Mechanical Energy

Kinetic Energy, energy of motion, $K = \frac{1}{2}mv^2$

Potential Energy, energy stored due to a conservative force, $\Delta U = -W = -\int \vec{F} \cdot d\vec{s}$

Potential Energy

Consider the Coulomb Force, $\vec{F} = k\frac{q_1 q_2}{r_{12}^2}\hat{r}$

$$\Delta U = -\int_{r_0}^{r_f} k\frac{q_1 q_2}{r^2}\hat{r} \cdot d\vec{r}$$

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$$\Delta U = -kq_1 q_2 \int_{r_0}^{r_f} \frac{dr}{r^2}$$

$$\Delta U = \Delta\left(k\frac{q_1 q_2}{r}\right)$$

Potential Energy

Potential Energy due to Coulomb Force,

$$U = k\frac{q_1 q_2}{r_{12}}$$

What happened to the Δ?
What does it imply about $U = 0$?

Potential Energy

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Note: Potential Energy is a scalar!

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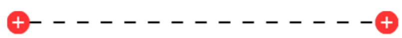
What does it mean that a set of charges has positive (negative) potential energy?

Potential Energy

$$\vec{F} = k\frac{q_1 q_2}{r_{12}^2}\hat{r} \qquad U = k\frac{q_1 q_2}{r_{12}}$$

$$F_x = -\frac{\partial U}{\partial x} \qquad \Delta U = -\int \vec{F}\cdot d\vec{s}$$

Example: Calculate the electric potential energy of two protons separated by a typical proton-proton intranuclear distance of 2×10^{-15}m.



Conservation of Energy

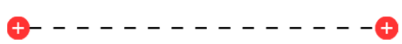
$$E_f - E_i = \left(W_{\text{other}}\right)_{i\to f}$$

(W_{other} refers to work done by non-conservative forces.)

If $W_{\text{other}} = 0$, then

$$U_0 + K_0 = U_f + K_f$$

Example: Consider releasing two protons with a typical intranuclear separation of 2×10^{-15}m. Assume they are only subject to the Coulomb Force. What maximum speed do the protons achieve? How far apart are the protons when they achieve maximum speed?



Point Charge in an Electric Field

$$\Delta U = -\int \vec{F}\cdot d\vec{s}$$

$$\Delta U = -q\int \vec{E}\cdot d\vec{s}$$

$$\frac{\Delta U}{q} = -\int \vec{E}\cdot d\vec{s}$$

Electric Potential

(Electric potential energy per charge)

$\Delta V = \frac{\Delta U}{q}$ $\quad$ $\Delta V = -\int \vec{E} \cdot d\vec{s}$

Electric Potential Energy (or Potential Energy)

$\vec{F} = k\frac{q_1 q_2}{r_{12}^2}\hat{r}$ $\quad$ $U = k\frac{q_1 q_2}{r_{12}}$

$F_x = -\frac{\partial U}{\partial x}$ $\quad$ $\Delta U = -\int \vec{F} \cdot d\vec{s}$

$\vec{F} = q\vec{E}$ $\quad$ $\Delta U = q\Delta V$

Electric Potential (or Potential)

$\vec{E} = k\frac{q}{r^2}\hat{r}$ $\quad$ $V = k\frac{q}{r}$

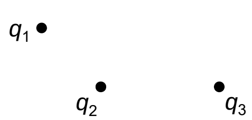
$E_x = -\frac{\partial V}{\partial x}$ $\quad$ $\Delta V = -\int \vec{E} \cdot d\vec{s}$

Example: A proton is released in a region of space where there is an electric potential. Describe the subsequent motion of the proton.

Example: A proton is released in a region of space where there is an electric potential. Describe the subsequent motion of the proton.

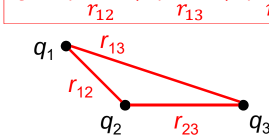
Example: An electron is released in a region of space where there is an electric potential. Describe the subsequent motion of the electron.

Example: Determine the potential energy of a set of three charges.



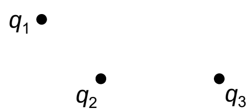
Example: Determine the potential energy of a set of three charges.

$$U = k\frac{q_1 q_2}{r_{12}} + k\frac{q_1 q_3}{r_{13}} + k\frac{q_2 q_3}{r_{23}}$$



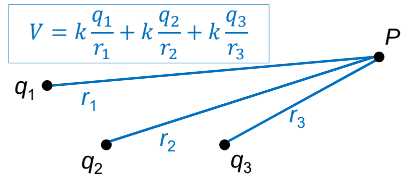
Electric potential energy, *U*, comes from the interaction of pairs of charges. Add potential energy of each pair of charges.

Example: Determine the potential of a set of three charges.



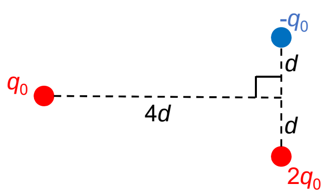
Example: Determine the potential of a set of three charges.

$$V = k\frac{q_1}{r_1} + k\frac{q_2}{r_2} + k\frac{q_3}{r_3}$$

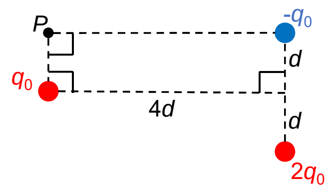


Electric potential, *V*, comes from individual charges. Add potential of each individual charge.

Example: Calculate the electric potential energy of the given set of three charges.



Example: Calculate the electric potential at point *P* due to the given set of three charges.



Units

- *U* is measured in joules (J).
- *V* is measured in volts (V) where $1\text{V} = \frac{1\text{J}}{1\text{C}}$
- The units of *E* are $\frac{\text{N}}{\text{C}}$ $\left(E = \frac{F}{q}\right)$ or $\frac{\text{V}}{\text{m}}$ $\left(E_x = -\frac{\partial V}{\partial x}\right)$.

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Units

- Electron volts (eV) are another unit of energy. An eV is the energy gained by a proton moving through a potential difference of 1 V.

$$\Delta U = q\Delta V$$
$$1\text{eV} = (1.6 \times 10^{-19}\text{C})(1\text{V})$$
$$1\text{eV} = 1.6 \times 10^{-19}\text{J}$$