We want to find the quasi-static work done by a gas whose pressure $p$ and volume $V$ vary along some adiabatic path according to

$$p V^\gamma = K,$$

where $\gamma$ and $K$ are constants. The $p - V$ work is defined as

$$W = \int_{V_i}^{V_f} p_{\text{op}} dV.$$

Because the work is to be carried out quasi-statically, we have $p_{\text{op}} = \bar{p}$, where $\bar{p}$ is given by Eq.(1). Then Eq.(2) becomes

$$W = \int_{V_i}^{V_f} \bar{p} dV = K \int_{V_i}^{V_f} V^{-\gamma} dV = \frac{K}{1 - \gamma} V^{1-\gamma}|_{V_i}^{V_f}$$

$$W = \frac{1}{1 - \gamma} [KV_f^{1-\gamma} - KV_i^{1-\gamma}].$$

We can evaluate $K$ using any point that lies on the expansion path. To get Reif's result we use the end points. This gives

$$K = \bar{p}_i V_i^\gamma = \bar{p}_f V_f^\gamma,$$

which may be substituted into Eq.(4) to yield

$$W = \frac{1}{1 - \gamma} [\bar{p}_f V_f - \bar{p}_i V_i].$$