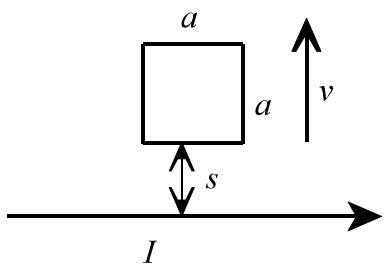
Phys 321–E&M II–Final Exam–Dec. 15, 1999

1. (25 pts) A square loop of wire (side a) lies on a table, a distance s from a very long straight wire, which carries a current I.



a) Find the magnetic flux through the loop.

b) If someone now pulls the loop directly away from the wire, at speed v, what *emf* is generated in the loop?

c) In what direction (clockwise or counterclockwise) does the current in the loop flow?

2. (25 pts) Consider a long solenoid of radius R of n turns per unit length, carrying a current I. The magnetic field due to the solenoid is given by

$$\vec{B} = \mu_0 n I \hat{z} \quad (s < R) \qquad \text{and} \qquad \vec{B} = 0 \quad (s > R) .$$

a) Determine the 3 × 3 matrix that represents the stress tensor in the region inside the solenoid.

b) Determine the force per unit length on the "right half" of the inside of the solenoid; that is, the region from $\phi = 0$ to $\phi = \pi$.

Recall: $T_{ij} = \varepsilon_0 (E_i E_j - \frac{1}{2}\delta_{ij} E^2) + \dfrac{1}{\mu_0}(B_i B_j - \frac{1}{2}\delta_{ij} B^2)$

$$\vec{F} = \oint_S \vec{\vec{T}} \cdot d\vec{a} - \varepsilon_0 \mu_0 \frac{d}{dt} \int_V \vec{S}\, d\tau$$

3. (25 pts) An insulating circular ring (radius R) lies in the xy plane, centered at the origin. It carries a linear charge density $\lambda = \lambda_0 (1 + \sin\phi)$, where λ_0 is constant and ϕ is the usual azimuthal angle. The ring is now set spinning at constant angular velocity ω about the z axis. Calculate the power radiated.

4. (25 pts) A rocket ship leaves earth at a speed of (4/5)c. When a clock on the rocket says one hour has elapsed, the rocket sends a light signal back to earth.

a) According to *earth* clocks, when was the signal sent?

b) According to *earth* clocks, how long after the rocket left did the signal arrive back on earth?

c) According to the *rocket* observer, how long after the rocket left did the signal arrive back on earth?

5. (25 pts) Event *A* happens at the point ($x = 6, y = 2, z = 1$) and at a time *t* given by $ct = 7$; event *B* occurs at (3,2,1) and $ct = 5$, both in system *S*.

a) Find the invariant interval between *A* and *B*.

b) Is the invariant interval timelike, spacelike, or lightlike?

c) Find the velocity of an inertial system relative to *S* so that the events occur simultaneously or the events are at the same spatial point.

d) Determine the coordinates of event A in the new inertial system, that is, $(\bar{x}, \bar{y}, \bar{z})$ and $c\bar{t}$.

6. (25 pts) If a particle's kinetic energy is twice its rest energy, what is its speed?

RECALL: Lorentz transformation:

$$\bar{x} = \gamma (x - vt) \quad \bar{y} = y \quad \bar{z} = z$$

$$\bar{t} = \gamma \left(t - \frac{v}{c^2}x\right) \quad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$\vec{p} = \gamma_u m\vec{u} \qquad E = \gamma_u mc^2 \qquad E^2 = p^2c^2 + m^2c^4 \qquad \frac{u}{c} = \frac{pc}{E}$$