Phys. 321 - E&M II - Test 3 - Nov.12, 2001

1. (25 pts) Suppose $V = 0$ and $\vec{A} = A_0 \sin(ky - \omega t)\hat{z}$, where A_0, ω, and k are constants.

a) Find $\vec{E}$ and $\vec{B}$.

b) Use the Maxwell equation for the Curl of $\vec{B}$ to determine the relation between ω and k.

2. (25 pts) A particle of charge q moves in a circle of radius R at constant angular velocity ω. Assume that the circle lies in the xy plane, centered at the origin, and at time $t = 0$ the charge is at $(R, 0)$, on the positive x axis. Find the Liénard-Wiechert scalar and vector potentials for points on the z axis.

Recall: $V(\vec{r},t) = k_e \dfrac{qc}{(rc - \vec{r} \bullet \vec{v})}$ and $\vec{A}(\vec{r},t) = \dfrac{\vec{v}}{c^2} V(\vec{r},t)$

3. (25 pts) Determine an expression for the radiation resistance of an oscillating electric dipole. This is the resistance that would give the same average power loss – to heat – as the oscillating dipole in fact puts out in the form of radiation. Give your answer in terms of d/λ, where d is the distance between the plus and minus charges of the dipole and λ is the wavelength of the radiation.

4. (25 pts) An insulating circular ring (radius R) lies in the xy plane, centered at the origin. It carries a linear charge density $\lambda = \lambda_0 \cos\varphi$, where λ_0 is constant and φ is the usual azimuthal angle. The ring is now set spinning at constant angular velocity ω about the z axis.

a) Determine the initial dipole moment of the ring and write down the dipole moment as a function of time.

b) Determine the power radiated by the spinning ring.

(10 pts) Bonus part of the problem:

c) As the ring radiates it will lose energy and therefore will spin slower and slower. The kinetic energy of the ring as a function of ω is given by $\frac{1}{2}I\omega^2$, where I is the moment of inertia of the ring. Determine an expression for the angular velocity as a function of time; that is, $\omega(t)$ if at time $t = 0$, it is spinning with an angular velocity ω_0.

Larmor formula: $P = \dfrac{\mu_0(\ddot{p})^2}{6\pi c}$