Phys 321 – E & M II – Test 2 – Nov. 1, 1999

1. Consider a parallel plate capacitor constructed of circular plates of radius a and separated by a distance $w << a$. Thin wires connect to the centers of the plates. Assume the current I in the wires is constant. Also assume the surface charge density on the plates is uniform, at any given time, and is zero at $t = 0$.

   a) Find the electric field between the plates, as a function of s and t.

   b) Find the displacement current between the plates, as a function of s and t.

   c) Find the magnetic field between the plates, as a function of s and t.

2. A long coaxial cable carries current I (the current flows down the surface of the inner cylinder, radius a, and back along the outer cylinder, radius b). The two cylindrical conductors are held at a potential difference V. The electric and magnetic fields due to the two cylindrical conductors are given by:

$$\vec{E} = \frac{V}{\ell n\left(\frac{b}{a}\right)} \frac{\hat{s}}{s} \qquad \vec{B} = \frac{\mu_0 I}{2\pi\, s}\hat{\phi} \quad a \le s \le b; \quad \vec{E} = \vec{B} = 0 \quad s < a\,; \quad s > b$$

   a) Determine the inductance per unit length, $\mathfrak{L}$, by finding the magnetic energy stored in a section of length ℓ.

   b) Calculate the power (energy per unit time) transported down the coaxial cable.

3. Write down the (real) electric and magnetic fields for a monochromatic plane wave of amplitude E_0 , frequency ω, and phase angle zero that is traveling in the direction from the origin to the point (1,2,2), with polarization parallel to the xy plane.

4. Consider a long solenoid of radius R of n turns per unit length, carrying a current I. The magnetic field due to the solenoid is given by

$$\vec{B} = \mu_0\, n\, I\, \hat{z} \quad (s < R) \qquad \text{and} \qquad \vec{B} = 0 \quad (s > R)\,.$$

   a) Determine the 3 × 3 matrix that represents the stress tensor in the region inside the solenoid.

   b) Determine the force per unit length on the "right half" of the inside of the solenoid; that is, the region from $\phi = 0$ to $\phi = \pi$.

Recall: $T_{ij} = \varepsilon_0 (E_i E_j - \tfrac{1}{2}\delta_{ij}E^2) + \dfrac{1}{\mu_0}(B_i B_j - \tfrac{1}{2}\delta_{ij}B^2)$

$$\vec{F} = \oint_S \vec{T}\cdot d\vec{a} - \varepsilon_0\mu_0 \frac{d}{dt}\int_V \vec{S}\, d\tau$$