Phys. 321 - E&M - Test 3 - Dec. 3, 1999

1. (25 pts) A monochromatic electromagnetic wave traveling in the z-direction is incident on a thick slab of conducting material. The (time averaged) energy density in the conductor is given by:

$$\langle U \rangle = \frac{k^2}{2\mu\omega^2} E_0^2 e^{-2\kappa z}$$

a) Determine the total energy per unit area delivered to the slab.
   Hint: assume the slab has a cross sectional area A and it is infinitely thick in order to be able to integrate from $z = 0$ to $z = \infty$.

b) What fraction of the total energy delivered to the slab is deposited within the skin depth?

2. (25 pts) A particle of charge q moves in a circle of radius R at constant angular velocity ω. Assume that the circle lies in the xy plane, centered at the origin, and at time $t = 0$ the charge is at (R, 0), on the positive x axis. Find the Liénard-Wiechert scalar and vector potentials for points on the z axis.

3. (25 pts) An insulating circular ring (radius R) lies in the xy plane, centered at the origin. It carries a linear charge density $\lambda = \lambda_0 \cos\varphi$, where λ_0 is constant and φ is the usual azimuthal angle. The ring is now set spinning at constant angular velocity ω about the z axis.

a) Determine the initial dipole moment of the ring and write down the dipole moment as a function of time.

b) Determine the power radiated by the spinning ring.

4. (25 pts) A positron of charge q and mass m is accelerated from rest through a distance D in a uniform electric field E. What fraction of the energy gained is lost by radiation?

($q = 1.6 \times 10^{-19}$ C, $m = 9.1 \times 10^{-31}$ kg)
$E = 3 \times 10^6$ V/m and $D = 1$ m.

5. (10 pts) Bonus Problem: The dipole radiation power distribution is given by:

$$\frac{d\langle P \rangle}{d\Omega} = \left(\frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c} \right) \sin^2\theta$$

Use this to explain

a) the blueness of the sky, and

b) the redness of the sunset.