

Errata to “Comparison of Smallest Eigenvalues for Fractional-Order Nonlocal Boundary Value Problems”

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Abstract

In the article “Comparison of smallest eigenvalues for fractional-order nonlocal boundary value problems”, published in *Advances in Dynamical Systems and Applications*, Volume 14, Number 2, pp. 189–199 (2019), an error was made concerning the left boundary condition. This errata addresses that error.

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Concerning the results of the paper [3], an error was brought to the attention of the authors by Professor Jeffrey Webb. In particular, for a Riemann–Liouville fractional derivative problem studied via an integral equation in the space $C[0, 1]$, only the initial value $y(0) = 0$ gives a well-posed problem. As a consequence, the phrase on [3, page 192] preceding the expression for $G(t, s)$ that states, “Extending arguments of Henderson and Luca [16, 19], we obtain by direct computation that the Green’s function for (3.1)–(1.3) is given by ...,” is in error. Namely, in each of [16] and [19], the boundary condition at $t = 0$ involves $y(0) = 0$, and the computation of the Green’s function in each of those papers does not lead to the expression for $G(t, s)$ given on [3, page

192]. By replacing the boundary condition in (1.3) at $t = 0$ by $y(0) = 0$ and then deleting the corresponding terms from the expression for $G(t, s)$ on [3, page 192] and employing the weighted Banach space $B = \{y : y = t^{\alpha-1}v, v \in C[0, 1]\}$ with norm $\|y\| = \sup_{t \in [0, 1]} |v(t)|$, standard arguments show $M, N : P \setminus \{0\} \rightarrow Q$, where

$$Q := \{y = t^{\alpha-1}v \in B : y(t) > 0, t \in (0, 1], v(0) > 0\} \subset P^\circ,$$

which provides a correction for the results of [3]. Useful references supporting this correction are [1, 2, 4–7].

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