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Remarks on: "Oscillation in Nonlinear Neutral Difference Equations with Positive and Negative Coefficients" [Int. J. Difference Equ. 5 (2010)]

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Abstract

This paper includes some examples that reveal certain inconsistencies in the main results of the paper mentioned in the title.

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1 Examples

In [1], the authors study the asymptotic behaviour of solutions to

$$\Delta^2\big[y(n)+p(n)y(n-m)\big]+q(n)G(y(n-\sigma))-r(n)H(y(n-\tau))=0,\quad n\geq 0,\quad (1.1)$$

where p, q, r are sequences of reals such that q is positive, r is nonnegative, G, H are continuous functions on reals satisfying the usual sign condition, and m, σ, τ are positive integers. In the study of (1.1), the following is the primary assumption:

(H$_0$) $\displaystyle\sum_{j=0}^{\infty} jr(j) < \infty.$

The study of oscillation of (1.1) requires different techniques from the study of

$$\Delta^2\big[y(n)+p(n)y(n-m)\big]+q(n)G(y(n-\sigma))=0,\quad n\geq 0$$

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with only nonnegative coefficient q since any nonoscillatory solution y of (1.1) does not imply that the neutral term $y(n) + p(n)y(n-m)$ is eventually monotonic.

Let us proceed with some examples showing that [1] includes incorrect results.

Example 1.1. Consider

$$\Delta^2 y(n) + \sqrt[3]{2^{2(n-2)}}\sqrt[3]{y(n-2)} - \frac{10}{2^{2n}}\big(y(n-1)\big)^3 = 0, \quad n \geq 0, \tag{1.2}$$

where $p(n) \equiv 0$, $q(n) = \sqrt[3]{2^{2(n-2)}}$ ($\geq 1/2^2$), $\sigma = 2$, $G(u) = \sqrt[3]{u}$, $r(n) = 10/2^{2n}$, $\tau = 1$ and $H(u) = u^3$. One can check that all the assumptions of [1, Theorem 2.1], [1, Theorem 2.3], [1, Corollary 2.5], [1, Theorem 2.6], [1, Theorem 2.7], [1, Theorem 2.8], [1, Theorem 2.12] and [1, Theorem 2.13] are satisfied. Due to authors' claim, every solution of (1.2) oscillates or tends to 0 asymptotically. One can check that $y(n) = 2^n$ for $n \geq 0$ is a solution of (1.2), which neither oscillates nor tends to 0 asymptotically, contradicting their claim.

Example 1.2. Consider

$$\Delta^2\big[y(n) - 2y(n-1)\big] + 2^7\big(y(n-4)\big)^3 - \frac{1}{2^{2n}}\big(y(n-1)\big)^5 = 0, \quad n \geq 0, \tag{1.3}$$

where $p(n) \equiv -2$, $m = 1$, $q(n) \equiv 2^7$, $\sigma = 4$, $G(u) = u^3$, $r(n) = 1/2^{2n}$, $\tau = 1$ and $H(u) = u^5$. Equation (1.3) satisfies all the assumptions of [1, Theorem 2.10], hence it should either oscillate or tend to 0 asymptotically due to authors' claim. But $y(n) = 2^n$ for $n \geq 0$ is a positive solution of (1.3), which diverges to ∞.

Example 1.3. Consider

$$\Delta^2\big[y(n) - y(n-1)\big] + 2^7\big(y(n-4)\big)^3 - \left(\frac{1}{2^{2n}} + \frac{1}{2^{4(n-1)}}\right)\big(y(n-1)\big)^5 = 0, \quad n \geq 0. \tag{1.4}$$

This example satisfies all assumptions of [1, Theorem 2.11], but it does not admit solutions which either oscillate or tend to zero asymptotically as a solution of (1.4) is $y(n) = 2^n$ for $n \geq 0$.

2 Final Comments

In the proof of [1, Theorem 2.1], the authors make the substitution

$$k(n) := \sum_{j=n}^{\infty}(j - n + 1)r(j)H\big(y(j-\tau)\big), \quad n \geq 0,$$

where y is an eventually positive solution of (1.1), and claim that $\Delta k(n) < 0$ and $\Delta^2 k(n) > 0$ for all sufficiently large n. However, the condition (H_0) does not imply that the sequence k is well defined (or convergent) since the sum is taken over infinite number of elements, and y is an unknown function. Indeed, we can see that the sequence k satisfies $k(n) = \infty$ for all $n \geq 0$ in Examples 1.1–1.3. The same erroneous method is also applied in [2].

It would be of significant importance if some answer can be delivered for the solutions of the nonlinear equation (1.1) and its forced form.

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