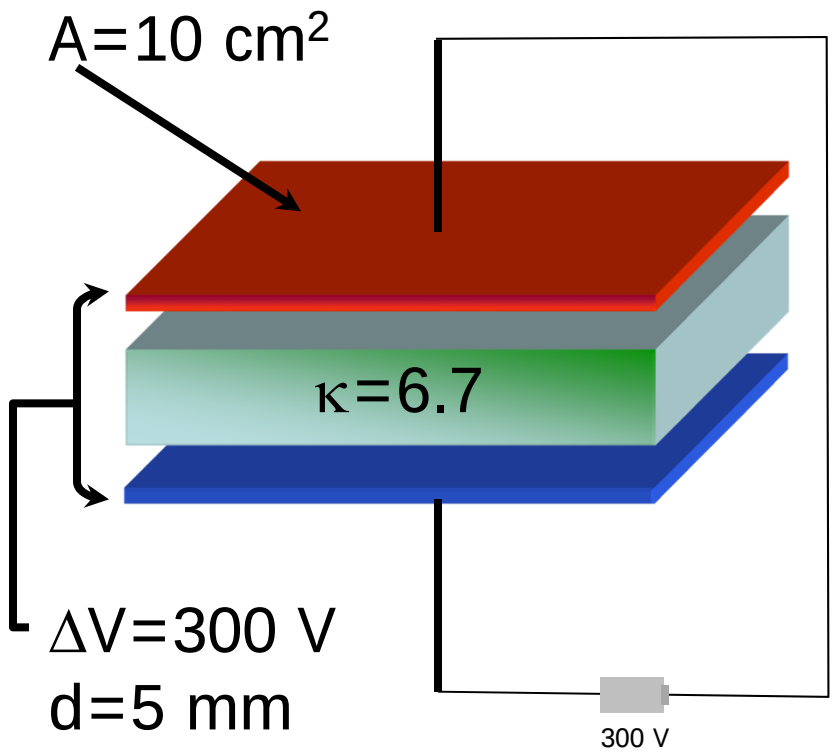
Example: find the energy stored in the capacitor.

$$C = 1.19 \times 10^{-11} \text{F}$$

$$U = \frac{1}{2} C (\Delta V)^2$$

$$U = \frac{1}{2} \left(1.19 \times 10^{-11}\right) (300)^2 \text{ J}$$

$$U = 5.36 \times 10^{-7} \text{ J}$$



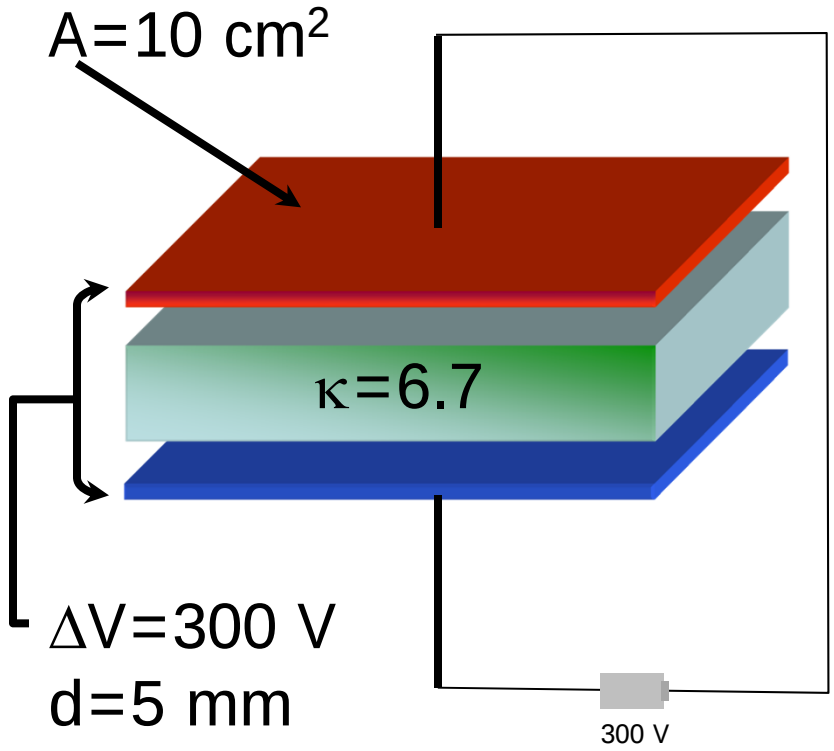
Example: the battery is now disconnected. What are the charge, capacitance, and energy stored in the capacitor?

The charge and capacitance are unchanged, so the voltage drop and energy stored are unchanged.

$Q = 3.56\,\text{nC}$

$C = 1.19 \times 10^{-11}\,\text{F}$

$U = 5.36 \times 10^{-7}\,\text{J}$

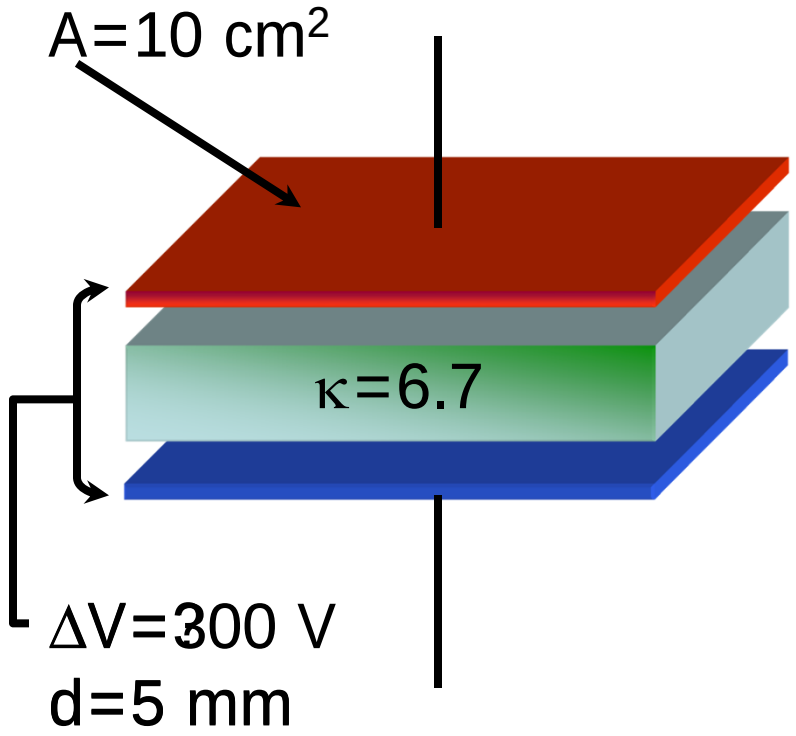


Example: the dielectric is removed without changing the plate separation. What are the capacitance, charge, potential difference, and energy stored in the capacitor?

$$C = \frac{\varepsilon_0 A}{d}$$

$$C = \frac{\left(8.85\times10^{-12}\right)\left(10\times10^{-4}\right)}{5\times10^{-3}}\,F$$

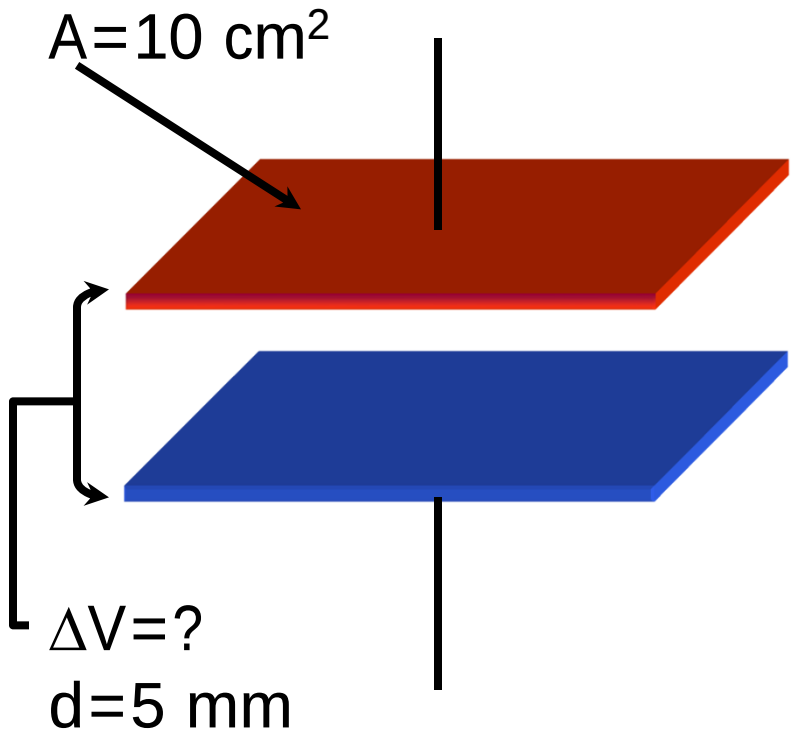
$$C = 1.78\times10^{-12}\,F$$



Example: the dielectric is removed without changing the plate separation. What are the capacitance, charge, potential difference, and energy stored in the capacitor?

The charge remains unchanged, because there is nowhere for it to go.

$Q = 3.56\ nC$

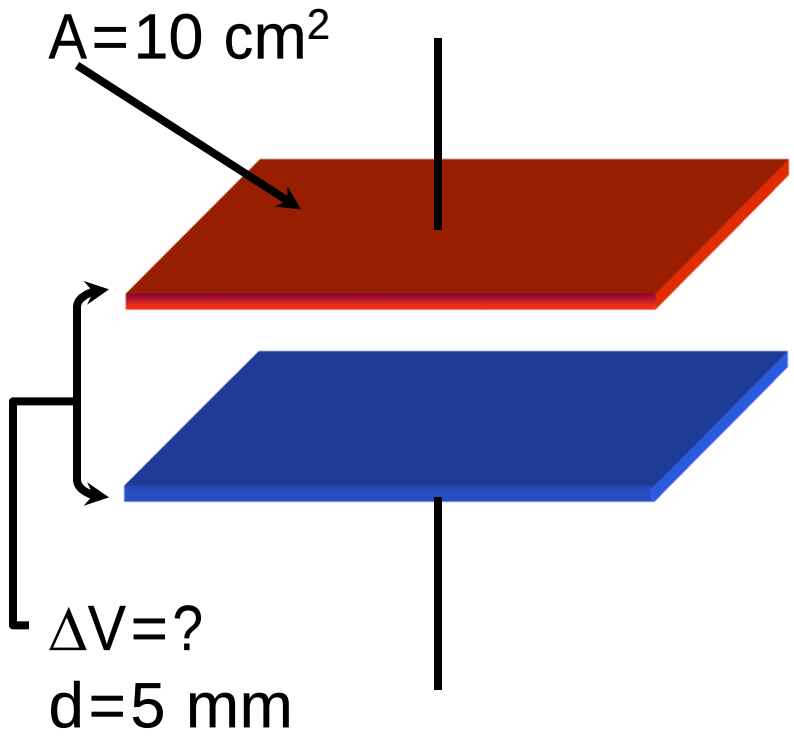


Example: the dielectric is removed without changing the plate separation. What are the capacitance, charge, potential difference, and energy stored in the capacitor?

Knowing C and Q we can calculate the new potential difference.

$$\Delta V = \frac{Q}{C} = \frac{\left(3.56\times10^{-9}\right)}{\left(1.78\times10^{-12}\right)} V$$

$$\Delta V = 2020 \text{ V}$$

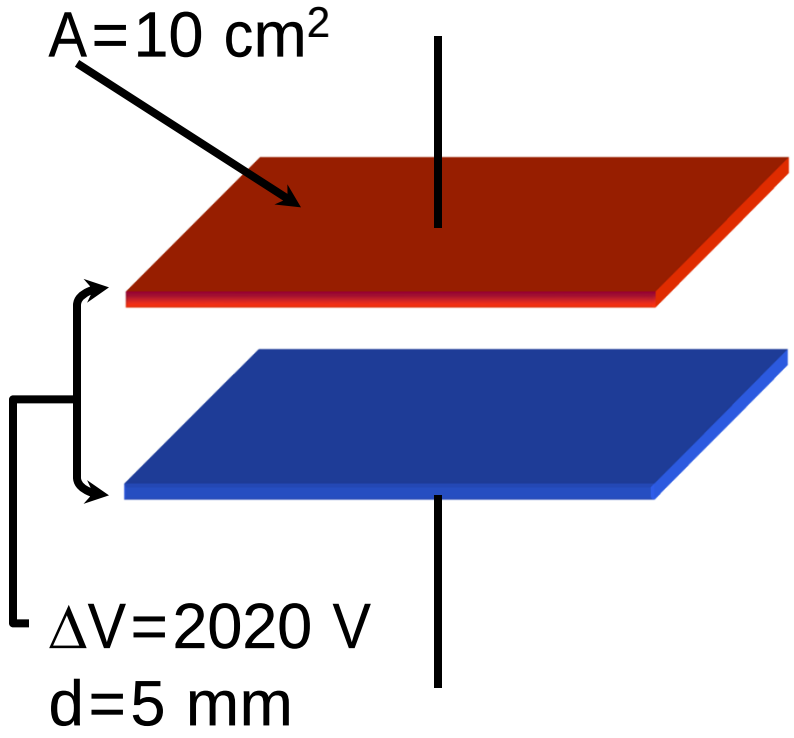


Example: the dielectric is removed without changing the plate separation. What are the capacitance, charge, potential difference, and energy stored in the capacitor?

$$U = \frac{1}{2} C (\Delta V)^2$$

$$U = \frac{1}{2}\left(1.78\times10^{-12}\right)(2020)^2 \text{ J}$$

$$U = 3.63\times10^{-6} \text{ J}$$



$$U_{before} = 5.36 \times 10^{-7}\,\text{J}$$

$$U_{after} = 3.63 \times 10^{-6}\,\text{J}$$

$$\frac{U_{after}}{U_{before}} = 6.7$$

Huh?? The energy stored increases by a factor of κ??

Sure. It took work to remove the dielectric. The stored energy increased by the amount of work done.

$$\Delta U = W_{external}$$

A “toy” to play with...

http://phet.colorado.edu/en/simulation/capacitor-lab

(You might even learn something.)