The gravitational potential and the "uniform" gravitational field near the earth's surface

The gravitational potential energy for two particles of mass $\mu$ and $\nu$ is

$$V = -GM\mu/\rho,$$  \hspace{1cm} (1)

where $G$ is the Newtonian constant of gravitation and $r$ is the distance between the two particles. Note that the minus sign implies that this is an attractive potential, so that the potential energy increases as the particle separation increases. By the usual convention, the potential is zero when the particles are infinitely far apart.

Now let's apply this formula to find the potential energy of a particle of mass $\mu$ near the surface of the earth. Let $M_e$ be the mass of the earth, and treat the earth as a sphere with a radius equal to the earth's mean radius $R_e$. Let $h$ equal the height of the particle above the earth's surface. Then we can write

$$r = R_e + h.$$  \hspace{1cm} (2)

Next, put Eq.(2) into Eq.(1) and rearrange slightly to obtain

$$V = -(GMm/R_e) \left( \frac{1}{1 + h/R_e} \right).$$  \hspace{1cm} (3)

Since $R_e \approx 6400$ km, for elevations less than 100 km or so, we will have $h/R_e << 1$. Hence, we can make a Taylor series expansion of Eq.(3) to find

$$V = -(GMm/R_e) \left( 1 - h/R_e + \cdots \right),$$  \hspace{1cm} (4)

where we have neglected the very small terms of second and higher order. Now, we can rewrite this result in the following instructive way

$$V = V_0 + mgh,$$  \hspace{1cm} (5)

where $g$ is the familiar gravitational acceleration (9.8 m/s$^2$), defined as

$$GMm/R_e^2 = mg,$$  \hspace{1cm} (6)

and $V_0$ is the particle's potential energy at $h = 0$, i.e., at the earth's surface,

$$V_0 = -(GMm/R_e).$$  \hspace{1cm} (7)

Two points are worth noting: (1) When we treat the motion of particles near the earth's surface, we usually regard the potential energy as zero at $h = 0$. This is equivalent to redefining the gravitational potential as $V'$,

$$V' = V - V_0 = mgh.$$  \hspace{1cm} (8)

(2) Equations (5) and (8) are only valid when $h/R_e << 1$. If your problem involves distances that are comparable to or greater than $R_e$, you must abandon Eq.(8) and revert to Eq.(1).