

1.(25 pts) Consider an infinite parallel-plate capacitor, with the lower plate (at  $z = 0$ ) and carrying the charge density  $+\sigma$ , and the upper plate (at  $z = d$ ) carrying the charge density  $-\sigma$ .

a) Determine the  $3 \times 3$  matrix that represents the stress tensor in the region between the plates.

b) Determine the force per unit area on the bottom plate.

Recall:  $T_{ij} = \epsilon_0 (E_i E_j - \frac{1}{2} \delta_{ij} E^2) + \frac{1}{\mu_0} (B_i B_j - \frac{1}{2} \delta_{ij} B^2)$

$$\vec{F} = \oint_S \vec{T} \cdot d\vec{a} - \epsilon_0 \mu_0 \frac{d}{dt} \int_V \vec{S} d\tau$$

2.(25 pts) A long coaxial cable carries current  $I$ . The current flows down the surface of the inner cylinder, radius  $a$ , and back along the outer cylinder, radius  $b$ . The inner and outer conductors are held at a potential difference  $V$ .

a) Use Ampere's Law to determine the magnetic field in the region between the conducting cylinders.

b) Use Gauss's Law to determine the electric field in the region between the conducting cylinders. Show that  $\vec{E} = \frac{V}{\ell \ln(\frac{b}{a})} \frac{\hat{s}}{s}$ .

c) Calculate the power (energy per unit time) transported down the coaxial cable.

3.(25 pts) NASA plans to build a spaceship and propel it to Mars by using a "sail" to reflect solar radiation. The sail is square with 1 km on a side and is totally reflecting. Assume an average intensity of  $900 \text{ W/m}^2$  for the sunlight between the orbits of earth and Mars.

a) Determine the average radiation force on the sail.

b) If the spaceship plus astronauts has a mass of 3 metric tons (3000 kg), estimate how long it will take the astronauts to get to Mars if the force is assumed to be constant. The distance from earth to Mars is about  $10^{11} \text{ m}$ . Neglect gravitational forces, of course.

4.(25 pts) Consider an electromagnetic wave at normal incidence on the interface between two non-magnetic media. The wave is traveling in medium #1 ( $n_1 = 1.5$ ) in the  $z$ -direction and polarized in the  $x$ -direction. The interface is in the  $xy$ -plane at  $z = 0$  and medium #2 has  $n_2 = 2.5$ .

a) Write down expressions for the electric and magnetic fields for the incident, reflected, and transmitted waves.

b) According to the boundary conditions what equations must be satisfied?

b) Determine the reflection coefficient,  $R$ . Is the reflected wave in-phase or out-of-phase with the incident wave?