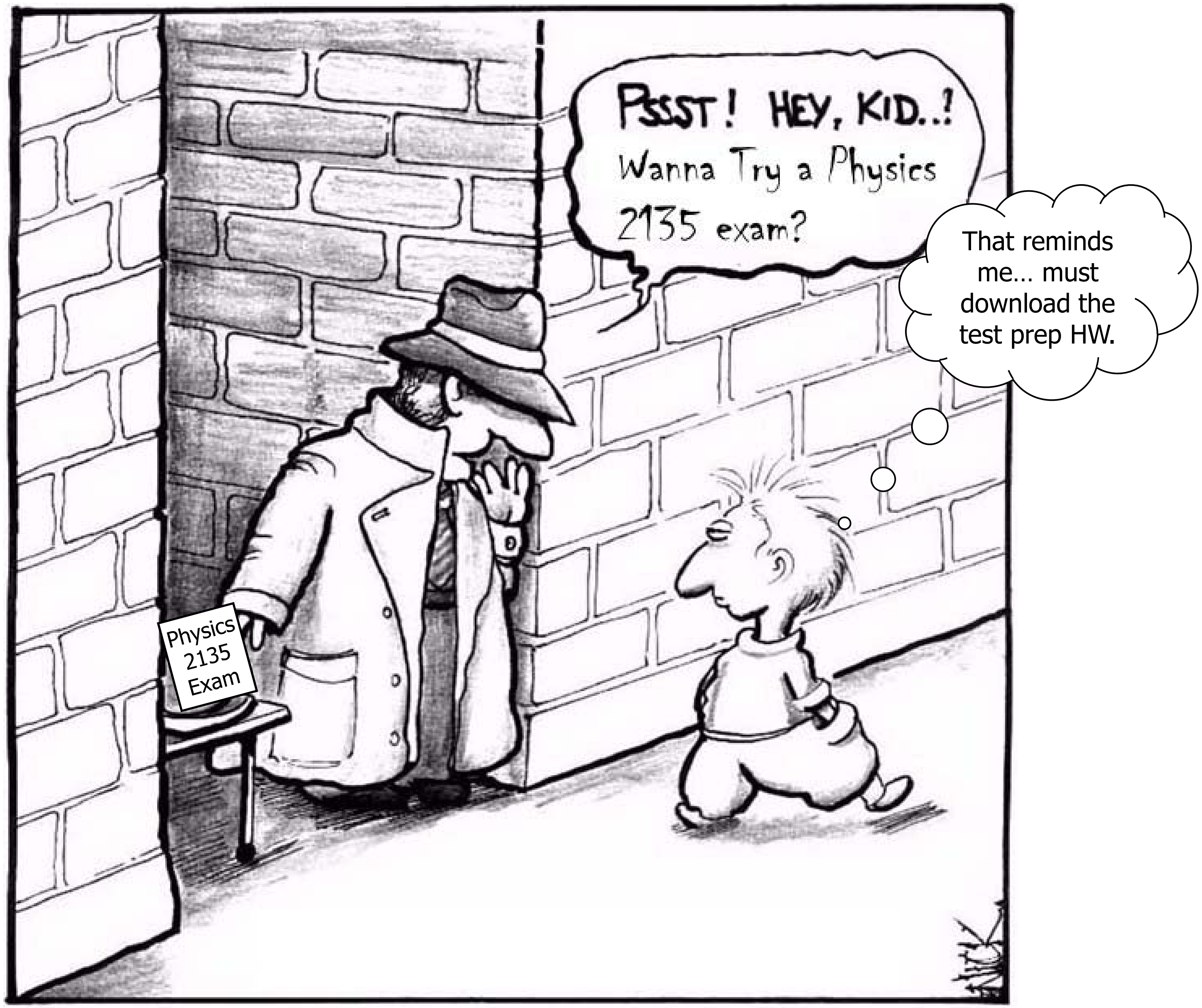


adapted from http://www.nearingzero.net (nz118.jpg)

Exam Reminders

- 5 multiple choice questions, 4 worked problems
- There will be one multiple choice question (#5) with no incorrect answer (free question)
- **Do not bring a calculator**
- no external communications, any use of a cell phone, tablet, smartwatch etc. will be considered **cheating**
- no headphones
- be on time, you will not be admitted after 5:15pm

Exam Reminders

- grade spreadsheets will be posted the **day after the exam**

- you will need your **PIN** to find your grade
 (PINs should have been distributed by recitation
instructors)

- test preparation homework 1 is posted on course website, will be discussed in recitation tomorrow

- problems on the test preparation home work are **NOT** guaranteed to cover all topics on the exam!!!

Exam 1 topics

Electric charge and electric force, Coulomb's Law

Electric field (calculating electric fields, motion of a charged particle in an electric field, dipoles)

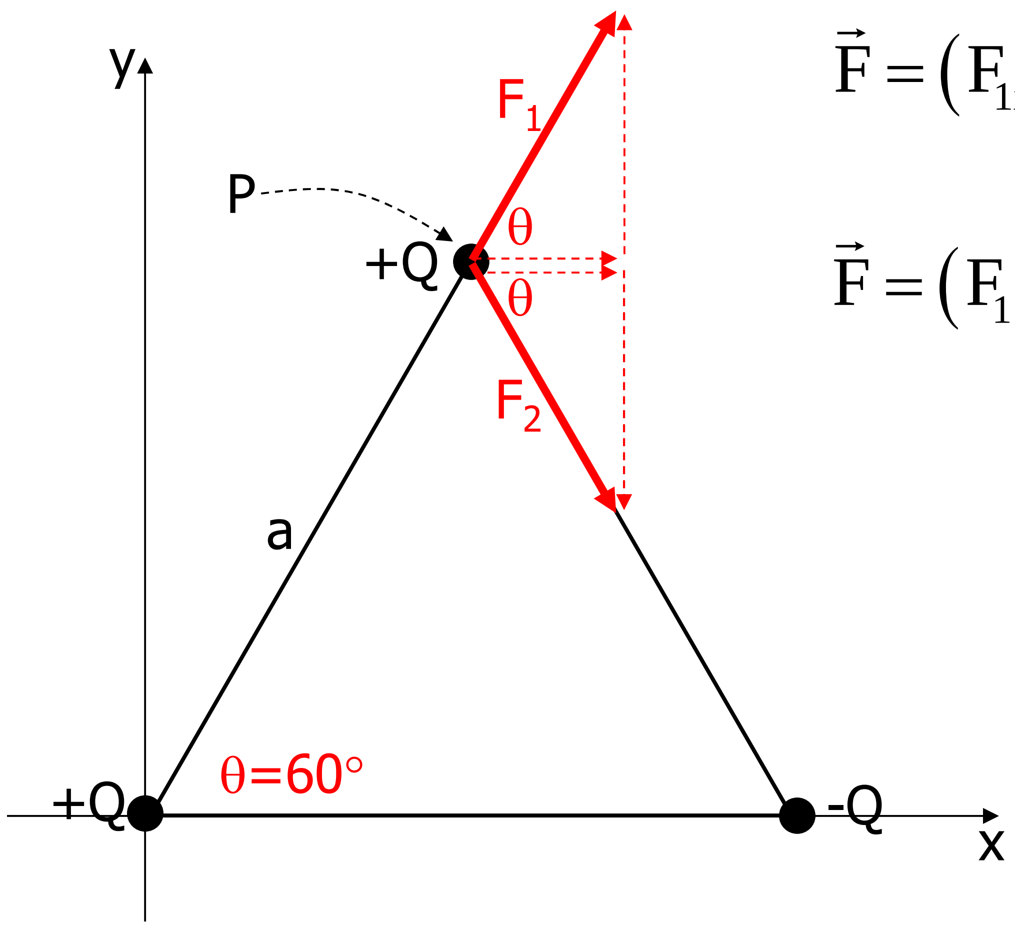
Gauss' Law (electric flux, calculating electric fields via Gaussian surfaces, fields and surface charges of conductors)

Electric potential and potential energy (calculating work, potential energy and potential, calculating fields from potentials, equipotentials, potentials of conductors)

Exam 1 topics

- don't forget the Physics 1135 concepts
- look at old tests (2014 to 2017 tests are on course website)
- exam problems **may come from topics not covered** in test preparation homework or test review lecture

Three charges +Q, +Q, and –Q, are located at the corners of an equilateral triangle with sides of length a. What is the force on the charge located at point P (see diagram)?



$$\vec{F} = \left(F_{1x} + F_{2x}\right)\hat{i} + \left(F_{1y} + F_{2y}\right)\hat{j}$$

$$\vec{F} = \left(F_1 \cos\theta + F_2 \cos\theta\right)\hat{i}$$

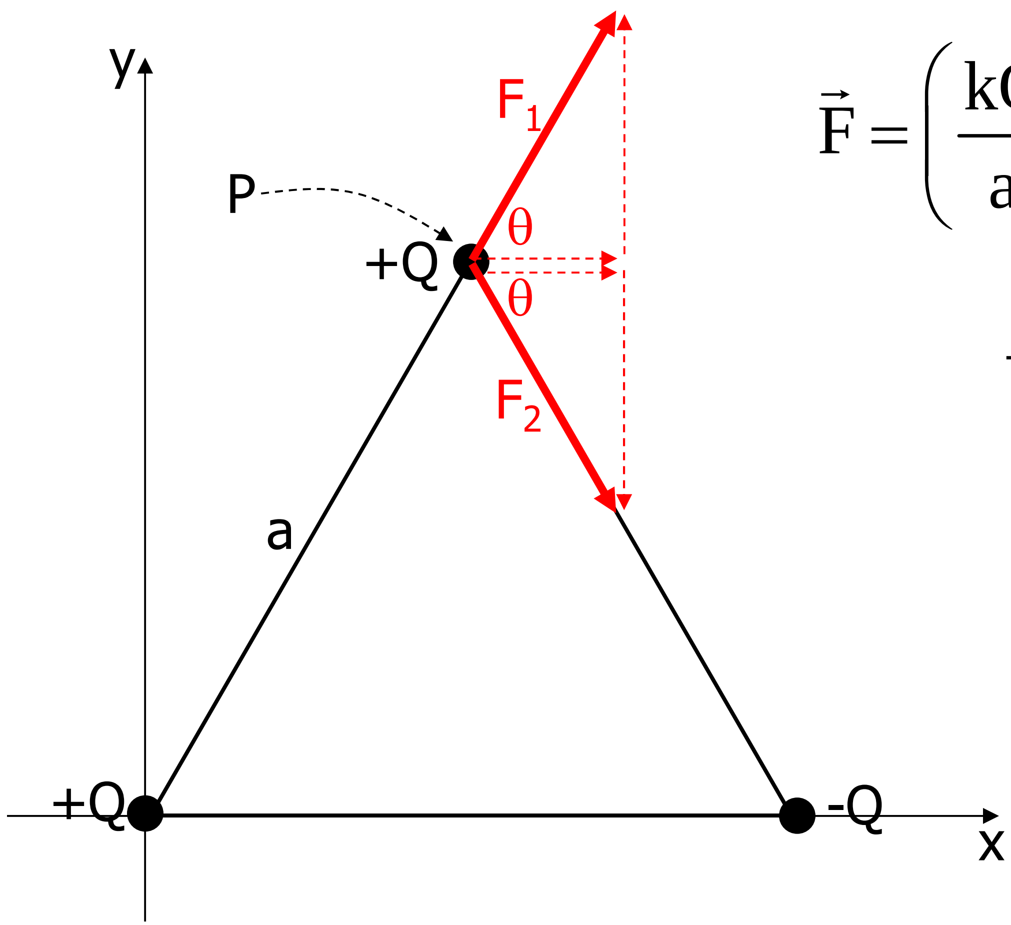
$$+ \left(F_1 \sin\theta - F_2 \sin\theta\right)\hat{j}$$

$$F_1 = k\frac{\left|(+Q)(+Q)\right|}{a^2} = k\frac{Q^2}{a^2}$$

$$F_2 = k\frac{\left|(-Q)(+Q)\right|}{a^2} = k\frac{Q^2}{a^2}$$

Note: if there is not a problem like this on Exam 1, there will be one on the Final!

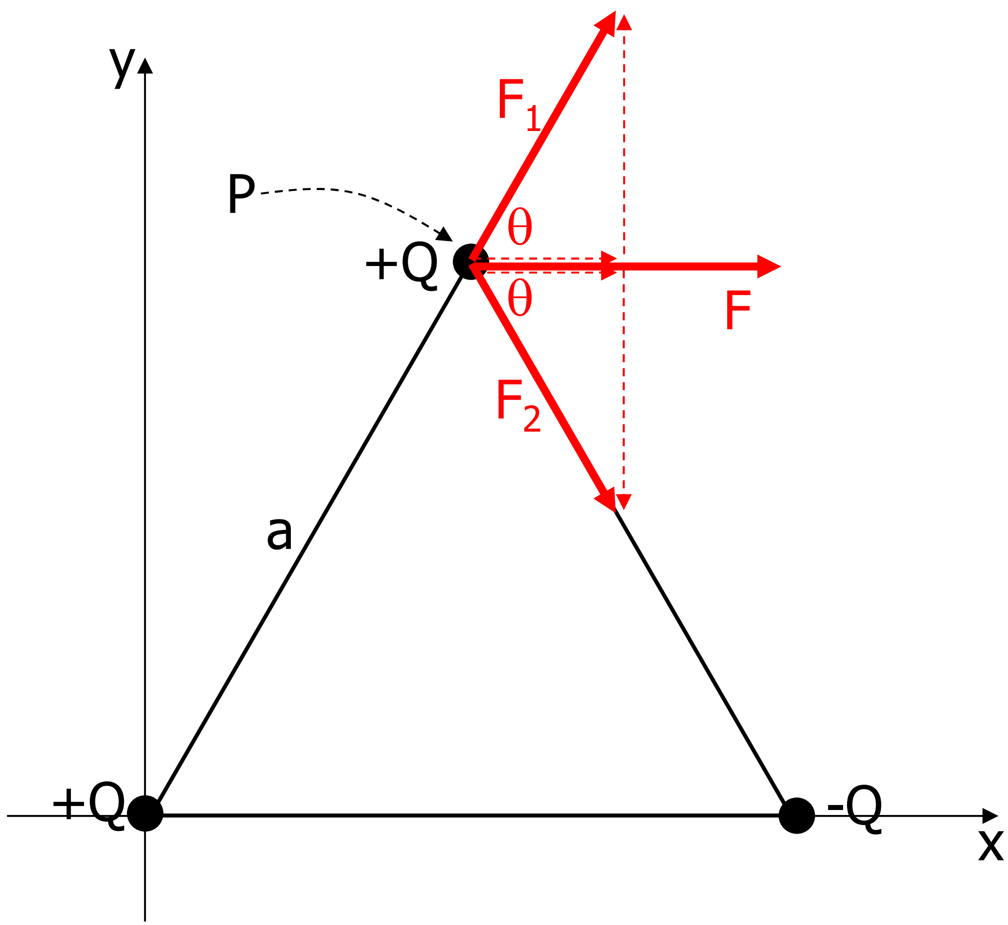
Three charges +Q, +Q, and –Q, are located at the corners of an equilateral triangle with sides of length a. What is the force on the charge located at point P (see diagram)?



$$\vec{F} = \left(\frac{kQ^2}{a^2}\cos 60^\circ + \frac{kQ^2}{a^2}\cos 60^\circ \right)\hat{i} + \left(\frac{kQ^2}{a^2}\sin 60^\circ - \frac{kQ^2}{a^2}\sin 60^\circ \right)\hat{j}$$

I could have stated that $F_y=0$ and $F_x=2F_{1x}$ by symmetry, but I decided to do the full calculation here.

Three charges +Q, +Q, and –Q, are located at the corners of an equilateral triangle with sides of length a. What is the force on the charge located at point P (see diagram)?



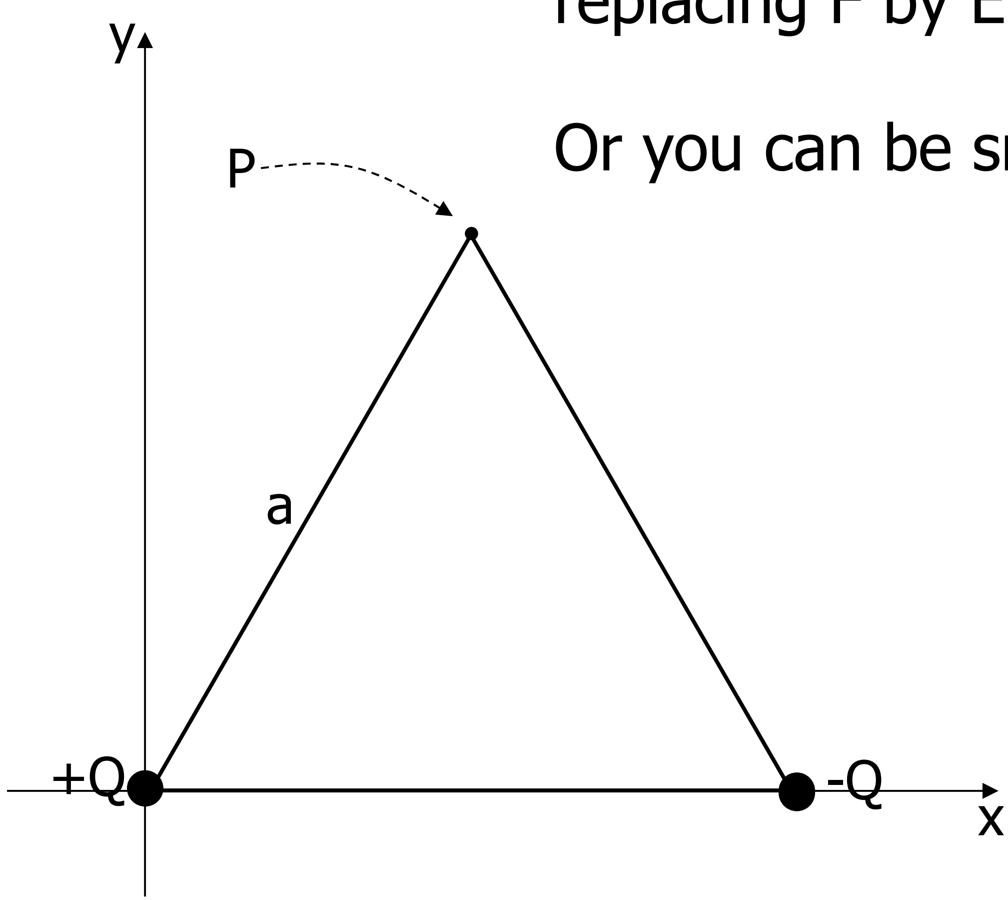
$$\vec{F} = 2\frac{kQ^2}{a^2}\cos 60^\circ\ \hat{i}$$

$$\boxed{\vec{F} = \frac{kQ^2}{a^2}\ \hat{i}}$$

What is the electric field at P due to the two charges at the base of the triangle?

You can "repeat" the above calculation, replacing F by E (and using Coulomb's Law).

Or you can be smart... $\vec{F} = q\vec{E}$

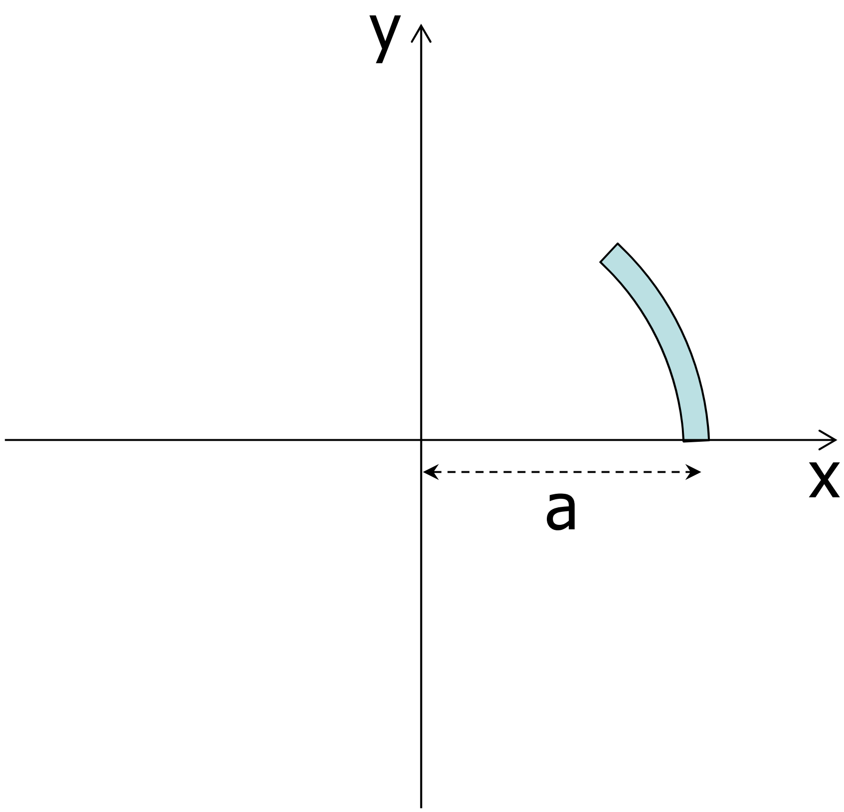


$$\vec{E} = \frac{\vec{F}}{q} = \frac{\frac{kQ^2}{a^2}\hat{i}}{Q} = \boxed{\frac{kQ}{a^2}\hat{i}}$$

This is the charge which had been at point P, "feeling" the force $\vec{F}$.

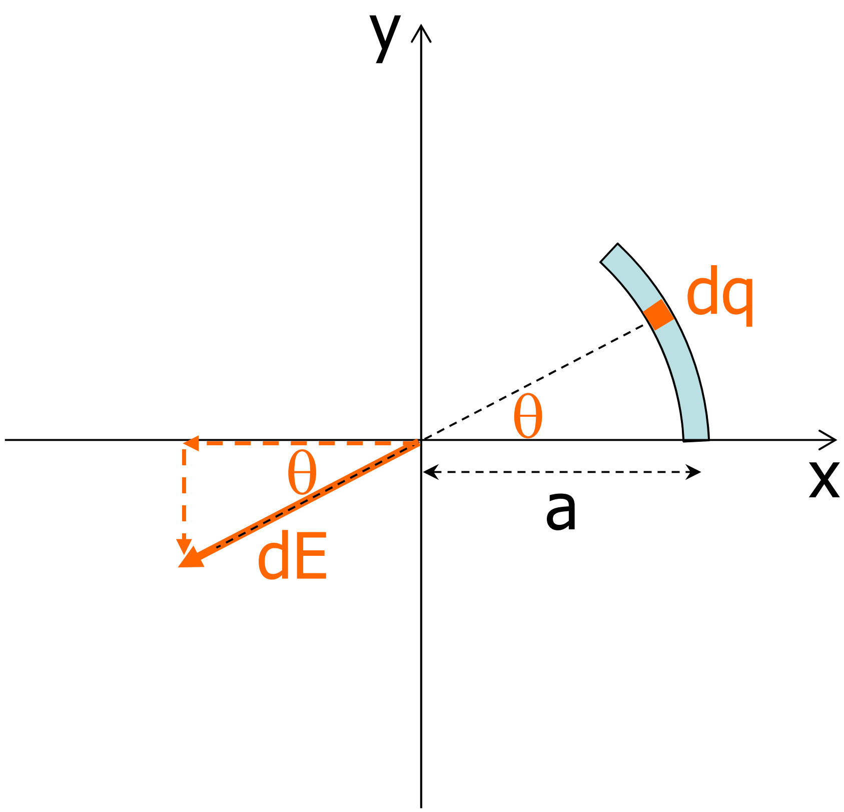
Caution: never write $q = \vec{E}/\vec{F}$. Why?

A rod is bent into an eighth of a circle of radius *a*, as shown. The rod carries a total positive charge +*Q* uniformly distributed over its length. What is the electric field at the origin?



$$dE = k\frac{|dq|}{r^2}$$

A rod is bent into an eighth of a circle of radius *a*, as shown. The rod carries a total positive charge $+Q$ uniformly distributed over its length. What is the electric field at the origin?



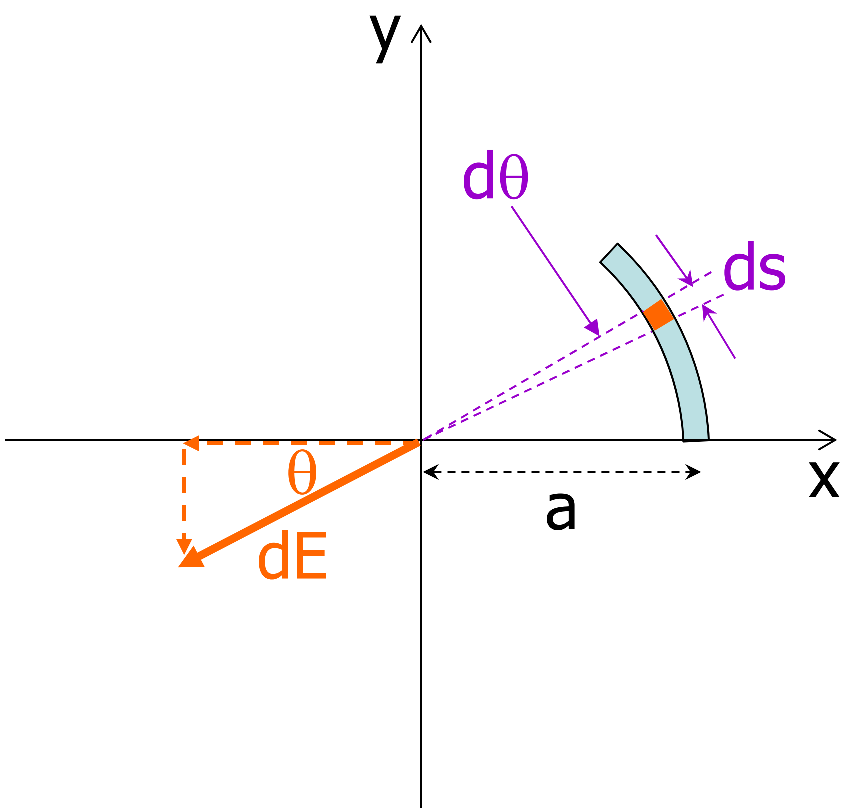
$$dE = k\frac{|dq|}{r^2} = k\frac{|dq|}{a^2}$$

$$|dq| = |\lambda|\ ds = \frac{|\text{charge}|}{\text{length}}\ ds$$

$$|dq| = \frac{|+Q|}{(\text{length of arc})}\ ds$$

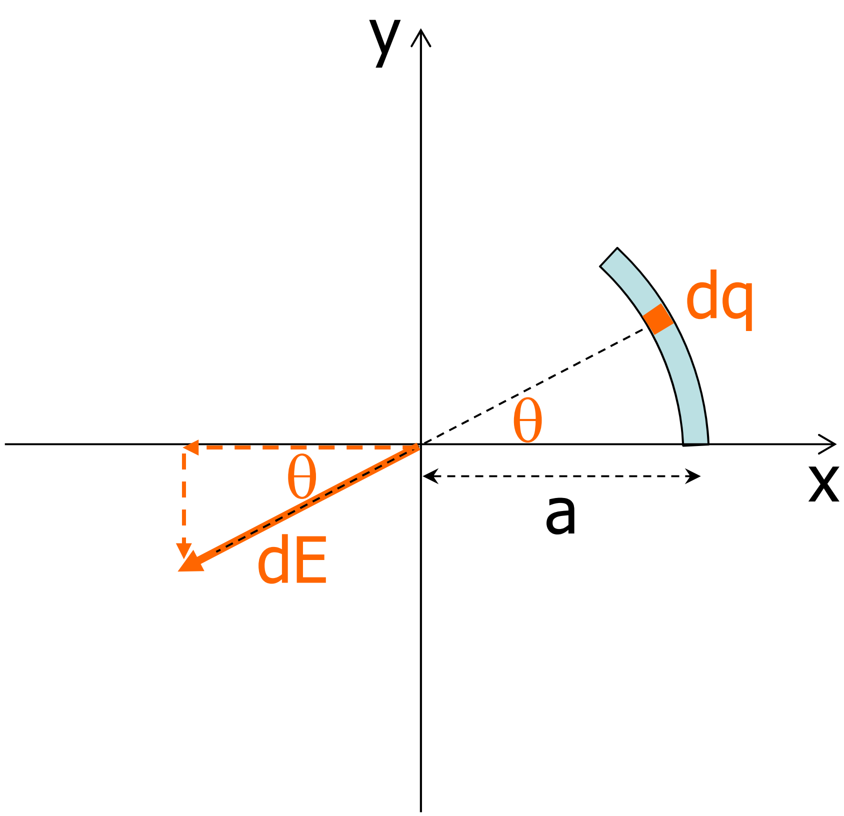
$$|dq| = \frac{|+Q|}{\left(2\pi a/8\right)}\ ds = \frac{4|+Q|}{\pi a}\ ds$$

A rod is bent into an eighth of a circle of radius *a*, as shown. The rod carries a total positive charge +*Q* uniformly distributed over its length. What is the electric field at the origin?



$$ds = a\,d\theta$$

A rod is bent into an eighth of a circle of radius *a*, as shown. The rod carries a total positive charge +*Q* uniformly distributed over its length. What is the electric field at the origin?



$$dE = \frac{k}{a^2}\left[\left(\frac{4|+Q|}{\pi a}\right)(a\, d\theta)\right]$$

$$dE_x = -\, dE \cos\theta$$

$$dE_y = -\, dE \sin\theta$$

$$E_x = -\int_0^{\pi/4} dE \cos\theta$$

$$\frac{1}{8}(2\pi) = \frac{\pi}{4}$$

$$E_y = -\int_0^{\pi/4} dE \sin\theta$$

A rod is bent into an eighth of a circle of radius *a*, as shown. The rod carries a total positive charge $+Q$ uniformly distributed over its length. What is the electric field at the origin?

$$dE = \frac{4k\left|+Q\right|}{\pi a^2}\ d\theta$$

$$E_x = -\int_0^{\pi/4} \frac{4k\left|+Q\right|}{\pi a^2}\cos\theta\ d\theta = -\frac{4k\left|+Q\right|}{\pi a^2}\int_0^{\pi/4}\cos\theta\ d\theta$$

$$E_x = -\frac{4k\left|+Q\right|}{\pi a^2}\left(\sin\theta\right)\Big|_0^{\pi/4} = -\frac{4k\left|+Q\right|}{\pi a^2}\left(\sin \pi/4 - \sin 0\right)$$

$$E_x = -\frac{4k\left|+Q\right|}{\pi a^2}\left(\frac{\sqrt{2}}{2} - 0\right) = -\frac{2\sqrt{2}k\left|+Q\right|}{\pi a^2}$$

A rod is bent into an eighth of a circle of radius *a*, as shown. The rod carries a total positive charge +*Q* uniformly distributed over its length. What is the electric field at the origin?

$$dE = \frac{4k\left|+Q\right|}{\pi a^2}\, d\theta$$

$$E_y = -\int_0^{\pi/4} \frac{4k\left|+Q\right|}{\pi a^2} \sin\theta \, d\theta = -\frac{4k\left|+Q\right|}{\pi a^2}\int_0^{\pi/4} \sin\theta \, d\theta$$

$$E_y = -\frac{4k\left|+Q\right|}{\pi a^2}\left(-\cos\theta\right)\Big|_0^{\pi/4} = -\frac{4k\left|+Q\right|}{\pi a^2}\left(-\cos\pi/4 + \cos 0\right)$$

$$E_y = -\frac{4k\left|+Q\right|}{\pi a^2}\left(-\frac{\sqrt{2}}{2} + 1\right) = -\frac{4k\left|+Q\right|}{\pi a^2}\left(1 - \frac{\sqrt{2}}{2}\right)$$

A rod is bent into an eighth of a circle of radius *a*, as shown. The rod carries a total positive charge $+Q$ uniformly distributed over its length. What is the electric field at the origin?

$$\vec{E} = -\left(\frac{2\sqrt{2}kQ}{\pi a^2}\right)\hat{i} - \left(\frac{4kQ}{\pi a^2}\left(1 - \frac{\sqrt{2}}{2}\right)\right)\hat{j}$$

$$\vec{E} = -\frac{2kQ}{\pi a^2}\left[\left(\sqrt{2}\right)\hat{i} + \left(2 - \sqrt{2}\right)\hat{j}\right]$$

You should provide reasonably simplified answers on exams, but remember, each algebra step is a chance to make a mistake.

What would be different if the charge were negative?

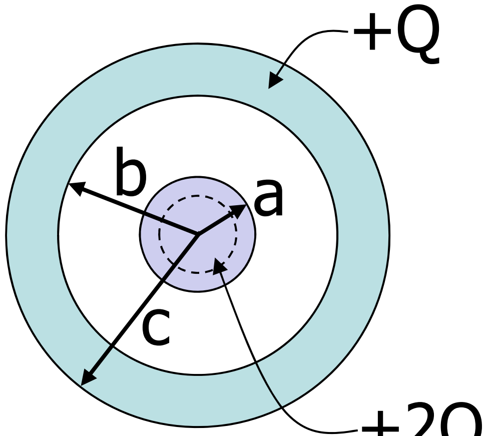
What would you do differently if you were asked to calculate the potential rather than the electric field?

How would you find the force on a test charge -*q* at the origin?

An insulating spherical shell has an inner radius b and outer radius c. The shell has a uniformly distributed total charge +Q. Concentric with the shell is a solid conducting sphere of total charge +2Q and radius $a<b$. Find the magnitude of the electric field for $r<a$.

An insulating spherical shell has an inner radius b and outer radius c. The shell has a uniformly distributed total charge +Q. Concentric with the shell is a solid conducting sphere of total charge +2Q and radius a<b. Find the magnitude of the electric field for r<a.

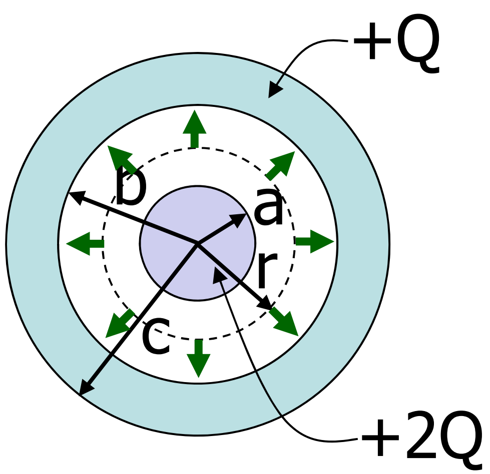
For 0<r<a, we are inside the conductor, so E=0.



If E=0 there is no need to specify a direction (and the problem doesn’t ask for one anyway).

An insulating spherical shell has an inner radius b and outer radius c. The shell has a uniformly distributed total charge +Q. Concentric with the shell is a solid conducting sphere of total charge +2Q and radius a<b. Use Gauss' Law to find the magnitude of the electric field for a<r<b.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$



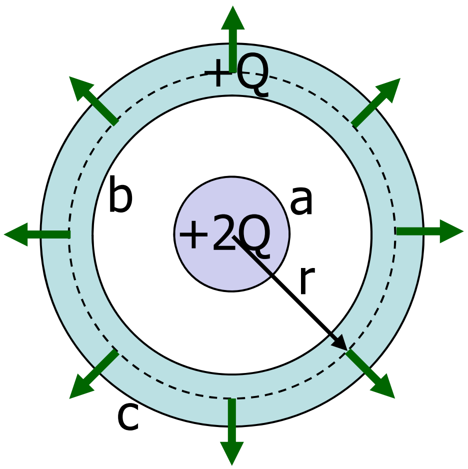
$$E\left(4\pi r^2\right) = \frac{2Q}{\varepsilon_o}$$

$$E = \frac{Q}{2\pi\varepsilon_o r^2}$$

Be able to do this: begin with a statement of Gauss's Law. Draw an appropriate Gaussian surface on the diagram and label its radius *r*. Justify the steps leading to your answer.

An insulating spherical shell has an inner radius b and outer radius c. The shell has a uniformly distributed total charge +Q. Concentric with the shell is a solid conducting sphere of total charge +2Q and radius a<b. Use Gauss' Law to find the magnitude of the electric field for b<r<c.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$

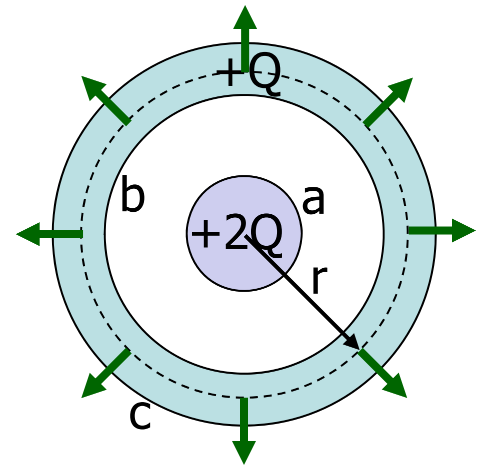


$$E\left(4\pi r^2\right) = \frac{q_{shell,enclosed} + q_{conductor,enclosed}}{\varepsilon_o}$$

$$q_{conductor,enclosed} = 2Q$$

$$q_{shell,enclosed} = \rho_{shell} V_{shell,enclosed} = \frac{Q_{shell}}{V_{shell}} V_{shell,enclosed}$$

$$q_{shell,enclosed} = \frac{Q_{shell}}{V_{shell}} V_{shell,enclosed}$$



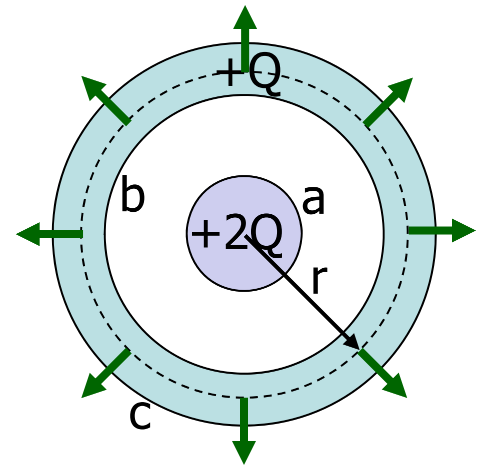
$$q_{shell,enclosed} = \frac{Q}{\left[\frac{4}{3}\pi c^3 - \frac{4}{3}\pi b^3\right]} \left[\frac{4}{3}\pi r^3 - \frac{4}{3}\pi b^3\right]$$

$$q_{shell,enclosed} = \frac{Q\left(r^3 - b^3\right)}{\left(c^3 - b^3\right)}$$

$$E\left(4\pi r^2\right) = \frac{\frac{Q\left(r^3 - b^3\right)}{\left(c^3 - b^3\right)} + 2Q}{\varepsilon_o}$$

The direction of $\vec{E}$ is shown in the diagram. Solving for the magnitude E (do it!) is "just" math.

$$E\left(4\pi r^2\right) = \frac{\dfrac{Q\left(r^3 - b^3\right)}{\left(c^3 - b^3\right)} + 2Q}{\varepsilon_o}$$



What would be different if we had concentric cylinders instead of concentric spheres?
What would be different if the outer shell were a conductor instead of an insulator?

An insulating spherical shell has an inner radius b and outer radius c. The shell has a uniformly distributed total charge +Q. Concentric with the shell is a solid conducting sphere of total charge +2Q and radius a<b. Find the magnitude of the electric field for b<r<c.

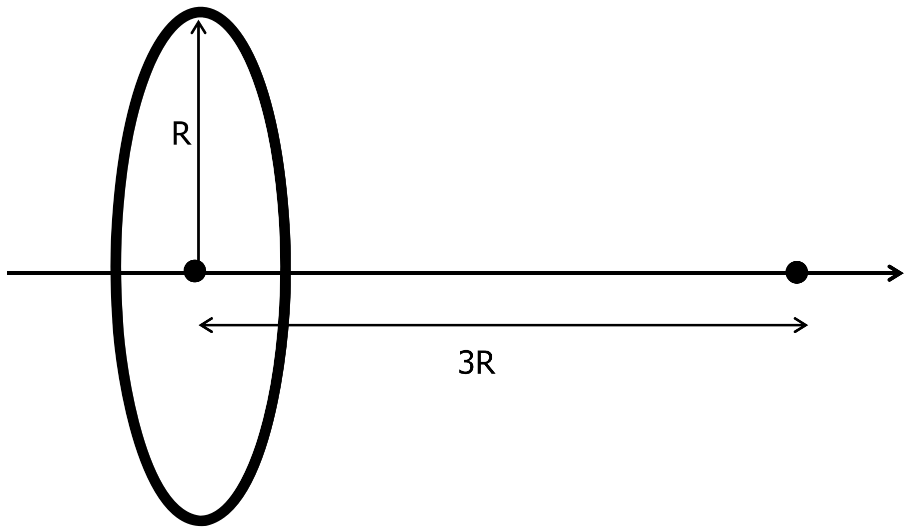
$$Q_{shell} = \rho\left[\frac{4}{3}\pi c^3 - \frac{4}{3}\pi b^3\right]$$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$

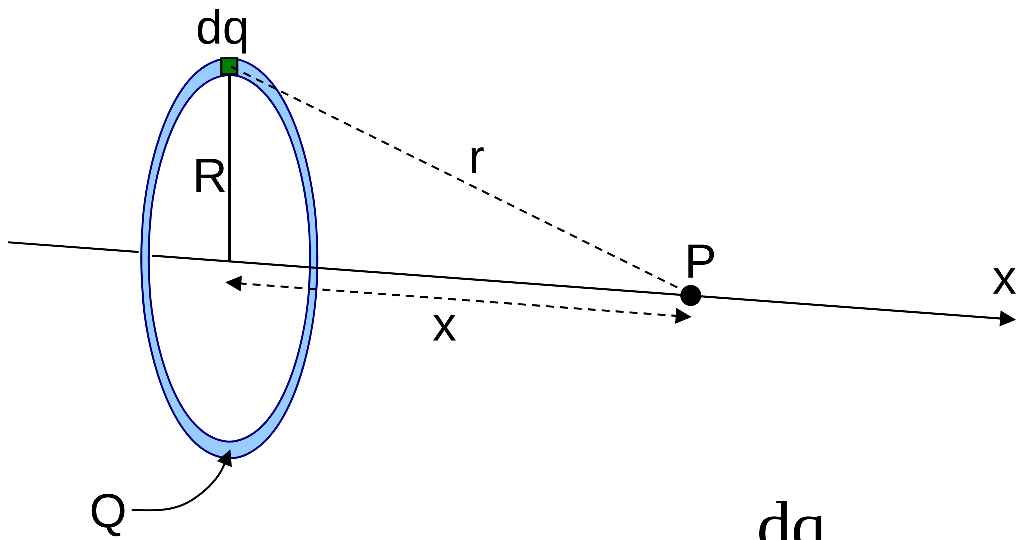
$$E\left(4\pi r^2\right) = \frac{\rho\left[\frac{4}{3}\pi r^3 - \frac{4}{3}\pi b^3\right] + 2Q}{\varepsilon_o}$$

What would be different if we had concentric cylinders instead of concentric spheres?
What would be different if the outer shell were a conductor instead of an insulator?

A ring with radius R has a uniform positive charge density λ. Calculate the potential difference between the point at the center of the ring and a point on the axis of the ring that is a distance of 3R from the center of the ring.



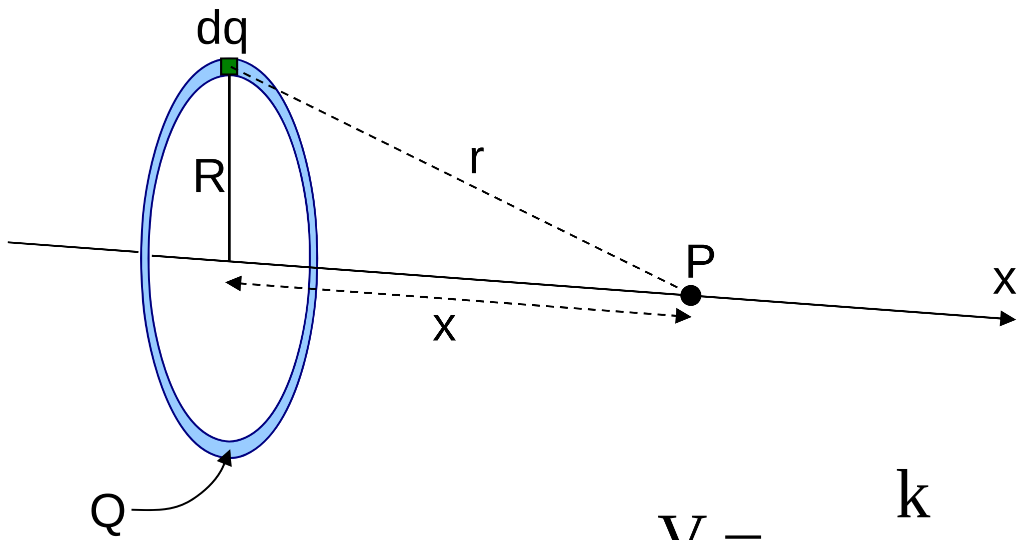
Begin by deriving the equation for the potential along the central axis of a ring of charge. We did this back in part 2 of lecture 6. I am going to be lazy… err, efficient… and just copy the appropriate slides.



Every dq of charge on the ring is the same distance from the point P.

$$dV = k\frac{dq}{r} = k\frac{dq}{\sqrt{x^2+R^2}}$$

$$V = \int_{ring} dV = \int_{ring} \frac{kdq}{\sqrt{x^2+R^2}} = \frac{k}{\sqrt{x^2+R^2}}\int_{ring} dq$$

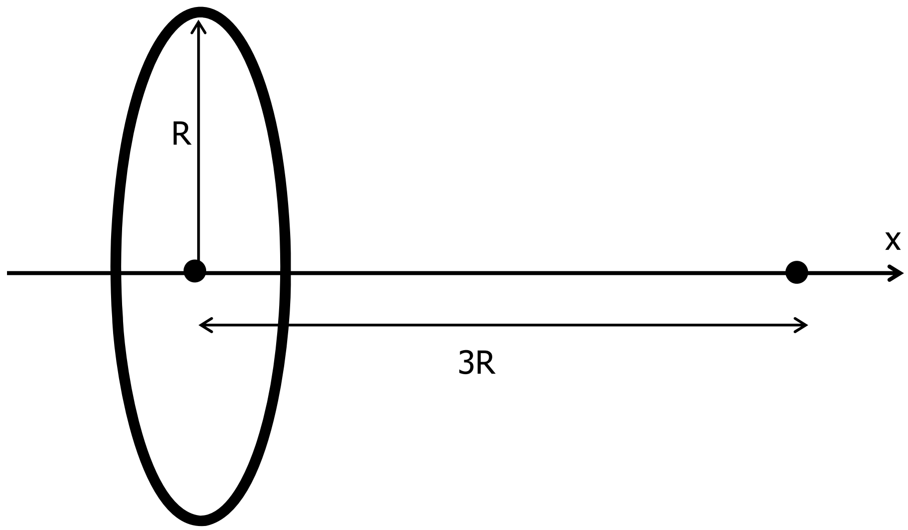


$$V = \frac{k}{\sqrt{x^2 + R^2}} \int_{ring} dq$$

$$V = \frac{kQ}{\sqrt{x^2 + R^2}} \qquad Q = \lambda\left(2\pi R\right)$$

$$V = \frac{2\pi\lambda k R}{\sqrt{x^2 + R^2}}$$

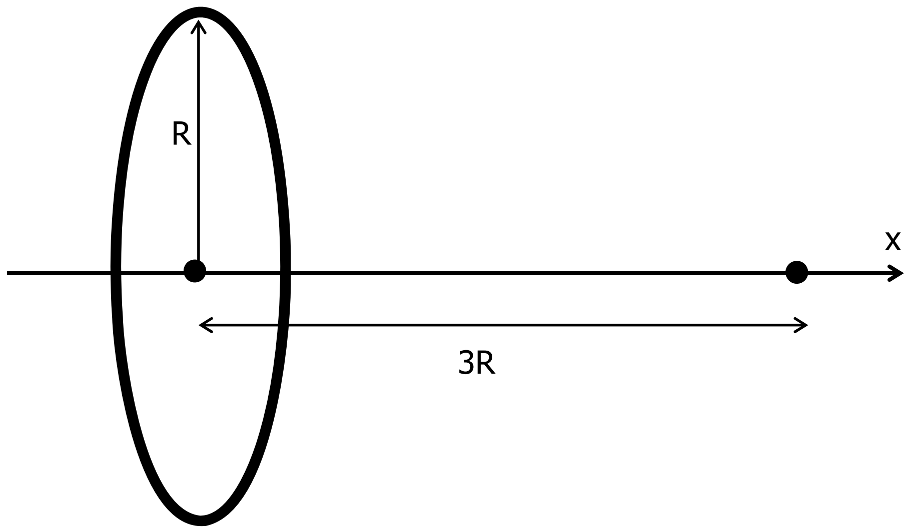
A ring with radius R has a uniform positive charge density λ. Calculate the potential difference between the point at the center of the ring and a point on the axis of the ring that is a distance of 3R from the center of the ring.



$$V(x) = \frac{2\pi\lambda kR}{\sqrt{x^2 + R^2}}$$

$$V(0) - V(3R) = \frac{2\pi\lambda kR}{\sqrt{0^2 + R^2}} - \frac{2\pi\lambda kR}{\sqrt{(3R)^2 + R^2}} = 2\pi\lambda kR\left(\frac{1}{R} - \frac{1}{R\sqrt{10}}\right)$$

A ring with radius R has a uniform positive charge density λ. Calculate the potential difference between the point at the center of the ring and a point on the axis of the ring that is a distance of 3R from the center of the ring.



$$V(0) - V(3R) = 2\pi\lambda k\left(\frac{\sqrt{10}-1}{\sqrt{10}}\right)$$

If a proton is released from rest at the center of the ring, how fast will it be at point P?