Phys 321 - E & M II - Final Exam - Dec. 15, 2003

1. (17 pts) A long solenoid with radius *a* and n_1 turns per unit length is surrounded coaxially by a short solenoid with radius b > a of length l_2 with n_2 turns per unit length.

a) Find the self-inductance per unit length of the long solenoid.

b) Find the mutual inductance of the long and short solenoids.

c) If the current in the long solenoid varies as $I(t) = I_0 \sin(\omega t)$, determine the emf (or voltage) induced in the long solenoid and the emf (or voltage) induced in the short solenoid.

d) What is the ratio of the two emf's (or voltages).

2.(17 pts) A long coaxial cable caries current *I*. The current flows down the surface of the inner cylinder, radius a, and back along the outer cylinder, radius b. The inner and outer conductors are held at a potential difference *V*.

a) Use Ampere's Law to determine the magnetic field in the region between the conducting cylinders.

b) Use Gauss's Law to determine the electric field in the region between the conducting cylinders. Show that $\vec{E} = \frac{V}{\ell n(\frac{b}{a})} \frac{\hat{s}}{s}$.

c) Calculate the power (energy per unit time) transported down the coaxial cable.

3. (17 pts) A particle of charge q and mass m, moving with a speed v, enters an external magnetic field B such that the direction of v is perpendicular to the direction of B.

a) The particle will execute cyclotron motion. Determine the radius of the cyclotron orbit.

b) Assuming no loss in the particle's speed what is the total power radiated by the charge?

c) As the charge radiates due to its acceleration, its speed will decrease as will its radius. Assuming this is a slow process, determine the radius of the particle as a function of time, R(t).

4. (17 pts) A truck is three times longer than a VW bug when they are both at rest. As they both are traveling through an intersection, a stationary pedestrian observes that the truck is only twice as long as the VW bug. If the VW bug is going at one-fourth the speed of light, *i.e.*, $u_{VW} = c/4$, how fast is the truck going? Leave your answer as a multiple of c.

5. (17 pts) Event *A* happens at the point (x = 9, y = 2, z = 1) and at a time *t* given by ct = 4; event *B* occurs at (5,2,1) and ct = 2, both in system *S*.

a) Find the invariant interval between A and B.

b) Is the invariant interval timelike, spacelike, or lightlike?

c) Find the velocity of an inertial system relative to *S* so that the events occur simultaneously or the events are at the same spatial point.

d) Determine the coordinates of event A in the new inertial system, that is, $(\bar{x}, \bar{y}, \bar{z})$ and $c\bar{t}$.

6. (17 pts) A particle of mass m whose total energy is three times its rest energy collides with an particle of mass 2m at rest. Assume the particles stick together during the collision.

- a) What is the mass of the resulting composite particle?
- b) What is the velocity of the resulting composite particle?

RECALL: Lorentz transformation:

$$\overline{x} = \gamma (x - vt) \quad \overline{y} = y \quad \overline{z} = z$$

$$\overline{t} = \gamma (t - \frac{v}{c^2}x) \quad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$\vec{p} = \gamma_u m \vec{u}$$
 $E = \gamma_u m c^2$ $E^2 = p^2 c^2 + m^2 c^4$ $\frac{u}{c} = \frac{pc}{E}$