Phys 321–E&M II–Final Exam–Dec. 14, 2001

1. (25 pts) A long solenoid with radius a and n turns per unit length carries a time-dependent current $I(t)$ in the $\hat{\phi}$ direction. Find the electric field (magnitude and direction) at a distance s from the axis (both inside and outside the solenoid), in the quasistatic approximation.

2. (25 pts) Write down the (real) electric and magnetic fields for a monochromatic plane wave of amplitude E_0 , frequency ω, and phase angle zero that is traveling in the direction from the origin to the point (2, 1, 1), with polarization parallel to the xz plane.

3. (25 pts) An insulating circular ring (radius R) lies in the xy plane, centered at the origin. It carries a linear charge density $\lambda = \lambda_0 \cos\varphi$, where λ_0 is constant and φ is the usual azimuthal angle. The ring is now set spinning at constant angular velocity ω about the z axis.

a) Determine the initial dipole moment of the ring and write down the dipole moment as a function of time.
b) Determine the power radiated by the spinning ring.

4. (25 pts) A Lincoln Continental is twice as long as a VW Beetle, when they are at rest. As the Continental overtakes the VW, going through a speed trap, a (stationary) policeman observes that they both have the same length. The VW is going at one-fourth the speed of light; that is, $u_{VW} = c/4$. How fast is the Lincoln going? Leave your answer as a multiple of c.

5. (25 pts) Event A happens at the point ($x = 6$, $y = 2$, $z = 1$) and at a time t given by $ct = 7$; event B occurs at (3,2,1) and $ct = 5$, both in system S.

a) Find the invariant interval between A and B.
b) Is the invariant interval timelike, spacelike, or lightlike?
c) Find the velocity of an inertial system relative to S so that the events occur simultaneously or the events are at the same spatial point.

6. (25 pts) A particle of mass m whose total energy is three times its rest energy collides with an identical particle at rest. Assume the particles stick together during the collision.

a) What is the mass of the resulting composite particle?
b) What is the velocity of the resulting composite particle?

RECALL: Lorentz transformation:

$$\bar{x} = \gamma\,(x - vt) \quad \bar{y} = y \quad \bar{z} = z$$

$$\bar{t} = \gamma\,(t - \frac{v}{c^2}x) \quad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$\vec{p} = \gamma_u m\vec{u} \qquad E = \gamma_u mc^2 \qquad E^2 = p^2c^2 + m^2c^4 \qquad \frac{u}{c} = \frac{pc}{E}$$