Continuous Charge Distributions

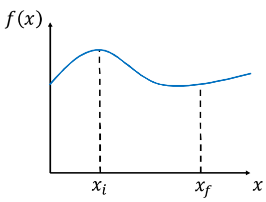
Coulomb's Law, as we have expressed it, only applies to discrete charges.

$$\vec{F} = k\frac{q_1 q_2}{r_{12}^2}\hat{r}_{12} \qquad \vec{E} = k\frac{q}{r^2}\hat{r}$$

To deal with continuous charge distributions, one must integrate.

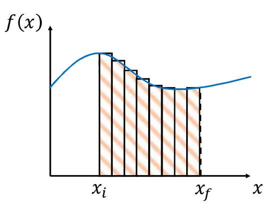
Integration

Recall finding the area under a curve between x_i and x_f using integration.



Integration

Recall finding the area under a curve between x_i and x_f using integration.



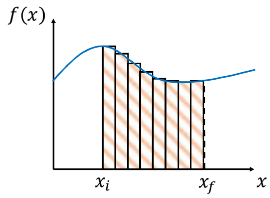
$$A_{Total} \approx \sum_{i}^{N} A_i = \sum_{i}^{N} f(x)\Delta x$$

$$A_{Total} = \int_{x_i}^{x_f} dA = \int_{x_i}^{x_f} f(x)dx$$

Integration

Key was being able to express dA as $(\text{some function}) \times dx$.

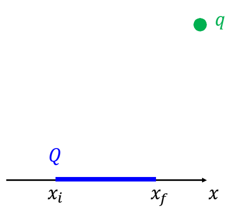
$$A_{Total} = \int_{x_i}^{x_f} dA = \int_{x_i}^{x_f} f(x)dx$$



Integration to find $\vec{F}_{Total}$

Key to finding force of Q on q is being able to express $d\vec{F}$ as $(\text{some function}) \times dx$.

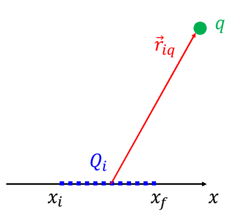
$$\vec{F}_{Total} = \int_{x_i}^{x_f} d\vec{F}$$



Integration to find $\vec{F}_{Total}$

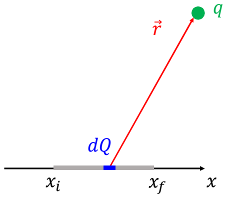
First consider a set of discrete charges Q_i along the x-axis.

$$\vec{F}_{Total} = \sum_{i}^{N} k \frac{Q_i q}{r_{iq}^2} \hat{r}_{iq}$$



Integration to find $\vec{F}_{Total}$

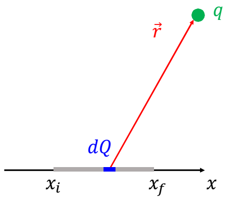
Replace the sum by an integral.



$$\vec{F}_{Total} = \sum_{i}^{N} k \frac{Q_i q}{r_{iq}^2} \hat{r}_{iq}$$

$$\vec{F}_{Total} = \int_{x_i}^{x_f} k \frac{dQq}{r^2} \hat{r}$$

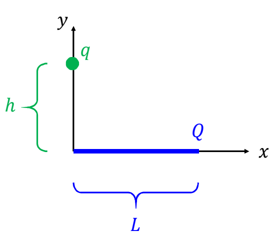
Integration to find $\vec{F}_{Total}$



$$\vec{F}_{Total} = \int_{x_i}^{x_f} k \frac{dQq}{r^2} \hat{r}$$

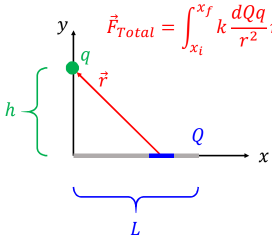
Must be able to express r^2 and $\hat{r}$ in terms of x and dQ in terms of dx.

Example: Determine the force on a charge q at $(0, h)$ due to a charge Q uniformly distributed along a line segment from the origin to $(L, 0)$.



Example: Determine the force on a charge q at $(0, h)$ due to a charge Q uniformly distributed along a line segment from the origin to $(L, 0)$.

$$\vec{F}_{Total} = \int_{x_i}^{x_f} k\frac{dQq}{r^2}\hat{r}$$



Need
r

$\hat{r}$

dQ

x_i and x_f

Example: Determine the force on a charge q at $(0, h)$ due to a charge Q uniformly distributed along a line segment from the origin to $(L, 0)$.

$$\vec{F}_{Total} = \int_{x_i}^{x_f} k\frac{dQq}{r^2}\hat{r}$$

$$r = \sqrt{x^2 + h^2}$$

$$\hat{r} = -\frac{x}{\sqrt{x^2+h^2}}\hat{\imath} + \frac{h}{\sqrt{x^2+h^2}}\hat{\jmath}$$

$$dQ = \frac{Q}{L}dx$$

$$x_i = 0 \text{ and } x_f = L$$

Example: Determine the force on a charge q at $(0, h)$ due to a charge Q uniformly distributed along a line segment from the origin to $(L, 0)$.

$$\vec{F}_{Total} = \frac{kQq}{L}\left[-\int_0^L \frac{xdx}{(x^2+h^2)^{3/2}}\hat{\imath} + h\int_0^L \frac{dx}{(x^2+h^2)^{3/2}}\hat{\jmath}\right]$$

$$\vec{F}_{Total} = \frac{kQq}{L}\left[\left(\frac{1}{\sqrt{L^2+h^2}} - \frac{1}{h}\right)\hat{\imath} + \frac{L}{h\sqrt{L^2+h^2}}\hat{\jmath}\right]$$

Example: Determine the electric field at $(x, 0)$ due to a line segment with a uniform charge density λ from $(0, -a)$ to $(0, a)$.

Example: Determine the electric field at $(x, 0)$ due to a line segment with a uniform charge density λ from $(0, -a)$ to $(0, a)$.

$$\vec{E} = k\lambda x \int_{-a}^{a} \frac{dy}{(x^2 + y^2)^{3/2}} \hat{\imath}$$

$$\vec{E} = \frac{2k\lambda a}{x\sqrt{x^2 + a^2}} \hat{\imath}$$

Example: Determine the electric field at $(x, 0)$ due to a line segment with a uniform charge density λ from $(0, -a)$ to $(0, a)$.

$$\vec{E} = k\lambda x \int_{-a}^{a} \frac{dy}{(x^2 + y^2)^{3/2}} \hat{\imath}$$

$$\vec{E} = \frac{2k\lambda a}{x\sqrt{x^2 + a^2}} \hat{\imath}$$

What happens if $a \to \infty$?