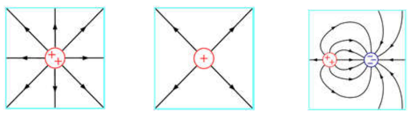
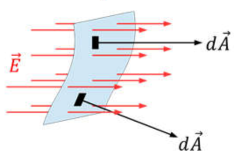
Electric Flux

- Field line density is proportional to electric field.
- Count field lines to determine strength.



Electric Flux
“Counting field lines” through a surface

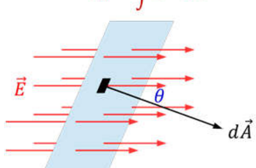
$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$



Area vector
- Magnitude of area
- Direction normal (perpendicular) to surface

Example: Flux through a rectangular surface due to a uniform field.

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$



$$\Phi_E = \int \vec{E} \cdot d\vec{A} = \int E(dA)\cos\theta = EA\cos\theta$$

Gauss's Law

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

Closed surface
Going out is defined as positive

Gauss's Law

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

Use in reverse to determine electric field IF it is possible to factor E out of the integral.

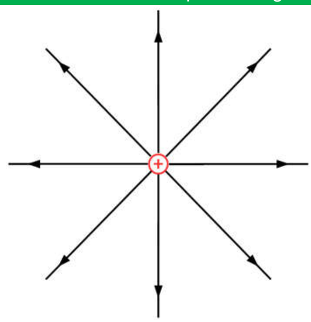
Usually this means that $\vec{E} \parallel d\vec{A}$ and constant in magnitude everywhere along the surface.

Gauss's Law

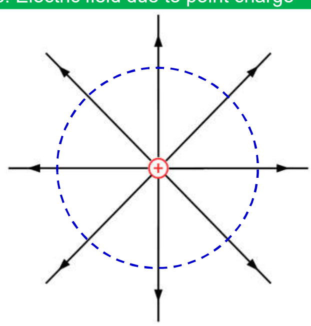
Three solvable symmetries

- Spherical - today's lecture
- Cylindrical } next lecture
- Planar

Example: Electric field due to point charge

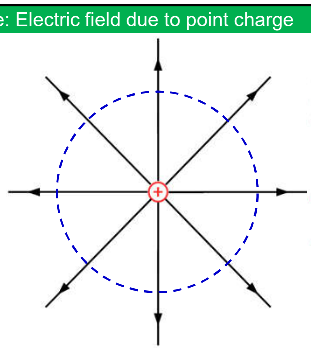


Example: Electric field due to point charge



Create a spherical Gaussian surface centered on the point charge.

Example: Electric field due to point charge

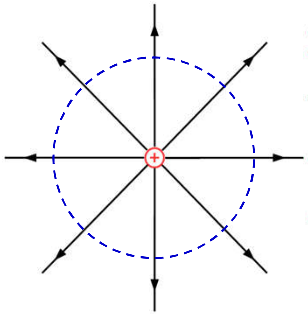


$\vec{E} \cdot d\vec{A}$ is constant everywhere along the sphere.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$E \oint dA = \frac{Q}{\varepsilon_0}$$

Example: Electric field due to point charge



$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

$$E \oint dA = \frac{Q}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\,\pi \epsilon_0 r^2}$$

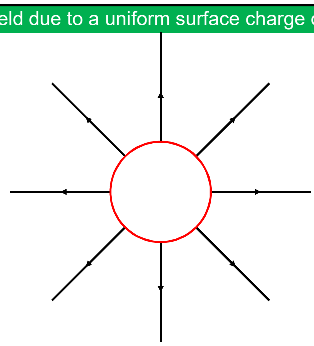
Example: Electric field due to point charge

Compare to result from Coulomb's Law

$$\vec{E} = \frac{Q}{4\,\pi \epsilon_0 r^2}\hat{r} \qquad \vec{E} = k\frac{Q}{r^2}\hat{r}$$

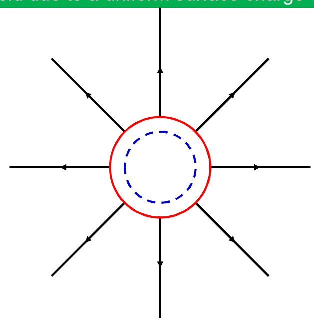
$$\frac{1}{4\pi\epsilon_0} = k$$

Example: Electric field due to a uniform surface charge on a sphere.



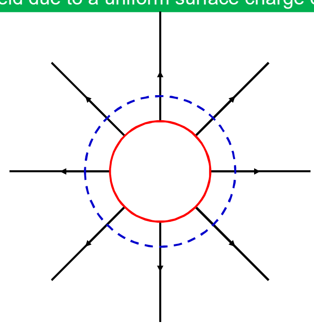
Example: Electric field due to a uniform surface charge on a sphere.

Inside, $r < R$



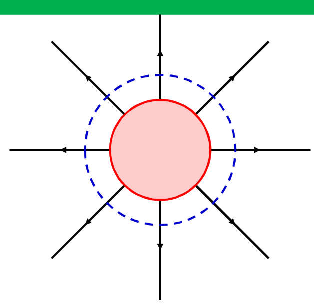
Example: Electric field due to a uniform surface charge on a sphere.

Outside, $r > R$



Example: Electric field due to a uniform volume charge , ρ, in a spherical solid.

Outside, $r > R$



Example: Electric field due to a uniform volume charge , ρ, in a spherical solid.

Inside, $r < R$